Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/195-7.3.5-u-a+b-arctanh-c+d-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [62]. This is test number [195].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (62)	0.00 (0)
Maple	98.39 (61)	1.61 (1)
Mathematica	96.77 (60)	3.23 (2)
Maxima	54.84 (34)	45.16 (28)
Fricas	27.42 (17)	72.58 (45)
Mupad	27.42 (17)	72.58 (45)
Giac	27.42 (17)	72.58 (45)
Sympy	24.19 (15)	75.81 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

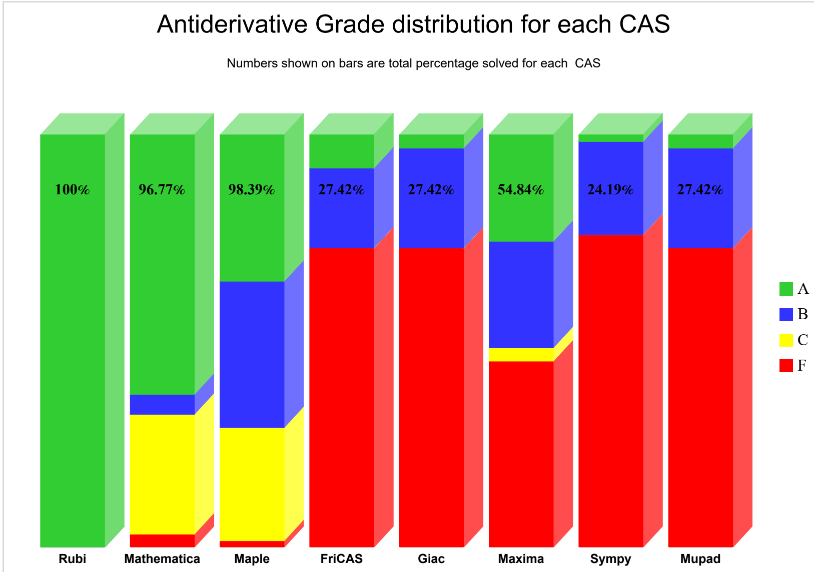
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

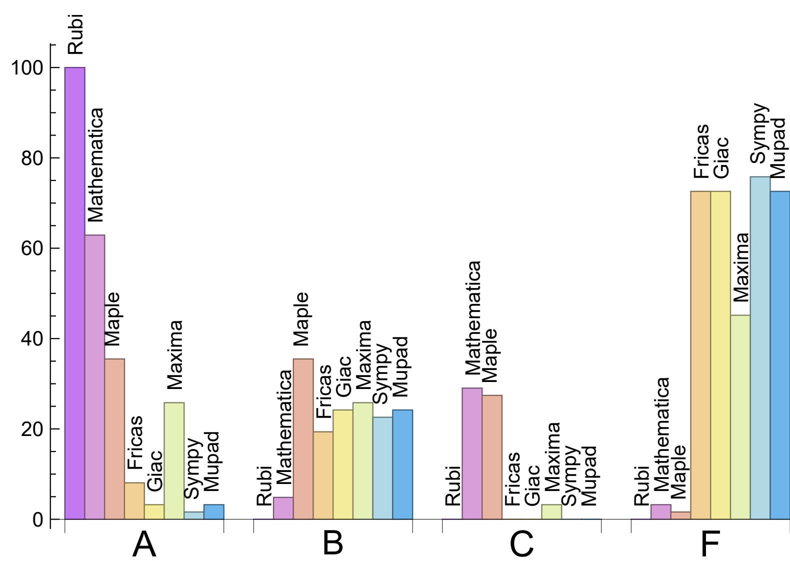
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	62.90	4.84	29.03	3.23
Maple	35.48	35.48	27.42	1.61
Maxima	25.81	25.81	3.23	45.16
Fricas	8.06	19.35	0.00	72.58
Mupad	N/A	24.19	0.00	72.58
Giac	3.23	24.19	0.00	72.58
Sympy	1.61	22.58	0.00	75.81

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	45	100.00 %	0.00 %	0.00 %
Giac	45	100.00 %	0.00 %	0.00 %
Maxima	28	96.43 %	0.00 %	3.57 %
Sympy	47	80.85 %	19.15 %	0.00 %
Mupad	45	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

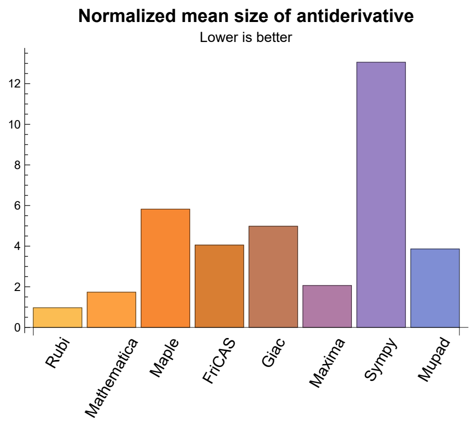
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

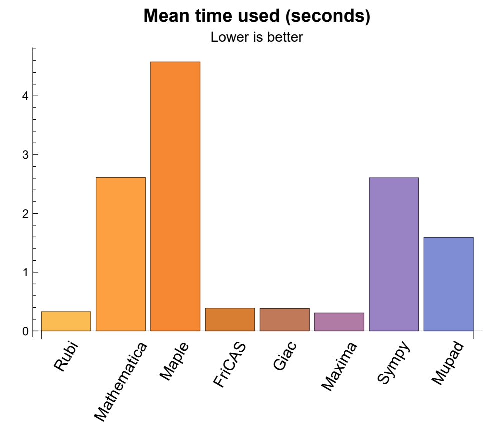
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.33	243.21	0.97	166.50	1.00
Mathematica	2.61	442.33	1.73	218.00	1.30
Maple	4.58	1663.69	5.82	427.00	2.18
Maxima	0.31	331.15	2.06	251.50	1.81
Fricas	0.39	496.71	4.05	348.00	3.51
Sympy	2.61	1952.87	13.06	313.00	3.65
Giac	0.38	605.53	4.98	351.00	4.12
Mupad	1.59	499.06	3.86	237.00	2.50

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{50, 51\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{42, 44, 45, 49, 57, 61\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

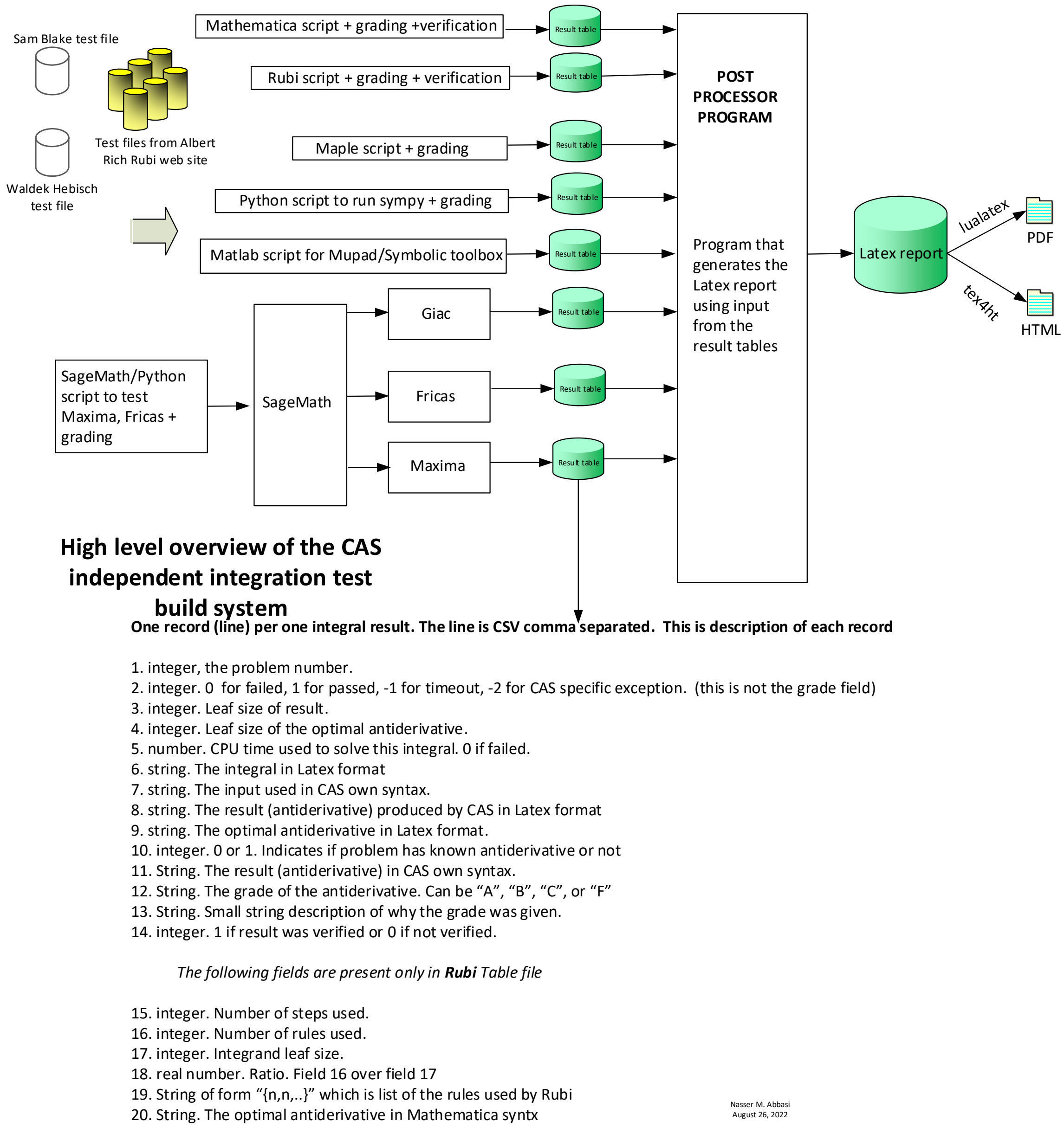
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 3, 4, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 29, 30, 31, 32, 33, 34, 36, 37, 38, 40, 41, 46, 47, 50, 51, 53, 54, 55, 59, 60, 62 }

B grade: { 2, 39, 45 }

C grade: { 5, 6, 7, 12, 18, 25, 26, 27, 28, 35, 42, 43, 44, 49, 56, 57, 58, 61 }

F grade: { 48, 52 }

2.1.3 Maple

A grade: { 4, 6, 7, 9, 10, 11, 12, 13, 14, 29, 34, 35, 36, 37, 41, 43, 50, 51, 55, 56, 59, 60 }

B grade: { 1, 2, 3, 15, 16, 17, 19, 20, 21, 22, 30, 31, 32, 33, 38, 39, 40, 44, 47, 54, 61, 62 }

C grade: { 5, 8, 18, 23, 24, 25, 26, 27, 28, 42, 45, 46, 48, 49, 53, 57, 58 }

F grade: { 52 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 7, 13, 32, 33, 34, 36, 37, 50, 51, 55, 56 }

B grade: { 9, 10, 11, 14, 15, 16, 17, 20, 22, 24, 29, 30, 31, 38, 39, 40 }

C grade: { 54, 57 }

F grade: { 5, 8, 12, 18, 19, 21, 23, 25, 26, 27, 28, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 58, 59, 60, 61, 62 }

2.1.5 FriCAS

A grade: { 13, 33, 34, 50, 51 }

B grade: { 9, 10, 11, 14, 15, 17, 20, 22, 31, 32, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.6 Sympy

A grade: { 34 }

B grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.7 Giac

A grade: { 50, 51 }

B grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.1.8 Mupad

A grade: { 50, 51 }

B grade: { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	187	887	320	0	0	0	-1
N.S.	1	1.00	0.71	3.37	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	1.285	2.180	0.265	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	463	666	259	0	0	0	-1
N.S.	1	1.00	2.27	3.26	1.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	1.595	0.082	0.260	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	312	202	0	0	0	-1
N.S.	1	1.00	0.72	2.29	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.204	0.070	0.262	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	91	139	0	0	0	-1
N.S.	1	1.00	0.68	1.12	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.050	1.846	0.262	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	634	931	0	0	0	0	-1
N.S.	1	1.00	4.28	6.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.169	20.375	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	208	356	244	0	0	0	-1
N.S.	1	1.00	0.83	1.42	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.020	2.449	0.260	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	271	449	360	0	0	0	-1
N.S.	1	1.00	0.73	1.21	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	2.091	0.213	0.270	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	160	0	0	0	0	-1
N.S.	1	1.00	1.34	2.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.071	10.561	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	78	88	349	500	231	363	414
N.S.	1	1.00	1.08	1.22	4.85	6.94	3.21	5.04	5.75
time (sec)	N/A	0.045	0.039	1.542	0.260	0.383	1.422	0.450	1.421

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	77	219	348	180	322	237
N.S.	1	1.00	0.86	1.12	3.17	5.04	2.61	4.67	3.43
time (sec)	N/A	0.044	0.032	0.562	0.257	0.385	1.074	0.436	0.618

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	77	65	117	121	95	180	73
N.S.	1	1.00	1.60	1.35	2.44	2.52	1.98	3.75	1.52
time (sec)	N/A	0.023	0.022	0.040	0.266	0.344	0.681	0.422	1.842

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	288	78	0	0	0	0	-1
N.S.	1	1.00	5.33	1.44	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.089	0.793	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	69	75	88	114	235	152	122
N.S.	1	1.00	1.10	1.19	1.40	1.81	3.73	2.41	1.94
time (sec)	N/A	0.039	0.078	0.591	0.262	0.432	1.204	0.412	1.369

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	100	77	121	166	313	194	67
N.S.	1	1.00	1.59	1.22	1.92	2.63	4.97	3.08	1.06
time (sec)	N/A	0.034	0.034	0.577	0.259	0.368	1.464	0.408	1.732

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	148	357	755	1288	581	733	1730
N.S.	1	1.00	0.93	2.25	4.75	8.10	3.65	4.61	10.88
time (sec)	N/A	0.177	0.058	2.698	0.459	0.379	2.384	0.462	2.124

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	150	342	572	0	0	0	-1
N.S.	1	1.00	0.84	1.91	3.20	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.290	4.122	0.452	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	134	272	320	333	238	351	432
N.S.	1	1.00	1.41	2.86	3.37	3.51	2.51	3.69	4.55
time (sec)	N/A	0.091	0.041	0.078	0.443	0.403	1.100	0.423	1.563

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	424	840	0	0	0	0	-1
N.S.	1	1.00	2.52	5.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.262	11.776	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	126	346	0	0	0	0	-1
N.S.	1	1.00	1.21	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.170	1.792	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	324	309	348	1102	375	776
N.S.	1	1.00	1.14	2.72	2.60	2.92	9.26	3.15	6.52
time (sec)	N/A	0.128	0.116	0.769	0.279	0.385	1.587	0.419	2.418

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	218	427	0	0	0	0	-1
N.S.	1	1.00	1.21	2.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.359	1.750	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	218	375	582	789	3516	730	2746
N.S.	1	1.00	1.27	2.18	3.38	4.59	20.44	4.24	15.97
time (sec)	N/A	0.189	0.158	0.790	0.307	0.388	5.387	0.428	3.424

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	336	1345	0	0	0	0	-1
N.S.	1	1.00	1.28	5.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.445	15.654	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	213	6414	628	0	0	0	-1
N.S.	1	1.00	1.33	40.09	3.92	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.760	2.888	0.457	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	599	1738	0	0	0	0	-1
N.S.	1	1.00	2.33	6.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.561	3.531	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	248	1867	0	0	0	0	-1
N.S.	1	1.00	1.73	13.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.626	2.885	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	335	5530	0	0	0	0	-1
N.S.	1	1.00	2.02	33.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.829	3.134	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	393	2080	0	0	0	0	-1
N.S.	1	1.00	1.46	7.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.914	6.384	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	34	58	0	0	0	-1
N.S.	1	1.00	1.48	1.62	2.76	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.005	0.934	0.251	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	62	132	0	0	0	-1
N.S.	1	1.00	1.62	1.94	4.12	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.007	0.874	0.260	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	270	982	331	818	644	2336	737
N.S.	1	1.00	1.61	5.85	1.97	4.87	3.83	13.90	4.39
time (sec)	N/A	0.238	0.160	1.395	0.256	0.408	2.073	0.460	1.724

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	174	590	207	421	369	976	381
N.S.	1	1.00	1.45	4.92	1.72	3.51	3.08	8.13	3.18
time (sec)	N/A	0.148	0.119	0.699	0.261	0.405	2.258	0.430	1.387

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	138	185	111	177	173	341	136
N.S.	1	1.00	1.42	1.91	1.14	1.82	1.78	3.52	1.40
time (sec)	N/A	0.122	0.074	0.194	0.255	0.368	1.037	0.429	1.337

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	44	36	61	46	200	48
N.S.	1	1.00	1.20	1.10	0.90	1.52	1.15	5.00	1.20
time (sec)	N/A	0.019	0.021	0.584	0.264	0.383	0.235	0.398	1.440

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	329	220	0	0	0	0	-1
N.S.	1	1.00	2.53	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.365	13.642	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	170	128	517	1658	474	170
N.S.	1	1.00	1.09	1.48	1.11	4.50	14.42	4.12	1.48
time (sec)	N/A	0.118	0.132	0.760	0.259	0.497	4.616	0.428	1.638

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	174	266	316	2443	19912	2567	417
N.S.	1	1.00	1.04	1.59	1.89	14.63	119.23	15.37	2.50
time (sec)	N/A	0.178	0.261	0.844	0.283	1.086	12.566	0.499	3.034

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	1082	4769	1438	0	0	0	-1
N.S.	1	1.00	1.93	8.49	2.56	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	5.988	1.096	0.464	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	795	2890	857	0	0	0	-1
N.S.	1	1.00	2.13	7.73	2.29	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	2.900	0.722	0.452	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	271	812	435	0	0	0	-1
N.S.	1	1.00	1.23	3.67	1.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.322	0.317	0.448	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	107	140	0	0	0	0	-1
N.S.	1	1.00	1.10	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.154	1.751	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	2404	1872	0	0	0	0	-1
N.S.	1	1.00	11.23	8.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	20.621	30.398	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	485	425	857	0	0	0	0	-1
N.S.	1	1.01	0.89	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.241	5.923	4.883	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	1968	1491	0	0	0	0	-1
N.S.	1	1.00	2.62	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.559	14.704	2.155	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	1646	12235	0	0	0	0	-1
N.S.	1	1.00	3.01	22.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	8.067	50.183	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	325	566	19935	0	0	0	0	-1
N.S.	1	1.00	1.74	61.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	0.718	3.832	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	205	284	0	0	0	0	-1
N.S.	1	1.00	1.55	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.178	0.496	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	3825	0	0	0	0	-1
N.S.	1	1.00	0.00	12.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	46.229	7.704	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1094	2701	5732	0	0	0	0	-1
N.S.	1	1.00	2.48	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.066	20.885	4.011	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	3.608	2.539	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.235	2.371	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.048	1.927	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	623	743	0	0	0	0	-1
N.S.	1	1.00	0.80	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.978	0.532	5.955	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	365	1289	591	0	0	0	-1
N.S.	1	1.00	0.76	2.68	1.23	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.222	4.041	0.591	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	138	184	192	0	0	0	-1
N.S.	1	1.00	1.15	1.53	1.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.011	12.585	0.259	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	759	305	192	0	0	0	-1
N.S.	1	1.00	4.08	1.64	1.03	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	3.163	8.188	0.265	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	1456	10288	651	0	0	0	-1
N.S.	1	1.00	2.67	18.88	1.19	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.649	20.077	2.897	0.555	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	832	832	917	638	0	0	0	0	-1
N.S.	1	1.00	1.10	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.103	4.662	1.975	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	549	773	0	0	0	0	-1
N.S.	1	1.00	0.94	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	0.380	2.066	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	598	1001	0	0	0	0	-1
N.S.	1	1.00	0.90	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.790	0.445	1.604	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	1208	2132	0	0	0	0	-1
N.S.	1	1.00	3.61	6.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	27.371	5.385	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	168	0	0	0	0	-1
N.S.	1	1.00	0.92	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.059	2.027	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [62] had the largest ratio of [32]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	19	15	1.00	12	1.250
2	A	15	13	1.00	12	1.083
3	A	12	10	1.00	10	1.000
4	A	6	6	1.00	8	0.750
5	A	2	2	1.00	12	0.167
6	A	17	15	1.00	12	1.250
7	A	21	16	1.00	12	1.333
8	A	4	5	1.00	12	0.417
9	A	6	5	1.00	21	0.238
10	A	6	5	1.00	21	0.238
11	A	5	5	1.00	19	0.263
12	A	3	3	1.00	21	0.143
13	A	7	7	1.00	21	0.333
14	A	5	5	1.00	21	0.238
15	A	13	9	1.00	23	0.391
16	A	11	10	1.00	23	0.435
17	A	8	7	1.00	21	0.333
18	A	8	7	1.00	23	0.304
19	A	6	6	1.00	23	0.261
20	A	10	9	1.00	23	0.391
21	A	10	9	1.00	23	0.391
22	A	15	10	1.00	23	0.435
23	A	14	11	1.00	23	0.478
24	A	10	10	1.00	21	0.476
25	A	10	8	1.00	23	0.348

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	8	1.00	23	0.348
27	A	9	8	1.00	23	0.348
28	A	16	13	1.00	23	0.565
29	A	3	3	1.00	12	0.250
30	A	3	3	1.00	19	0.158
31	A	7	5	1.00	18	0.278
32	A	7	5	1.00	18	0.278
33	A	7	5	1.00	16	0.312
34	A	4	3	1.00	10	0.300
35	A	5	5	1.00	18	0.278
36	A	7	5	1.00	18	0.278
37	A	5	4	1.00	18	0.222
38	A	20	15	1.00	20	0.750
39	A	16	13	1.00	20	0.650
40	A	13	10	1.00	18	0.556
41	A	6	6	1.00	12	0.500
42	A	2	2	1.00	20	0.100
43	A	21	19	1.01	20	0.950
44	A	26	18	1.00	20	0.900
45	A	21	14	1.00	20	0.700
46	A	15	11	1.00	18	0.611
47	A	6	7	1.00	12	0.583
48	A	2	2	1.00	20	0.100
49	A	30	18	1.00	20	0.900
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	6	4	1.00	18	0.222
53	A	23	5	1.00	16	0.312
54	A	17	5	1.00	16	0.312
55	A	5	5	1.00	14	0.357
56	A	15	7	1.00	16	0.438
57	A	25	7	1.00	16	0.438
58	A	31	7	1.00	16	0.438
59	A	31	13	1.00	18	0.722
60	A	37	16	1.00	18	0.889

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	12	8	1.00	19	0.421
62	A	6	5	1.00	32	0.156

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \tanh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=263

$$-\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}+\frac{a\tanh^{-1}(a+bx)}{b^4}+\frac{(1+6a^2)\,(a+bx)\tanh^{-1}(a+bx)}{2b^4}-\frac{a(a+bx)^2\tanh^{-1}(a+bx)}{b^4}+\frac{(a+b}{}$$

```
[Out] -a*x/b^3+1/12*(b*x+a)^2/b^4+a*arctanh(b*x+a)/b^4+1/2*(6*a^2+1)*(b*x+a)*arct
anh(b*x+a)/b^4-a*(b*x+a)^2*arctanh(b*x+a)/b^4+1/6*(b*x+a)^3*arctanh(b*x+a)/
b^4-a*(a^2+1)*arctanh(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*arctanh(b*x+a)^2/b^4+1
/4*x^4*arctanh(b*x+a)^2+2*a*(a^2+1)*arctanh(b*x+a)*ln(2/(-b*x-a+1))/b^4+1/1
2*ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*polylog(2
,(-b*x-a-1)/(-b*x-a+1))/b^4
```

Rubi [A]
time = 0.24, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6246, 6065, 6021, 266, 6037, 327, 212, 272, 45, 6195, 6095, 6131, 6055, 2449, 2352}

$$\frac{a(a^2+1)\,\mathrm{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^4}+\frac{(6a^2+1)\log\left(1-(a+bx)^2\right)}{4b^4}-\frac{a(a^2+1)\tanh^{-1}(a+bx)^2}{b^4}+\frac{(6a^2+1)\,(a+bx)\tanh^{-1}(a+bx)}{2b^4}+\frac{2a(a^2+1)\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{b^4}-\frac{(a^4+6a^2+1)\tanh^{-1}(a+bx)^2}{4b^4}+\frac{(a+bx)^2}{12b^4}+\frac{\log\left(1-(a+bx)^2\right)}{12b^4}+\frac{(a+bx)^3\tanh^{-1}(a+bx)}{6b^4}-\frac{a(a+bx)^2\tanh^{-1}(a+bx)}{b^4}+\frac{a\tanh^{-1}(a+bx)}{b^4}-\frac{ax}{b^3}+\frac{1}{4}x^4\tanh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + (a*ArcTanh[a + b*x])/b^4 + ((1 + 6*a^
2)*(a + b*x)*ArcTanh[a + b*x])/(2*b^4) - (a*(a + b*x)^2*ArcTanh[a + b*x])/b
^4 + ((a + b*x)^3*ArcTanh[a + b*x])/(6*b^4) - (a*(1 + a^2)*ArcTanh[a + b*x]
^2)/b^4 - ((1 + 6*a^2 + a^4)*ArcTanh[a + b*x]^2)/(4*b^4) + (x^4*ArcTanh[a +
 b*x]^2)/4 + (2*a*(1 + a^2)*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Lo
g[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) +
(a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \tanh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \tanh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{(1+6a^2)\tanh^{-1}(x)}{b^4} + \frac{4ax\tanh^{-1}(x)}{b^4} - \right.\right.$$

$$= \frac{1}{4}x^4 \tanh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int x^2 \tanh^{-1}(x)\,dx, x, a+bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{(1+6a^2+}{}\right.}{}$$

$$= \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\tanh^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\tan}{6}$$

$$= -\frac{ax}{b^3} + \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\tanh^{-1}(a+bx)}{b^4} + \frac{(a+}{}$$

$$= -\frac{ax}{b^3} + \frac{a\tanh^{-1}(a+bx)}{b^4} + \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\tan}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{a\tanh^{-1}(a+bx)}{b^4} + \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a}{}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{a\tanh^{-1}(a+bx)}{b^4} + \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a}{}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{a\tanh^{-1}(a+bx)}{b^4} + \frac{(1+6a^2)(a+bx)\tanh^{-1}(a+bx)}{2b^4} - \frac{a}{}$$

<u>**Mathematica**</u> [**A**]
time = 1.28, size = 187, normalized size = 0.71

$$-\frac{1+11a^2+10abx-b^2x^2+3(1-4a+6a^2-4a^3+a^4-b^4x^4)\tanh^{-1}(a+bx)^2-2\tanh^{-1}(a+bx)\left(9a+13a^3+3bx+9a^2bx-3ab^2x^2+b^3x^3+12(a+a^3)\log\left(1+e^{-2\tanh^{-1}(a+bx)}\right)\right)+8\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)+36a^2\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)+12(a+a^3)\,\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(a+bx)}\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[a + b*x]^2,x]
```

```
[Out] -1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 -
 b^4*x^4)*ArcTanh[a + b*x]^2 - 2*ArcTanh[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9
*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 + E^(-2*ArcTanh[a + b
*x])]) + 8*Log[1/Sqrt[1 - (a + b*x)^2]] + 36*a^2*Log[1/Sqrt[1 - (a + b*x)^2
]] + 12*(a + a^3)*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/b^4
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(251) = 502.
time = 2.18, size = 887, normalized size = 3.37

method	result
derivativedivides	Expression too large to display
default	Expression too large to display
risch	$-\frac{1}{12b^4}-\frac{5ax}{6b^3}-\frac{\ln(-bx-a+1)(-bx-a+1)^3}{12b^4}+\frac{\ln(-bx-a+1)(-bx-a+1)^2}{8b^4}+\frac{13\ln(-bx-a-1)a^3}{12b^4}-\frac{(-b^4x^4+a^4+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/3*ln(b*x+a+1)+1/12*(b*x+a)^2-arctanh(b*x+a)*ln(b*x+a-1)*a+3/2*arct
anh(b*x+a)*ln(b*x+a-1)*a^2+1/4*arctanh(b*x+a)*ln(b*x+a-1)*a^4-arctanh(b*x+a
)*ln(b*x+a-1)*a^3+3*arctanh(b*x+a)*a^2*(b*x+a)-arctanh(b*x+a)*(b*x+a)^2*a+1
/3*ln(b*x+a-1)+1/16*ln(b*x+a+1)^2+1/16*ln(b*x+a-1)^2-(b*x+a)*a+3/2*ln(b*x+a
-1)*a^2+3/2*ln(b*x+a+1)*a^2-1/2*ln(b*x+a-1)*a+1/2*ln(b*x+a+1)*a+1/6*arctanh
(b*x+a)*(b*x+a)^3+1/2*arctanh(b*x+a)*(b*x+a)+1/4*arctanh(b*x+a)*ln(b*x+a-1)
-1/4*arctanh(b*x+a)*ln(b*x+a+1)+1/4*arctanh(b*x+a)^2*a^4+1/4*arctanh(b*x+a)
^2*(b*x+a)^4-1/8*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/8*ln(-1/2*b*x-1/2*a+1
/2)*ln(1/2*b*x+1/2*a+1/2)+1/16*ln(b*x+a+1)^2*a^4+1/4*ln(b*x+a+1)^2*a^3+3/8*
ln(b*x+a+1)^2*a^2+1/4*ln(b*x+a+1)^2*a-1/8*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)
+1/16*ln(b*x+a-1)^2*a^4-1/4*ln(b*x+a-1)^2*a^3+3/8*ln(b*x+a-1)^2*a^2-1/4*ln(
b*x+a-1)^2*a+dilog(1/2*b*x+1/2*a+1/2)*a^3+dilog(1/2*b*x+1/2*a+1/2)*a-1/4*ar
ctanh(b*x+a)*ln(b*x+a+1)*a^4-arctanh(b*x+a)*ln(b*x+a+1)*a^3-3/2*arctanh(b*x
+a)*ln(b*x+a+1)*a^2-arctanh(b*x+a)*ln(b*x+a+1)*a-arctanh(b*x+a)^2*a^3*(b*x+
a)+3/2*arctanh(b*x+a)^2*a^2*(b*x+a)^2-arctanh(b*x+a)^2*a*(b*x+a)^3-1/8*ln(b
*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^4-1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a
^3-3/4*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^2-1/2*ln(b*x+a+1)*ln(-1/2*b*x-1
/2*a+1/2)*a+1/8*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)*a^4+1/2*ln(-1/
2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)*a^3+3/4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/
2*b*x+1/2*a+1/2)*a^2+1/2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)*a-1/8
*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^4+1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2
)*a^3-3/4*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^2+1/2*ln(b*x+a-1)*ln(1/2*b*x+
1/2*a+1/2)*a)
```

Maxima [A]
time = 0.27, size = 320, normalized size = 1.22

$$\frac{1}{4}x^4\operatorname{artanh}(bx+a)^2+\frac{1}{48}b^2\left(\frac{48(a^3+a)\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^6}+\frac{4(13a^3+18a^2+9a+4)\log(bx+a+1)}{b^6}+\frac{4b^2x^2-40abx+3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)^2-6(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)\log(bx+a-1)+3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)^2-4(13a^3-18a^2+9a-4)\log(bx+a-1)}{b^6}\right)+\frac{1}{12}b\left(\frac{2(b^2x^3-3abx^2+3(3a^2+1)x)}{b^4}-\frac{3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)}{b^5}+\frac{3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)}{b^5}\right)\operatorname{artanh}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(1
/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 + 18
```

```
*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 + 4*a
^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1
)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log
(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6) + 1/
12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^
2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b
*x + a - 1)/b^5)*arctanh(b*x + a)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^3*arctanh(b*x + a)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{atanh}^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(b*x+a)**2,x)
[Out] Integral(x**3*atanh(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*arctanh(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{atanh}(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a + b*x)^2,x)
[Out] int(x^3*atanh(a + b*x)^2, x)
```

3.2 $\int x^2 \tanh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=204

$$\frac{x}{3b^2}-\frac{\tanh^{-1}(a+bx)}{3b^3}-\frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3}+\frac{(a+bx)^2\tanh^{-1}(a+bx)}{3b^3}+\frac{a(3+a^2)\tanh^{-1}(a+bx)^2}{3b^3}+\cdots$$

```
[Out] 1/3*x/b^2-1/3*arctanh(b*x+a)/b^3-2*a*(b*x+a)*arctanh(b*x+a)/b^3+1/3*(b*x+a)
^2*arctanh(b*x+a)/b^3+1/3*a*(a^2+3)*arctanh(b*x+a)^2/b^3+1/3*(3*a^2+1)*arct
anh(b*x+a)^2/b^3+1/3*x^3*arctanh(b*x+a)^2-2/3*(3*a^2+1)*arctanh(b*x+a)*ln(2
/(-b*x-a+1))/b^3-a*ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*polylog(2,(-b*x-a-1)/(
-b*x-a+1))/b^3
```

Rubi [A]

time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6246, 6065, 6021, 266, 6037, 327, 212, 6195, 6095, 6131, 6055, 2449, 2352}

$$-\frac{(3a^2+1)\,\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3}+\frac{a(a^2+3)\tanh^{-1}(a+bx)^2}{3b^3}+\frac{(3a^2+1)\tanh^{-1}(a+bx)^2}{3b^3}-\frac{2(3a^2+1)\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{3b^3}-\frac{a\log\left(1-(a+bx)^2\right)}{b^3}+\frac{(a+bx)^2\tanh^{-1}(a+bx)}{3b^3}-\frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3}-\frac{\tanh^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\tanh^{-1}(a+bx)^2+\frac{x}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[a + b*x]^2,x]
```

```
[Out] x/(3*b^2) - ArcTanh[a + b*x]/(3*b^3) - (2*a*(a + b*x)*ArcTanh[a + b*x])/b^3
 + ((a + b*x)^2*ArcTanh[a + b*x])/(3*b^3) + (a*(3 + a^2)*ArcTanh[a + b*x]^2
)/(3*b^3) + ((1 + 3*a^2)*ArcTanh[a + b*x]^2)/(3*b^3) + (x^3*ArcTanh[a + b*x
]^2)/3 - (2*(1 + 3*a^2)*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
 - b*x))])/(3*b^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \tanh^{-1}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{3}x^3\tanh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \left(\frac{3a\tanh^{-1}(x)}{b^3} - \frac{x\tanh^{-1}(x)}{b^3} - \frac{(a(3+a^2)\right.\right. \\
&= \frac{1}{3}x^3\tanh^{-1}(a+bx)^2 + \frac{2\text{Subst}\left(\int x\tanh^{-1}(x)\,dx, x, a+bx\right)}{3b^3} + \frac{2\text{Subst}\left(\int \frac{(a(3+a}{}\right.}{} \\
&= -\frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\tanh^{-1}(a+bx) \\
&= \frac{x}{3b^2} - \frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\tanh^{-1}(a \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a+bx)}{3b^3} - \frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a+bx)}{3b^3} - \frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a+bx)}{3b^3} - \frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx}{3b^3} \\
&= \frac{x}{3b^2} - \frac{\tanh^{-1}(a+bx)}{3b^3} - \frac{2a(a+bx)\tanh^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\tanh^{-1}(a+bx}{3b^3}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 463 vs. $2(204) = 408$.
time = 1.60, size = 463, normalized size = 2.27

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[a + b*x]^2,x]
```

```
[Out] -1/12*((1 - (a + b*x)^2)^(3/2)*(-((a + b*x)/Sqrt[1 - (a + b*x)^2]) + (6*a*(
a + b*x)*ArcTanh[a + b*x])/Sqrt[1 - (a + b*x)^2] + (3*(a + b*x)*ArcTanh[a +
 b*x]^2)/Sqrt[1 - (a + b*x)^2] - (3*a^2*(a + b*x)*ArcTanh[a + b*x]^2)/Sqrt[
1 - (a + b*x)^2] + ArcTanh[a + b*x]^2*Cosh[3*ArcTanh[a + b*x]] + 3*a^2*ArcT
anh[a + b*x]^2*Cosh[3*ArcTanh[a + b*x]] + 2*ArcTanh[a + b*x]*Cosh[3*ArcTanh
[a + b*x]]*Log[1 + E^(-2*ArcTanh[a + b*x])] + 6*a^2*ArcTanh[a + b*x]*Cosh[3
*ArcTanh[a + b*x]]*Log[1 + E^(-2*ArcTanh[a + b*x])] - 6*a*Cosh[3*ArcTanh[a
+ b*x]]*Log[1/Sqrt[1 - (a + b*x)^2]] + (3*(1 - 4*a + 3*a^2)*ArcTanh[a + b*x
```

```
]^2 + 2*ArcTanh[a + b*x]*(2 + (3 + 9*a^2)*Log[1 + E^(-2*ArcTanh[a + b*x])])
 - 18*a*Log[1/Sqrt[1 - (a + b*x)^2]])/Sqrt[1 - (a + b*x)^2] - (4*(1 + 3*a^2
)*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/(1 - (a + b*x)^2)^(3/2) - Sinh[3*Ar
cTanh[a + b*x]] + 6*a*ArcTanh[a + b*x]*Sinh[3*ArcTanh[a + b*x]] - ArcTanh[a
 + b*x]^2*Sinh[3*ArcTanh[a + b*x]] - 3*a^2*ArcTanh[a + b*x]^2*Sinh[3*ArcTan
h[a + b*x]]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(190) = 380$.
time = 0.08, size = 666, normalized size = 3.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*a-1/6*ln(b*x+a+1)+1/3*b*x-arctanh(b*x+a)*ln(b*x+a-1)*a+arctanh(b
*x+a)*ln(b*x+a-1)*a^2-1/3*arctanh(b*x+a)*ln(b*x+a-1)*a^3+1/6*ln(b*x+a-1)-1/
12*ln(b*x+a+1)^2+1/12*ln(b*x+a-1)^2-1/3*dilog(1/2*b*x+1/2*a+1/2)-ln(b*x+a-1
)*a-ln(b*x+a+1)*a+1/3*arctanh(b*x+a)*ln(b*x+a-1)+1/3*arctanh(b*x+a)*ln(b*x+
a+1)+1/6*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/6*ln(-1/2*b*x-1/2*a+1/2)*ln(1
/2*b*x+1/2*a+1/2)-1/12*ln(b*x+a+1)^2*a^3-1/4*ln(b*x+a+1)^2*a^2-1/4*ln(b*x+a
+1)^2*a-1/6*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)-1/12*ln(b*x+a-1)^2*a^3+1/4*ln
(b*x+a-1)^2*a^2-1/4*ln(b*x+a-1)^2*a+1/3*arctanh(b*x+a)*ln(b*x+a+1)*a^3+arct
anh(b*x+a)*ln(b*x+a+1)*a^2+arctanh(b*x+a)*ln(b*x+a+1)*a+1/6*ln(b*x+a+1)*ln(
-1/2*b*x-1/2*a+1/2)*a^3+1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^2+1/2*ln(b
*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a-1/6*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*
a+1/2)*a^3-1/2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)*a^2-1/2*ln(-1/2
*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)*a+1/6*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/
2)*a^3-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^2+1/2*ln(b*x+a-1)*ln(1/2*b*x
+1/2*a+1/2)*a-2*arctanh(b*x+a)*(b*x+a)*a+arctanh(b*x+a)^2*a^2*(b*x+a)-arcta
nh(b*x+a)^2*a*(b*x+a)^2-dilog(1/2*b*x+1/2*a+1/2)*a^2+1/3*arctanh(b*x+a)*(b*
x+a)^2-1/3*arctanh(b*x+a)^2*a^3+1/3*arctanh(b*x+a)^2*(b*x+a)^3)
```

Maxima [A]
time = 0.26, size = 259, normalized size = 1.27

$$\frac{1}{3}\,x^{3}\,\operatorname{artanh}\left(bx+a\right)^{2}-\frac{1}{12}\,b^{2}\left(\frac{4\,(3\,a^{2}+1)\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{b^{5}}+\frac{2\,(5\,a^{2}+6\,a+1)\log\left(bx+a+1\right)}{b^{5}}+\frac{(a^{3}+3\,a^{2}+3\,a+1)\log\left(bx+a+1\right)^{2}-2\,(a^{3}+3\,a^{2}+3\,a+1)\log\left(bx+a+1\right)\log\left(bx+a-1\right)+(a^{3}-3\,a^{2}+3\,a-1)\log\left(bx+a-1\right)^{2}-4\,bx-2\,(5\,a^{2}-6\,a+1)\log\left(bx+a-1\right)}{b^{5}}\right)+\frac{1}{3}\,b\left(\frac{bx^{2}-4\,ax}{b^{3}}+\frac{(a^{3}+3\,a^{2}+3\,a+1)\log\left(bx+a+1\right)}{b^{4}}-\frac{(a^{3}-3\,a^{2}+3\,a-1)\log\left(bx+a-1\right)}{b^{4}}\right)\operatorname{artanh}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 + 6*
a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^2 -
 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3*a^2
 + 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x + a -
1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b*x + a
 + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arctanh(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^2*arctanh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(b*x+a)**2,x)
[Out] Integral(x**2*atanh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*arctanh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a + b*x)^2,x)
[Out] int(x^2*atanh(a + b*x)^2, x)
```

3.3 $\int x \tanh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=136

$$\frac{(a+bx)\tanh^{-1}(a+bx)}{b^2}-\frac{a\tanh^{-1}(a+bx)^2}{b^2}-\frac{(1+a^2)\tanh^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\tanh^{-1}(a+bx)^2+\frac{2a\tanh^{-1}(a-}{}$$

```
[Out] (b*x+a)*arctanh(b*x+a)/b^2-a*arctanh(b*x+a)^2/b^2-1/2*(a^2+1)*arctanh(b*x+a
)^2/b^2+1/2*x^2*arctanh(b*x+a)^2+2*a*arctanh(b*x+a)*ln(2/(-b*x-a+1))/b^2+1/
2*ln(1-(b*x+a)^2)/b^2+a*polylog(2,(-b*x-a-1)/(-b*x-a+1))/b^2
```

Rubi [A]

time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6246, 6065, 6021, 266, 6195, 6095, 6131, 6055, 2449, 2352}

$$-\frac{(a^2+1)\tanh^{-1}(a+bx)^2}{2b^2}+\frac{a\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^2}+\frac{\log\left(1-(a+bx)^2\right)}{2b^2}-\frac{a\tanh^{-1}(a+bx)^2}{b^2}+\frac{(a+bx)\tanh^{-1}(a+bx)}{b^2}+\frac{2a\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{b^2}+\frac{1}{2}x^2\tanh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcTanh[a + b*x])/b^2 - (a*ArcTanh[a + b*x]^2)/b^2 - ((1 + a^2)*
ArcTanh[a + b*x]^2)/(2*b^2) + (x^2*ArcTanh[a + b*x]^2)/2 + (2*a*ArcTanh[a +
 b*x]*Log[2/(1 - a - b*x)])/b^2 + Log[1 - (a + b*x)^2]/(2*b^2) + (a*PolyLog
[2, -((1 + a + b*x)/(1 - a - b*x))])/b^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int x\tanh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\tanh^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\tanh^{-1}(a+bx)^2 - \text{Subst}\left(\int\left(-\frac{\tanh^{-1}(x)}{b^2}+\frac{(1+a^2-2ax)\tanh^{-1}(x)}{b^2(1-x^2)}\right)dx$$

$$= \frac{1}{2}x^2\tanh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int\tanh^{-1}(x)\,dx,x,a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\frac{(1+a^2-2ax)\tan}{1-x^2}\right.}{b^2}$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\tanh^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int\frac{x}{1-x^2}\,dx,x,a+bx\right)}{b^2} -$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\tanh^{-1}(a+bx)^2 + \frac{\log\left(1-(a+bx)^2\right)}{2b^2} + \frac{(2a)\text{Subs}}{}$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} - \frac{a\tanh^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\tanh^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\,\text{t}$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} - \frac{a\tanh^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\tanh^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\,\text{t}$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} - \frac{a\tanh^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\tanh^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\,\text{t}$$

$$= \frac{(a+bx)\tanh^{-1}(a+bx)}{b^2} - \frac{a\tanh^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\tanh^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\,\text{t}$$

Mathematica [A]
time = 0.20, size = 98, normalized size = 0.72

$$\frac{(-1+2a-a^2+b^2x^2)\tanh^{-1}(a+bx)^2+2\tanh^{-1}(a+bx)\left(a+bx+2a\log\left(1+e^{-2\tanh^{-1}(a+bx)}\right)\right)-2\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)-2a\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[a + b*x]^2,x]
```

```
[Out] ((-1 + 2*a - a^2 + b^2*x^2)*ArcTanh[a + b*x]^2 + 2*ArcTanh[a + b*x]*(a + b*
x + 2*a*Log[1 + E^(-2*ArcTanh[a + b*x])]) - 2*Log[1/Sqrt[1 - (a + b*x)^2]]
- 2*a*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(132) = 264$.
time = 0.07, size = 312, normalized size = 2.29

method	result

derivativedivides	$\frac{\frac{\operatorname{arctanh}(bx+a)^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2(bx+a)a+\operatorname{arctanh}(bx+a)(bx+a)-\operatorname{arctanh}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arctanh}(bx+a)}{2}}{}$
default	$\frac{\frac{\operatorname{arctanh}(bx+a)^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2(bx+a)a+\operatorname{arctanh}(bx+a)(bx+a)-\operatorname{arctanh}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arctanh}(bx+a)}{2}}{}$
risch	$-\frac{(-x^2b^2+a^2+2a+1)\ln(bx+a+1)^2}{8b^2}+\left(-\frac{\ln(-bx-a+1)x^2}{4}-\frac{-\ln(-bx-a+1)a^2+2\ln(-bx-a+1)a-2bx-\ln(-bx}{4b^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arctanh(b*x+a)^2*(b*x+a)^2-arctanh(b*x+a)^2*(b*x+a)*a+arctanh(b*
x+a)*(b*x+a)-arctanh(b*x+a)*ln(b*x+a-1)*a+1/2*arctanh(b*x+a)*ln(b*x+a-1)-ar
ctanh(b*x+a)*ln(b*x+a+1)*a-1/2*arctanh(b*x+a)*ln(b*x+a+1)+1/2*ln(b*x+a-1)+1
/2*ln(b*x+a+1)-1/4*ln(b*x+a-1)^2*a+dilog(1/2*b*x+1/2*a+1/2)*a+1/2*ln(b*x+a-
1)*ln(1/2*b*x+1/2*a+1/2)*a+1/8*ln(b*x+a-1)^2-1/4*ln(b*x+a-1)*ln(1/2*b*x+1/2
*a+1/2)-1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a+1/2*ln(-1/2*b*x-1/2*a+1/2)
*ln(1/2*b*x+1/2*a+1/2)*a+1/4*ln(b*x+a+1)^2*a-1/4*ln(-1/2*b*x-1/2*a+1/2)*ln(
b*x+a+1)+1/4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)+1/8*ln(b*x+a+1)^2
)
```

Maxima [A]
time = 0.26, size = 202, normalized size = 1.49

$$\frac{1}{2}x^2\operatorname{artanh}(bx+a)^2+\frac{1}{8}b^2\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)a}{b^4}+\frac{4(a+1)\log(bx+a+1)}{b^4}+\frac{(a^2+2a+1)\log(bx+a+1)^2-2(a^2+2a+1)\log(bx+a+1)\log(bx+a-1)+(a^2-2a+1)\log(bx+a-1)^2-4(a-1)\log(bx+a-1)}{b^4}\right)+\frac{1}{2}b\left(\frac{2x}{b^2}-\frac{(a^2+2a+1)\log(bx+a+1)}{b^3}+\frac{(a^2-2a+1)\log(bx+a-1)}{b^3}\right)\operatorname{artanh}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2
*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a +
1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x +
a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1)*lo
g(b*x + a - 1))/b^4) + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1)/b^
3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arctanh(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arctanh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(b*x+a)**2,x)
```

```
[Out] Integral(x*atanh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}(a + b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a + b*x)^2,x)
```

```
[Out] int(x*atanh(a + b*x)^2, x)
```

3.4 $\int \tanh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=81

$$\frac{\tanh^{-1}(a+bx)^2}{b}+\frac{(a+bx)\tanh^{-1}(a+bx)^2}{b}-\frac{2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b}$$

```
[Out] arctanh(b*x+a)^2/b+(b*x+a)*arctanh(b*x+a)^2/b-2*arctanh(b*x+a)*ln(2/(-b*x-a
+1))/b-polylog(2,(-b*x-a-1)/(-b*x-a+1))/b
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6238, 6021, 6131, 6055, 2449, 2352}

$$-\frac{\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b}+\frac{(a+bx)\tanh^{-1}(a+bx)^2}{b}+\frac{\tanh^{-1}(a+bx)^2}{b}-\frac{2\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]^2,x]
```

```
[Out] ArcTanh[a + b*x]^2/b + ((a + b*x)*ArcTanh[a + b*x]^2)/b - (2*ArcTanh[a + b*
x]*Log[2/(1 - a - b*x)])/b - PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))]/b
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \tanh^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \tanh^{-1}(x)^2\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{x\tanh^{-1}(x)}{1-x^2}\,dx, x, a+bx\right)}{b}\\
&= \frac{\tanh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{1-x}\,dx, x, a+bx\right)}{b}\\
&= \frac{\tanh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} - \frac{2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2\text{S}}{}\\
&= \frac{\tanh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} - \frac{2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2\text{S}}{}\\
&= \frac{\tanh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\tanh^{-1}(a+bx)^2}{b} - \frac{2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}}{}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.68

$$\frac{\tanh^{-1}(a+bx)\left((-1+a+bx)\tanh^{-1}(a+bx)-2\log\left(1+e^{-2\tanh^{-1}(a+bx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]^2,x]
```

```
[Out] (ArcTanh[a + b*x]*((-1 + a + b*x)*ArcTanh[a + b*x] - 2*Log[1 + E^(-2*ArcTan
h[a + b*x])]) + PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/b
```

Maple [A]
time = 1.85, size = 91, normalized size = 1.12

method	result
derivativedivides	$\frac{\operatorname{arctanh}(bx+a)^2(bx+a-1)+2\operatorname{arctanh}(bx+a)^2-2\operatorname{arctanh}(bx+a)\ln\left(1+\frac{(bx+a+1)^2}{-(bx+a)^2+1}\right)-\operatorname{polylog}\left(2,-\frac{(bx+a+1)^2}{-(bx+a)^2+1}\right)}{b}$
default	$\frac{\operatorname{arctanh}(bx+a)^2(bx+a-1)+2\operatorname{arctanh}(bx+a)^2-2\operatorname{arctanh}(bx+a)\ln\left(1+\frac{(bx+a+1)^2}{-(bx+a)^2+1}\right)-\operatorname{polylog}\left(2,-\frac{(bx+a+1)^2}{-(bx+a)^2+1}\right)}{b}$
risch	$\frac{(bx+a+1)\ln(bx+a+1)^2}{4b}+\left(-\frac{\ln(-bx-a+1)x}{2}+\frac{-\ln(-bx-a+1)a+\ln(-bx-a+1)}{2b}\right)\ln(bx+a+1)+\frac{x\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(b*x+a)^2*(b*x+a-1)+2*arctanh(b*x+a)^2-2*arctanh(b*x+a)*ln(1+(b
*x+a+1)^2/(-(b*x+a)^2+1))-polylog(2,-(b*x+a+1)^2/(-(b*x+a)^2+1)))
```

Maxima [A]
time = 0.26, size = 139, normalized size = 1.72

$$-\frac{1}{4}b^2\left(\frac{(a+1)\log(bx+a+1)^2-2(a+1)\log(bx+a+1)\log(bx+a-1)+(a-1)\log(bx+a-1)^2}{b^3}+\frac{4\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^3}\right)+b\left(\frac{(a+1)\log(bx+a+1)}{b^2}-\frac{(a-1)\log(bx+a-1)}{b^2}\right)\operatorname{artanh}(bx+a)+x\operatorname{artanh}(bx+a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x
+ a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*b*
x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(b*x
 + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arctanh(b*x + a) + x*arctanh(
b*x + a)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)**2,x)
```

```
[Out] Integral(atanh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(a + b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)^2,x)
```

```
[Out] int(atanh(a + b*x)^2, x)
```

3.5 $\int \frac{\tanh^{-1}(a+bx)^2}{x} dx$

Optimal. Leaf size=148

$$-\tanh^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right)+\tanh^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+\tanh^{-1}(a+bx)\text{PolyLog}\Big(2$$

```
[Out] -arctanh(b*x+a)^2*ln(2/(b*x+a+1))+arctanh(b*x+a)^2*ln(2*b*x/(1-a)/(b*x+a+1)
)+arctanh(b*x+a)*polylog(2,1-2/(b*x+a+1))-arctanh(b*x+a)*polylog(2,1-2*b*x/
(1-a)/(b*x+a+1))+1/2*polylog(3,1-2/(b*x+a+1))-1/2*polylog(3,1-2*b*x/(1-a)/(
b*x+a+1))
```

Rubi [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6246, 6059}

$$\frac{1}{2}\text{Li}_3\left(1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{Li}_3\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)-\text{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)\tanh^{-1}(a+bx)-\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)^2+\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\tanh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]^2/x,x]
```

```
[Out] -(ArcTanh[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcTanh[a + b*x]^2*Log[(2*b*x)
/((1 - a)*(1 + a + b*x))] + ArcTanh[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*x)
] - ArcTanh[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + Poly
Log[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*
x))]/2
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)^2}{x}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$
$$= -\tanh^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right) + \tanh^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.17, size = 634, normalized size = 4.28

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]^2/x,x]
```

```
[Out] (-4*ArcTanh[a + b*x]^3)/3 - (2*ArcTanh[a + b*x]^3)/(3*a) + (2*Sqrt[1 - a^2]
*E^ArcTanh[a]*ArcTanh[a + b*x]^3)/(3*a) - ArcTanh[a + b*x]^2*Log[1 + E^(-2*
ArcTanh[a + b*x])] - I*Pi*ArcTanh[a + b*x]*Log[(E^(-ArcTanh[a + b*x]) + E^A
rcTanh[a + b*x])/2] + ArcTanh[a + b*x]^2*Log[(1 + a - E^(2*ArcTanh[a + b*x]
) + a*E^(2*ArcTanh[a + b*x]))/(2*E^ArcTanh[a + b*x])] - ArcTanh[a + b*x]^2*
Log[1 + ((-1 + a)*E^(2*ArcTanh[a + b*x]))/(1 + a)] + ArcTanh[a + b*x]^2*Log
[1 - E^(-ArcTanh[a] + ArcTanh[a + b*x])] + ArcTanh[a + b*x]^2*Log[1 + E^(-A
rcTanh[a] + ArcTanh[a + b*x])] - 2*ArcTanh[a]*ArcTanh[a + b*x]*Log[(I/2)*(-
E^(ArcTanh[a] - ArcTanh[a + b*x]) + E^(-ArcTanh[a] + ArcTanh[a + b*x]))] +
ArcTanh[a + b*x]^2*Log[1 - E^(-2*ArcTanh[a] + 2*ArcTanh[a + b*x])] + I*Pi*A
rcTanh[a + b*x]*Log[1/Sqrt[1 - (a + b*x)^2]] - ArcTanh[a + b*x]^2*Log[-((b*
x)/Sqrt[1 - (a + b*x)^2])] + 2*ArcTanh[a]*ArcTanh[a + b*x]*Log[(-I)*Sinh[Ar
cTanh[a] - ArcTanh[a + b*x]]] + ArcTanh[a + b*x]*PolyLog[2, -E^(-2*ArcTanh[
a + b*x])] - ArcTanh[a + b*x]*PolyLog[2, -(((-1 + a)*E^(2*ArcTanh[a + b*x])
)/(1 + a))] + 2*ArcTanh[a + b*x]*PolyLog[2, -E^(-ArcTanh[a] + ArcTanh[a + b
*x])] + 2*ArcTanh[a + b*x]*PolyLog[2, E^(-ArcTanh[a] + ArcTanh[a + b*x])] +
 ArcTanh[a + b*x]*PolyLog[2, E^(-2*ArcTanh[a] + 2*ArcTanh[a + b*x])] + Poly
Log[3, -E^(-2*ArcTanh[a + b*x])]/2 + PolyLog[3, -(((-1 + a)*E^(2*ArcTanh[a
+ b*x]))/(1 + a))]/2 - 2*PolyLog[3, -E^(-ArcTanh[a] + ArcTanh[a + b*x])] -
2*PolyLog[3, E^(-ArcTanh[a] + ArcTanh[a + b*x])] - PolyLog[3, E^(-2*ArcTanh
[a] + 2*ArcTanh[a + b*x])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.38, size = 931, normalized size = 6.29

method	result	size

derivativedivides	Expression too large to display	931
default	Expression too large to display	931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-b*x)*arctanh(b*x+a)^2-arctanh(b*x+a)^2*ln(-(b*x+a+1)^2/(-(b*x+a)^2+1)+1
+a*(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))+1/2*I*Pi*csgn(I*(-(b*x+a+1)^2/(-(b*x+a)^
2+1)+1+a*(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))/(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))*(c
sgn(I*(-(b*x+a+1)^2/(-(b*x+a)^2+1)+1+a*(1+(b*x+a+1)^2/(-(b*x+a)^2+1))))*csg
n(I/(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))-csgn(I*(-(b*x+a+1)^2/(-(b*x+a)^2+1)+1+a
*(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))/(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))*csgn(I/(1+
(b*x+a+1)^2/(-(b*x+a)^2+1)))-csgn(I*(-(b*x+a+1)^2/(-(b*x+a)^2+1)+1+a*(1+(b*
x+a+1)^2/(-(b*x+a)^2+1))))*csgn(I*(-(b*x+a+1)^2/(-(b*x+a)^2+1)+1+a*(1+(b*x+
a+1)^2/(-(b*x+a)^2+1)))/(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))+csgn(I*(-(b*x+a+1)^
2/(-(b*x+a)^2+1)+1+a*(1+(b*x+a+1)^2/(-(b*x+a)^2+1)))/(1+(b*x+a+1)^2/(-(b*x+
a)^2+1)))^2)*arctanh(b*x+a)^2-arctanh(b*x+a)*polylog(2,-(b*x+a+1)^2/(-(b*x+
a)^2+1))+1/2*polylog(3,-(b*x+a+1)^2/(-(b*x+a)^2+1))+a/(a-1)*arctanh(b*x+a)^
2*ln(1-(a-1)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-a-1))+a/(a-1)*arctanh(b*x+a)*poly
log(2,(a-1)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-a-1))-1/2*a/(a-1)*polylog(3,(a-1)*
(b*x+a+1)^2/(-(b*x+a)^2+1)/(-a-1))-1/(a-1)*arctanh(b*x+a)^2*ln(1-(a-1)*(b*x
+a+1)^2/(-(b*x+a)^2+1)/(-a-1))-1/(a-1)*arctanh(b*x+a)*polylog(2,(a-1)*(b*x+
a+1)^2/(-(b*x+a)^2+1)/(-a-1))+1/2/(a-1)*polylog(3,(a-1)*(b*x+a+1)^2/(-(b*x+
a)^2+1)/(-a-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(b*x + a)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)**2/x,x)
[Out] Integral(atanh(a + b*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(arctanh(b*x + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)^2/x,x)
[Out] int(atanh(a + b*x)^2/x, x)
```

3.6 $\int \frac{\tanh^{-1}(a+bx)^2}{x^2} \, dx$

Optimal. Leaf size=251

$$-\frac{\tanh^{-1}(a+bx)^2}{x}+\frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}+\frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\tanh^{-1}(a+bx)\log\left(\right.}{1-a^2}$$

```
[Out] -arctanh(b*x+a)^2/x+b*arctanh(b*x+a)*ln(2/(-b*x-a+1))/(1-a)+b*arctanh(b*x+a
)*ln(2/(b*x+a+1))/(1+a)-2*b*arctanh(b*x+a)*ln(2/(b*x+a+1))/(-a^2+1)+2*b*arc
tanh(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)+1/2*b*polylog(2,(-b*x-a-1)/(
-b*x-a+1))/(1-a)-1/2*b*polylog(2,1-2/(b*x+a+1))/(1+a)+b*polylog(2,1-2/(b*x+
a+1))/(-a^2+1)-b*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)
```

Rubi [A]
time = 0.49, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6244, 378, 720, 31, 647, 6873, 6256, 6820, 12, 6857, 6057, 2449, 2352, 2497, 6055}

$$\frac{b\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{1-a^2}-\frac{b\text{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2}-\frac{2b\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)}{1-a^2}+\frac{2b\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\tanh^{-1}(a+bx)}{1-a^2}+\frac{b\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)}-\frac{b\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{2(a+1)}-\frac{\tanh^{-1}(a+bx)^2}{x}+\frac{b\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{1-a}+\frac{b\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcTanh[a + b*x]^2/x) + (b*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/(1 - a)
 + (b*ArcTanh[a + b*x]*Log[2/(1 + a + b*x)])/(1 + a) - (2*b*ArcTanh[a + b*x
]*Log[2/(1 + a + b*x)])/(1 - a^2) + (2*b*ArcTanh[a + b*x]*Log[(2*b*x)/((1 -
 a)*(1 + a + b*x))])/(1 - a^2) + (b*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x
))])/(2*(1 - a)) - (b*PolyLog[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*Pol
yLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*PolyLog[2, 1 - (2*b*x)/((1 - a
)*(1 + a + b*x))])/(1 - a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(a+bx)^2}{x^2}\,dx &= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b)\int \frac{\tanh^{-1}(a+bx)}{x\left(1-(a+bx)^2\right)}\,dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b)\int \frac{\tanh^{-1}(a+bx)}{x\left(1-a^2-2abx-b^2x^2\right)}\,dx \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)(1-x^2)}\,dx, x, a+bx\right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{b\tanh^{-1}(x)}{(-a+x)(1-x^2)}\,dx, x, a+bx\right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{(-a+x)(1-x^2)}\,dx, x, a+bx\right) \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \left(\frac{\tanh^{-1}(x)}{(-1+a^2)(a-x)} + \frac{\tanh^{-1}(x)}{2(-1+a)(-1+x)} - \frac{}{2} \right. \right. \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} - \frac{b\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{-1+x}\,dx, x, a+bx\right)}{1-a} - \frac{b\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{1+x}\,dx, x, \right.}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right.}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right.}{1+a} \\
&= -\frac{\tanh^{-1}(a+bx)^2}{x} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right.}{1+a}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.02, size = 208, normalized size = 0.83

$$
\frac{-\left(\left(-a+a^3+a^2bx+b\left(-1+\sqrt{1-a^2}\,e^{\tanh^{-1}(a)}\right)x\right)\tanh^{-1}(a+bx)^2\right)+abx\tanh^{-1}(a+bx)\left(-i\pi+2\tanh^{-1}(a)-2\log\left(1-e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)\right)+abx\left(i\pi\left(\log\left(1+e^{2\tanh^{-1}(a+bx)}\right)-\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)\right)+2\tanh^{-1}(a)\left(\log\left(1-e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)-\log\left(-i\sinh\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\right)\right)\right)+abx\text{PolyLog}\left(2,e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)}{a\left(-1+a^2\right)x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]^2/x^2,x]
```

```
[Out] (-((-a + a^3 + a^2*b*x + b*(-1 + Sqrt[1 - a^2]*E^ArcTanh[a])*x)*ArcTanh[a +
 b*x]^2) + a*b*x*ArcTanh[a + b*x]*((-I)*Pi + 2*ArcTanh[a] - 2*Log[1 - E^(2*
ArcTanh[a] - 2*ArcTanh[a + b*x])]) + a*b*x*(I*Pi*(Log[1 + E^(2*ArcTanh[a +
b*x])] - Log[1/Sqrt[1 - (a + b*x)^2]]) + 2*ArcTanh[a]*(Log[1 - E^(2*ArcTanh
[a] - 2*ArcTanh[a + b*x])] - Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]])
) + a*b*x*PolyLog[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])])/(a*(-1 + a^2)*
x)
```

Maple [A]
time = 2.45, size = 356, normalized size = 1.42

method	result
derivativedivides	$b\left(-\frac{\operatorname{arctanh}(bx+a)^2}{bx}-\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2a+2}-\frac{2\operatorname{arctanh}(bx+a)\ln(-bx)}{(a-1)(a+1)}+\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a-1}{2a-2}\right.$
default	$b\left(-\frac{\operatorname{arctanh}(bx+a)^2}{bx}-\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2a+2}-\frac{2\operatorname{arctanh}(bx+a)\ln(-bx)}{(a-1)(a+1)}+\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a-1}{2a-2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arctanh(b*x+a)^2/b/x-2*arctanh(b*x+a)/(2*a+2)*ln(b*x+a+1)-2*arctanh(b*x
+a)/(a-1)/(a+1)*ln(-b*x)+2*arctanh(b*x+a)/(2*a-2)*ln(b*x+a-1)+1/4/(a-1)*ln(
b*x+a-1)^2-1/2/(a-1)*dilog(1/2*b*x+1/2*a+1/2)-1/2/(a-1)*ln(b*x+a-1)*ln(1/2*
b*x+1/2*a+1/2)-1/2/(a+1)*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/2/(a+1)*ln(-1
/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)+1/2/(a+1)*dilog(1/2*b*x+1/2*a+1/2)+
1/4/(a+1)*ln(b*x+a+1)^2-1/(a-1)/(a+1)*dilog(1/(-a+1)*(-b*x-a+1))-1/(a-1)/(a
+1)*ln(-b*x)*ln(1/(-a+1)*(-b*x-a+1))+1/(a-1)/(a+1)*dilog((-b*x-a-1)/(-a-1))
+1/(a-1)/(a+1)*ln(-b*x)*ln((-b*x-a-1)/(-a-1)))
```

Maxima [A]
time = 0.26, size = 244, normalized size = 0.97

$$\frac{1}{4}\,b^2\left(\frac{(a-1)\log(bx+a+1)^2-2\,(a-1)\log(bx+a+1)\log(bx+a-1)+(a+1)\log(bx+a-1)^2}{a^2b-b}-\frac{4\left(\log(bx+a-1)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{a^2b-b}+\frac{4\left(\log\left(\frac{bx}{a+1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a+1}\right)\right)}{a^2b-b}-\frac{4\left(\log\left(\frac{bx}{a-1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a-1}\right)\right)}{a^2b-b}\right)-b\left(\frac{\log(bx+a+1)}{a+1}-\frac{\log(bx+a-1)}{a-1}+\frac{2\,\log(x)}{a^2-1}\right)\operatorname{artanh}(bx+a)-\frac{\operatorname{artanh}(bx+a)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x +
 a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4*(l
og(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(b*x/
(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b)) - b*(log(b*x + a +
1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arctanh(b*x + a
) - arctanh(b*x + a)^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(atanh(a + b*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)^2/x^2,x)
```

```
[Out] int(atanh(a + b*x)^2/x^2, x)
```

3.7 $\int \frac{\tanh^{-1}(a+bx)^2}{x^3} \, dx$

Optimal. Leaf size=370

$$-\frac{b\tanh^{-1}(a+bx)}{(1-a^2)\,x}-\frac{\tanh^{-1}(a+bx)^2}{2x^2}+\frac{b^2\log(x)}{(1-a^2)^2}+\frac{b^2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2}-\frac{b^2\log(1-a-bx)}{2(1-a)^2(1+a)}-\frac{b^2\,\mathrm{t}}{}$$

```
[Out] -b*arctanh(b*x+a)/(-a^2+1)/x-1/2*arctanh(b*x+a)^2/x^2+b^2*ln(x)/(-a^2+1)^2+
1/2*b^2*arctanh(b*x+a)*ln(2/(-b*x-a+1))/(1-a)^2-1/2*b^2*ln(-b*x-a+1)/(1-a)^
2/(1+a)-1/2*b^2*arctanh(b*x+a)*ln(2/(b*x+a+1))/(1+a)^2-2*a*b^2*arctanh(b*x+
a)*ln(2/(b*x+a+1))/(-a^2+1)^2+2*a*b^2*arctanh(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+
1))/(-a^2+1)^2-1/2*b^2*ln(b*x+a+1)/(1-a)/(1+a)^2+1/4*b^2*polylog(2,(-b*x-a-
1)/(-b*x-a+1))/(1-a)^2+1/4*b^2*polylog(2,1-2/(b*x+a+1))/(1+a)^2+a*b^2*polyl
og(2,1-2/(b*x+a+1))/(-a^2+1)^2-a*b^2*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a
^2+1)^2
```

Rubi [A]

time = 0.59, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6244, 378, 724, 815, 6873, 6256, 6857, 6063, 720, 31, 647, 6057, 2449, 2352, 2497, 6055}

$$\frac{ab^2\mathrm{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{(1-a^2)^2}-\frac{ab^2\mathrm{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2}+\frac{b^2\log(x)}{(1-a^2)^2}-\frac{2ab^2\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)}{(1-a^2)^2}+\frac{2ab^2\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\tanh^{-1}(a+bx)}{(1-a^2)^2}-\frac{b\tanh^{-1}(a+bx)}{(1-a^2)\,x}+\frac{b^2\mathrm{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2}+\frac{b^2\mathrm{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{4(a+1)^2}-\frac{b^2\log(-a-bx+1)}{2(1-a)^2(a+1)}-\frac{b^2\log(a+bx+1)}{2(1-a)(a+1)^2}+\frac{b^2\log\left(\frac{2}{-a-bx+1}\right)\tanh^{-1}(a+bx)}{2(1-a)^2}-\frac{b^2\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)}{2(a+1)^2}-\frac{\tanh^{-1}(a+bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]^2/x^3,x]
```

```
[Out] -((b*ArcTanh[a + b*x])/((1 - a^2)*x)) - ArcTanh[a + b*x]^2/(2*x^2) + (b^2*L
og[x])/(1 - a^2)^2 + (b^2*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a)
^2) - (b^2*Log[1 - a - b*x])/(2*(1 - a)^2*(1 + a)) - (b^2*ArcTanh[a + b*x]*
Log[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2*ArcTanh[a + b*x]*Log[2/(1 +
a + b*x)])/(1 - a^2)^2 + (2*a*b^2*ArcTanh[a + b*x]*Log[(2*b*x)/((1 - a)*(1
+ a + b*x))])/(1 - a^2)^2 - (b^2*Log[1 + a + b*x])/(2*(1 - a)*(1 + a)^2) +
(b^2*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2*PolyL
og[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2*PolyLog[2, 1 - 2/(1 + a
+ b*x)])/(1 - a^2)^2 - (a*b^2*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x)
)])/(1 - a^2)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)^2}{x^3}\,dx = -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + b\int \frac{\tanh^{-1}(a+bx)}{x^2\left(1-(a+bx)^2\right)}\,dx$$

$$= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + b\int \frac{\tanh^{-1}(a+bx)}{x^2\left(1-a^2-2abx-b^2x^2\right)}\,dx$$

$$= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\left(1-x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \left(-\frac{b^2\tanh^{-1}(x)}{\left(-1+a^2\right)(a-x)^2} - \frac{2ab^2\tanh^{-1}(x)}{\left(-1+a^2\right)^2(a-x)} - \right.\right.$$

$$= -\frac{\tanh^{-1}(a+bx)^2}{2x^2} - \frac{b^2\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{-1+x}\,dx, x, a+bx\right)}{2(1-a)^2} + \frac{b^2\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{1+x}\,dx, \right.}{2(1+a)^2}$$

$$= -\frac{b\tanh^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} -$$

$$= -\frac{b\tanh^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\tanh^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} -$$

$$= -\frac{b\tanh^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{\left(1-a^2\right)^2} +$$

$$= -\frac{b\tanh^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\tanh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{\left(1-a^2\right)^2} +$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 2.09, size = 271, normalized size = 0.73

$$-\frac{\left(\left(1+a^4-b^2\left(-1+2\sqrt{1-a^2}\,e^{\tanh^{-1}(a)}\right)x^2-a^2\left(2+b^2x^2\right)\right)\tanh^{-1}(a+bx)^2\right)+2bx\tanh^{-1}(a+bx)\left(-1+a^2+abx+iab\pi x-2abx\tanh^{-1}(a)+2abx\log\left(1-e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)\right)+2b^2x^2\left(-ia\pi\log\left(1+e^{2\tanh^{-1}(a+bx)}\right)+ia\pi\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)+\log\left(-\frac{bx}{\sqrt{1-(a+bx)^2}}\right)-2a\tanh^{-1}(a)\left(\log\left(1-e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)-\log\left(-i\sinh\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\right)\right)\right)-2ab^2x^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)}{2\left(-1+a^2\right)^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]^2/x^3,x]
```

```
[Out] (-((1 + a^4 - b^2*(-1 + 2*Sqrt[1 - a^2]*E^ArcTanh[a])*x^2 - a^2*(2 + b^2*x^
2))*ArcTanh[a + b*x]^2) + 2*b*x*ArcTanh[a + b*x]*(-1 + a^2 + a*b*x + I*a*b*
Pi*x - 2*a*b*x*ArcTanh[a] + 2*a*b*x*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a +
 b*x])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcTanh[a + b*x])] + I*a*Pi*Lo
g[1/Sqrt[1 - (a + b*x)^2]] + Log[-((b*x)/Sqrt[1 - (a + b*x)^2])] - 2*a*ArcT
anh[a]*(Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - Log[(-I)*Sinh[ArcT
anh[a] - ArcTanh[a + b*x]]])) - 2*a*b^2*x^2*PolyLog[2, E^(2*ArcTanh[a] - 2*
ArcTanh[a + b*x])])/(2*(-1 + a^2)^2*x^2)
```

Maple [A]
time = 0.21, size = 449, normalized size = 1.21

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arctanh}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2(a+1)^2}+\frac{\operatorname{arctanh}(bx+a)}{(a-1)(a+1)bx}+\frac{2\operatorname{arctanh}(bx+a)a\ln(-bx)}{(a-1)^2(a+1)^2}-\operatorname{arctanh}\right.$
default	$b^2\left(-\frac{\operatorname{arctanh}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2(a+1)^2}+\frac{\operatorname{arctanh}(bx+a)}{(a-1)(a+1)bx}+\frac{2\operatorname{arctanh}(bx+a)a\ln(-bx)}{(a-1)^2(a+1)^2}-\operatorname{arctanh}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arctanh(b*x+a)^2/b^2/x^2+1/2*arctanh(b*x+a)/(a+1)^2*ln(b*x+a+1)+a
rctanh(b*x+a)/(a-1)/(a+1)/b/x+2*arctanh(b*x+a)*a/(a-1)^2/(a+1)^2*ln(-b*x)-1
/2*arctanh(b*x+a)/(a-1)^2*ln(b*x+a-1)-1/8/(a-1)^2*ln(b*x+a-1)^2+1/4/(a-1)^2
*dilog(1/2*b*x+1/2*a+1/2)+1/4/(a-1)^2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)+1/4
/(a+1)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4/(a+1)^2*ln(-1/2*b*x-1/2*a+1
/2)*ln(1/2*b*x+1/2*a+1/2)-1/4/(a+1)^2*dilog(1/2*b*x+1/2*a+1/2)-1/8/(a+1)^2*
ln(b*x+a+1)^2+1/(a-1)/(a+1)/(2*a+2)*ln(b*x+a+1)+1/(a-1)^2/(a+1)^2*ln(-b*x)-
1/(a-1)/(a+1)/(2*a-2)*ln(b*x+a-1)+a/(a-1)^2/(a+1)^2*dilog(1/(-a+1)*(-b*x-a+
1))+a/(a-1)^2/(a+1)^2*ln(-b*x)*ln(1/(-a+1)*(-b*x-a+1))-a/(a-1)^2/(a+1)^2*di
log((-b*x-a-1)/(-a-1))-a/(a-1)^2/(a+1)^2*ln(-b*x)*ln((-b*x-a-1)/(-a-1)))
```

Maxima [A]
time = 0.27, size = 360, normalized size = 0.97

$$\frac{1}{8}\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)a}{a^4-2a^2+1}-\frac{8\left(\log\left(\frac{bx}{a+1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a+1}\right)\right)a}{a^4-2a^2+1}+\frac{8\left(\log\left(\frac{bx}{a-1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a-1}\right)\right)a}{a^4-2a^2+1}-\frac{(a^2-2a+1)\log(bx+a+1)^2-2(a^2-2a+1)\log(bx+a+1)\log(bx+a-1)+(a^2+2a+1)\log(bx+a-1)^2}{a^4-2a^2+1}+\frac{4\log(bx+a+1)}{a^3+a^2-a-1}-\frac{4\log(bx+a-1)}{a^3-a^2-a+1}+\frac{8\log(x)}{a^4-2a^2+1}\right)b^2+\frac{1}{2}\left(\frac{4ab\log(x)}{a^4-2a^2+1}+\frac{b\log(bx+a+1)}{a^2+2a+1}-\frac{b\log(bx+a-1)}{a^2-2a+1}+\frac{2}{(a^2-1)x}\right)b\,\mathrm{artanh}(bx+a)-\frac{\mathrm{artanh}(bx+a)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*
a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x
/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(-b*
x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 - 2*(
a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log(b*x
```

```
+ a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1) - 4*
log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b^2 + 1/
2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) - b*
log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arctanh(b*x + a) - 1/
2*arctanh(b*x + a)^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="fricas")
[Out] integral(arctanh(b*x + a)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)**2/x**3,x)
[Out] Integral(atanh(a + b*x)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)^2/x^3,x, algorithm="giac")
[Out] integrate(arctanh(b*x + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)^2/x^3,x)
[Out] int(atanh(a + b*x)^2/x^3, x)
```

3.8 $\int \frac{\tanh^{-1}(1+bx)^2}{x}\,dx$

Optimal. Leaf size=56

$$-\tanh^{-1}(1+bx)^2\log\left(-\frac{2}{bx}\right)-\tanh^{-1}(1+bx)\text{PolyLog}\left(2,1+\frac{2}{bx}\right)+\frac{1}{2}\text{PolyLog}\left(3,1+\frac{2}{bx}\right)$$

```
[Out] -arctanh(b*x+1)^2*ln(-2/b/x)-arctanh(b*x+1)*polylog(2,1+2/b/x)+1/2*polylog(
3,1+2/b/x)
```

Rubi [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6246, 6055, 6095, 6205, 6745}

$$\frac{1}{2}\text{Li}_3\left(1+\frac{2}{bx}\right)-\text{Li}_2\left(1+\frac{2}{bx}\right)\tanh^{-1}(bx+1)-\log\left(-\frac{2}{bx}\right)\tanh^{-1}(bx+1)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + b*x]^2/x,x]
```

```
[Out] -(ArcTanh[1 + b*x]^2*Log[-2/(b*x)]) - ArcTanh[1 + b*x]*PolyLog[2, 1 + 2/(b*
x)] + PolyLog[3, 1 + 2/(b*x)]/2
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+bx)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{-\frac{1}{b}+\frac{x}{b}}\,dx, x, 1+bx\right)}{b}$$
$$= -\tanh^{-1}(1+bx)^2\log\left(-\frac{2}{bx}\right) + 2\text{Subst}\left(\int \frac{\tanh^{-1}(x)\log\left(\frac{2}{1-x}\right)}{1-x^2}\,dx, x, 1+bx\right)$$
$$= -\tanh^{-1}(1+bx)^2\log\left(-\frac{2}{bx}\right) - \tanh^{-1}(1+bx)\text{Li}_2\left(1+\frac{2}{bx}\right) + \text{Subst}\left(\int \frac{\text{Li}_2(1-}{1-}\right.$$
$$= -\tanh^{-1}(1+bx)^2\log\left(-\frac{2}{bx}\right) - \tanh^{-1}(1+bx)\text{Li}_2\left(1+\frac{2}{bx}\right) + \frac{1}{2}\text{Li}_3\left(1+\frac{2}{bx}\right)$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 1.34

$$-\frac{2}{3}\tanh^{-1}(1+bx)^3 - \tanh^{-1}(1+bx)^2\log\left(1+e^{-2\tanh^{-1}(1+bx)}\right) + \tanh^{-1}(1+bx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(1+bx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(1+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + b*x]^2/x,x]
```

```
[Out] (-2*ArcTanh[1 + b*x]^3)/3 - ArcTanh[1 + b*x]^2*Log[1 + E^(-2*ArcTanh[1 + b*
x])] + ArcTanh[1 + b*x]*PolyLog[2, -E^(-2*ArcTanh[1 + b*x])] + PolyLog[3, -
E^(-2*ArcTanh[1 + b*x])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.56, size = 160, normalized size = 2.86

method	result

derivativedivides	$\ln(bx)\operatorname{arctanh}(bx+1)^2 - \operatorname{arctanh}(bx+1)\operatorname{polylog}\left(2, -\frac{(bx+2)^2}{-(bx+1)^2+1}\right) + \frac{\operatorname{polylog}\left(3, -\frac{(bx+}{-(bx+}\right.}{2}$
default	$\ln(bx)\operatorname{arctanh}(bx+1)^2 - \operatorname{arctanh}(bx+1)\operatorname{polylog}\left(2, -\frac{(bx+2)^2}{-(bx+1)^2+1}\right) + \frac{\operatorname{polylog}\left(3, -\frac{(bx+}{-(bx+}\right.}{2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+1)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x)*arctanh(b*x+1)^2-arctanh(b*x+1)*polylog(2,-(b*x+2)^2/(-(b*x+1)^2+1)
)+1/2*polylog(3,-(b*x+2)^2/(-(b*x+1)^2+1))-(I*Pi*csgn(I/(1+(b*x+2)^2/(-(b*x
+1)^2+1)))^3-I*Pi*csgn(I/(1+(b*x+2)^2/(-(b*x+1)^2+1)))^2+I*Pi+ln(2))*arctan
h(b*x+1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="maxima")
```

```
[Out] 1/12*log(-b*x)^3 + 1/4*log(b*x + 2)^2*log(-x) - 1/4*integrate(2*(b*x*log(b)
 + 2*(b*x + 1)*log(-x) + 2*log(b))*log(b*x + 2)/(b*x^2 + 2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + 1)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(bx+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+1)**2/x,x)
```

```
[Out] Integral(atanh(b*x + 1)**2/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+1)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + 1)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}(b\,x+1)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(b*x + 1)^2/x,x)
```

```
[Out] int(atanh(b*x + 1)^2/x, x)
```

3.9 $\int (ce+dex)^3 \left(a+b\tanh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=72

$$\frac{1}{4}be^3x+\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tanh^{-1}(c+dx)}{4d}+\frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4d}$$

```
[Out] 1/4*b*e^3*x+1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctanh(d*x+c)/d+1/4*e^3*(d*x+
c)^4*(a+b*arctanh(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6242, 12, 6037, 308, 212}

$$\frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4d}+\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tanh^{-1}(c+dx)}{4d}+\frac{1}{4}be^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*e^3*x)/4 + (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTanh[c + d*x])/(4*d) +
 (e^3*(c + d*x)^4*(a + b*ArcTanh[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
```

```
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tanh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,c\right.}{4d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \left(-1-x^2+\right.\right.}{4d}$$
$$= \frac{1}{4}be^3x + \frac{be^3(c+dx)^3}{12d} + \frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4d} - \frac{(b}{}$$
$$= \frac{1}{4}be^3x + \frac{be^3(c+dx)^3}{12d} - \frac{be^3\tanh^{-1}(c+dx)}{4d} + \frac{e^3(c+dx)^4\left(a+b\right.}{4d}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 1.08

$$\frac{e^3\left(6b(c+dx)+2b(c+dx)^3+6a(c+dx)^4+6b(c+dx)^4\tanh^{-1}(c+dx)+3b\log(1-c-dx)-3b\log(1+c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (e^3*(6*b*(c + d*x) + 2*b*(c + d*x)^3 + 6*a*(c + d*x)^4 + 6*b*(c + d*x)^4*A
rcTanh[c + d*x] + 3*b*Log[1 - c - d*x] - 3*b*Log[1 + c + d*x]))/(24*d)
```

Maple [A]
time = 1.54, size = 88, normalized size = 1.22

method	result

derivativedivides	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{b\,e^3(dx+c)^4 \operatorname{arctanh}(dx+c)}{4}+\frac{e^3(dx+c)^3 b}{12}+\frac{b\,e^3(dx+c)}{4}+\frac{b\,e^3 \ln(dx+c-1)}{8}-\frac{b\,e^3 \ln(dx+c+1)}{8}}{d}$
default	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{b\,e^3(dx+c)^4 \operatorname{arctanh}(dx+c)}{4}+\frac{e^3(dx+c)^3 b}{12}+\frac{b\,e^3(dx+c)}{4}+\frac{b\,e^3 \ln(dx+c-1)}{8}-\frac{b\,e^3 \ln(dx+c+1)}{8}}{d}$
risch	$\frac{e^3(dx+c)^4 b \ln(dx+c+1)}{8d}-\frac{e^3 d^3 b\,x^4 \ln(-dx-c+1)}{8}-\frac{e^3 d^2 bc\,x^3 \ln(-dx-c+1)}{2}+\frac{e^3 d^3 a\,x^4}{4}-\frac{3e^3 db\,c^2 x^2 \ln(-dx-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a+1/4*b*e^3*(d*x+c)^4*arctanh(d*x+c)+1/12*e^3*(d*x+c
)^3*b+1/4*b*e^3*(d*x+c)+1/8*b*e^3*ln(d*x+c-1)-1/8*b*e^3*ln(d*x+c+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(60) = 120$.
time = 0.26, size = 349, normalized size = 4.85

$$\frac{1}{4}ad^3x^4e^3+acd^2x^3e^3+\frac{3}{2}ac^2dx^2e^3+\frac{3}{4}\left(2x^2\operatorname{artanh}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bc^2de^3+\frac{1}{2}\left(2x^3\operatorname{artanh}(dx+c)+d\left(\frac{dx^2-4cx}{d^3}+\frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4}-\frac{(c^3-3c^2+3c-1)\log(dx+c-1)}{d^4}\right)\right)bcd^2e^3+\frac{1}{24}\left(6x^4\operatorname{artanh}(dx+c)+d\left(\frac{2(d^2x^3-3cdx^2+3(3c^2+1)x)}{d^4}-\frac{3(c^4+4c^3+6c^2+4c+1)\log(dx+c+1)}{d^5}+\frac{3(c^4-4c^3+6c^2-4c+1)\log(dx+c-1)}{d^5}\right)\right)bd^3e^3+ac^3xe^3+\frac{(2(dx+c)\operatorname{artanh}(dx+c)+\log(-(dx+c)^2+1))bc^3e^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4*e^3 + a*c*d^2*x^3*e^3 + 3/2*a*c^2*d*x^2*e^3 + 3/4*(2*x^2*arct
anh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2
*c + 1)*log(d*x + c - 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctanh(d*x + c) +
d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^
3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*b*c*d^2*e^3 + 1/24*(6*x^4*arcta
nh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4
*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c
 + 1)*log(d*x + c - 1)/d^5))*b*d^3*e^3 + a*c^3*x*e^3 + 1/2*(2*(d*x + c)*arc
tanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*c^3*e^3/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(60) = 120$.
time = 0.38, size = 500, normalized size = 6.94

$$\frac{2(3ad^4x^4+(12ac+b)d^3x^3+3(6ac^2+bc)d^2x^2+3(4ac^3+bc^2+b)dx)\cosh(1)^3+6(3ad^4x^4+(12ac+b)d^3x^3+3(6ac^2+bc)d^2x^2+3(4ac^3+bc^2+b)dx)\cosh(1)^2\sinh(1)+6(3ad^4x^4+(12ac+b)d^3x^3+3(6ac^2+bc)d^2x^2+3(4ac^3+bc^2+b)dx)\cosh(1)\sinh(1)^2+2(3ad^4x^4+(12ac+b)d^3x^3+3(6ac^2+bc)d^2x^2+3(4ac^3+bc^2+b)dx)\sinh(1)^3+3((bd^4x^4+4bcd^3x^3+6bc^2d^2x^2+4bc^3dx+bc^4-b)\cosh(1)^3+3(bd^4x^4+4bcd^3x^3+6bc^2d^2x^2+4bc^3dx+bc^4-b)\cosh(1)^2\sinh(1)+3(bd^4x^4+4bcd^3x^3+6bc^2d^2x^2+4bc^3dx+bc^4-b)\cosh(1)\sinh(1)^2+(bd^4x^4+4bcd^3x^3+6bc^2d^2x^2+4bc^3dx+bc^4-b)\sinh(1)^3)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*(3*a*d^4*x^4 + (12*a*c + b)*d^3*x^3 + 3*(6*a*c^2 + b*c)*d^2*x^2 + 3
*(4*a*c^3 + b*c^2 + b)*d*x)*cosh(1)^3 + 6*(3*a*d^4*x^4 + (12*a*c + b)*d^3*x
^3 + 3*(6*a*c^2 + b*c)*d^2*x^2 + 3*(4*a*c^3 + b*c^2 + b)*d*x)*cosh(1)^2*sin
h(1) + 6*(3*a*d^4*x^4 + (12*a*c + b)*d^3*x^3 + 3*(6*a*c^2 + b*c)*d^2*x^2 +
```

```
3*(4*a*c^3 + b*c^2 + b)*d*x)*cosh(1)*sinh(1)^2 + 2*(3*a*d^4*x^4 + (12*a*c +
 b)*d^3*x^3 + 3*(6*a*c^2 + b*c)*d^2*x^2 + 3*(4*a*c^3 + b*c^2 + b)*d*x)*sinh
(1)^3 + 3*((b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c
^4 - b)*cosh(1)^3 + 3*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^
3*d*x + b*c^4 - b)*cosh(1)^2*sinh(1) + 3*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c
^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 - b)*cosh(1)*sinh(1)^2 + (b*d^4*x^4 + 4*b*
c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 - b)*sinh(1)^3)*log(-(d*x
 + c + 1)/(d*x + c - 1)))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(61) = 122$.

time = 1.42, size = 231, normalized size = 3.21

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{atanh}(c+dx)}{4d} + bc^3e^3x\operatorname{atanh}(c+dx) + \frac{3bc^2de^3x^2\operatorname{atanh}(c+dx)}{2} + \frac{bc^2e^3x}{4} + bcd^2e^3x^3\operatorname{atanh}(c+dx) + \frac{bcde^3x^2}{4} + \frac{bd^3e^3x^4\operatorname{atanh}(c+dx)}{4} + \frac{bd^2e^3x^3}{12} + \frac{be^3x}{4} - \frac{be^3\operatorname{atanh}(c+dx)}{4d} & \text{for } d \neq 0 \\ c^3e^3x(a + b\operatorname{atanh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*atanh(c + d*x)/(4*d) + b*c**3*e**3*x*atanh(c
 + d*x) + 3*b*c**2*d*e**3*x**2*atanh(c + d*x)/2 + b*c**2*e**3*x/4 + b*c*d**
2*e**3*x**3*atanh(c + d*x) + b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atanh(c +
 d*x)/4 + b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atanh(c + d*x)/(4*d), N
e(d, 0)), (c**3*e**3*x*(a + b*atanh(c)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(64) = 128$.

time = 0.45, size = 363, normalized size = 5.04

$$\frac{1}{6}\left((c+1)d - (c-1)d\right)\left(\frac{3\left(\frac{(dx+c+1)^3be^3}{(dx+c-1)^3} + \frac{(dx+c+1)be^3}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4} - \frac{4(dx+c+1)^3d^2}{(dx+c-1)^3} + \frac{6(dx+c+1)^2d^2}{(dx+c-1)^2} - \frac{4(dx+c+1)d^2}{dx+c-1} + d^2} + \frac{\frac{6(dx+c+1)^3ae^3}{(dx+c-1)^3} + \frac{6(dx+c+1)ae^3}{dx+c-1} + \frac{3(dx+c+1)^3be^3}{(dx+c-1)^3} - \frac{6(dx+c+1)^2be^3}{(dx+c-1)^2} + \frac{5(dx+c+1)be^3}{dx+c-1} - 2be^3}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4} - \frac{4(dx+c+1)^3d^2}{(dx+c-1)^3} + \frac{6(dx+c+1)^2d^2}{(dx+c-1)^2} - \frac{4(dx+c+1)d^2}{dx+c-1} + d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*e^3/(d*x + c - 1)^3 + (d*
x + c + 1)*b*e^3/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c
 + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*
x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2)
 + (6*(d*x + c + 1)^3*a*e^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a*e^3/(d*x +
c - 1) + 3*(d*x + c + 1)^3*b*e^3/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*e^3/
(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*e^3/(d*x + c - 1) - 2*b*e^3)/((d*x + c
+ 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x
 + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2))
```

Mupad [**B**]

time = 1.42, size = 414, normalized size = 5.75

$$x^3\left(\frac{d^2e^3(b+20ac)}{12} - \frac{2acd^2e^3}{3}\right) + \ln(c+dx+1)\left(\frac{bc^3e^3x}{2} + \frac{3bc^2de^3x^2}{4} + \frac{bcd^2e^3x^3}{2} + \frac{bd^3e^3x^4}{8}\right) - \ln(1-dx-c)\left(\frac{bc^3e^3x}{2} + \frac{3bc^2de^3x^2}{4} + \frac{bcd^2e^3x^3}{2} + \frac{bd^3e^3x^4}{8}\right) - x^2\left(\frac{c\left(\frac{d^2e^3(b+20ac)}{4} - 2acd^2e^3\right)}{d} - \frac{de^3(10ac^2+bc-a)}{2} + \frac{ade^3(4c^2-4)}{8}\right) + x\left(\frac{ce^3(20ac^2+3bc-6a)}{2} - \frac{(4c^2-4)\left(\frac{d^2e^3(b+20ac)}{4} - 2acd^2e^3\right)}{4d^2} + \frac{2c\left(\frac{2c\left(\frac{d^2e^3(b+20ac)}{4} - 2acd^2e^3\right)}{d} - de^3(10ac^2+bc-a) + \frac{ade^3(4c^2-4)}{4}\right)}{d}\right) + \frac{\ln(c+dx-1)(be^3-bc^4e^3)}{8d} + \frac{ad^3e^3x^4}{4} + \frac{be^3\ln(c+dx+1)(c^2+1)(c-1)(c+1)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atanh(c + d*x)),x)
```

```
[Out] x^3*((d^2*e^3*(b + 20*a*c))/12 - (2*a*c*d^2*e^3)/3) + log(c + d*x + 1)*((b*
d^3*e^3*x^4)/8 + (b*c^3*e^3*x)/2 + (3*b*c^2*d*e^3*x^2)/4 + (b*c*d^2*e^3*x^3
)/2) - log(1 - d*x - c)*((b*d^3*e^3*x^4)/8 + (b*c^3*e^3*x)/2 + (3*b*c^2*d*e
^3*x^2)/4 + (b*c*d^2*e^3*x^3)/2) - x^2*((c*((d^2*e^3*(b + 20*a*c))/4 - 2*a*
c*d^2*e^3))/d - (d*e^3*(b*c - a + 10*a*c^2))/2 + (a*d*e^3*(4*c^2 - 4))/8) +
 x*((c*e^3*(3*b*c - 6*a + 20*a*c^2))/2 - ((4*c^2 - 4)*((d^2*e^3*(b + 20*a*c
))/4 - 2*a*c*d^2*e^3))/(4*d^2) + (2*c*((2*c*((d^2*e^3*(b + 20*a*c))/4 - 2*a
*c*d^2*e^3))/d - d*e^3*(b*c - a + 10*a*c^2) + (a*d*e^3*(4*c^2 - 4))/4))/d)
+ (log(c + d*x - 1)*(b*e^3 - b*c^4*e^3))/(8*d) + (a*d^3*e^3*x^4)/4 + (b*e^3
*log(c + d*x + 1)*(c^2 + 1)*(c - 1)*(c + 1))/(8*d)
```

3.10 $\int (ce+dex)^2 \left(a+b\tanh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=69

$$\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d}+\frac{be^2\log\left(1-(c+dx)^2\right)}{6d}$$

```
[Out] 1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctanh(d*x+c))/d+1/6*b*e^2*ln
(1-(d*x+c)^2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6242, 12, 6037, 272, 45}

$$\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d}+\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left(1-(c+dx)^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*e^2*(c + d*x)^2)/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcTanh[c + d*x]))/(3*d
) + (b*e^2*Log[1 - (c + d*x)^2])/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tanh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\tanh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\tanh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{1-x^2}\,dx, x,\right.}{3d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, (\right.}{6d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right.\right.}{6d}$$
$$= \frac{be^2(c+dx)^2}{6d} + \frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3d} + \frac{be^2\log(1}{}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.86

$$\frac{e^2\left((c+dx)^2(b+2a(c+dx))+2b(c+dx)^3\tanh^{-1}(c+dx)+b\log\left(1-(c+dx)^2\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (e^2*((c + d*x)^2*(b + 2*a*(c + d*x)) + 2*b*(c + d*x)^3*ArcTanh[c + d*x] +
b*Log[1 - (c + d*x)^2]))/(6*d)
```

Maple [A]
time = 0.56, size = 77, normalized size = 1.12

method	result

derivativedivides	$\frac{\frac{e^2(dx+c)^3a}{3}+\frac{b\,e^2(dx+c)^3\operatorname{arctanh}(dx+c)}{3}+\frac{e^2(dx+c)^2b}{6}+\frac{b\,e^2\ln(dx+c-1)}{6}+\frac{b\,e^2\ln(dx+c+1)}{6}}{d}$
default	$\frac{\frac{e^2(dx+c)^3a}{3}+\frac{b\,e^2(dx+c)^3\operatorname{arctanh}(dx+c)}{3}+\frac{e^2(dx+c)^2b}{6}+\frac{b\,e^2\ln(dx+c-1)}{6}+\frac{b\,e^2\ln(dx+c+1)}{6}}{d}$
risch	$\frac{e^2(dx+c)^3b\ln(dx+c+1)}{6d}-\frac{e^2d^2b\,x^3\ln(-dx-c+1)}{6}-\frac{e^2dbc\,x^2\ln(-dx-c+1)}{2}+\frac{e^2a\,d^2x^3}{3}-\frac{e^2b\,c^2x\ln(-dx-c+1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a+1/3*b*e^2*(d*x+c)^3*arctanh(d*x+c)+1/6*e^2*(d*x+c)
^2*b+1/6*b*e^2*ln(d*x+c-1)+1/6*b*e^2*ln(d*x+c+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(60) = 120$.
time = 0.26, size = 219, normalized size = 3.17

$$\frac{1}{3}\,ad^2x^3e^2+acdx^2e^2+\frac{1}{2}\left(2\,x^2\operatorname{artanh}\left(dx+c\right)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log\left(dx+c+1\right)}{d^3}+\frac{(c^2-2\,c+1)\log\left(dx+c-1\right)}{d^3}\right)\right)bcde^2+\frac{1}{6}\left(2\,x^3\operatorname{artanh}\left(dx+c\right)+d\left(\frac{dx^2-4\,cx}{d^3}+\frac{(c^3+3\,c^2+3\,c+1)\log\left(dx+c+1\right)}{d^4}-\frac{(c^3-3\,c^2+3\,c-1)\log\left(dx+c-1\right)}{d^4}\right)\right)bd^2e^2+ac^2xe^2+\frac{(2\,(dx+c)\operatorname{artanh}\left(dx+c\right)+\log\left(-(dx+c)^2+1\right))bc^2e^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3*e^2 + a*c*d*x^2*e^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^
2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)
/d^3))*b*c*d*e^2 + 1/6*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (
c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d
*x + c - 1)/d^4))*b*d^2*e^2 + a*c^2*x*e^2 + 1/2*(2*(d*x + c)*arctanh(d*x +
c) + log(-(d*x + c)^2 + 1))*b*c^2*e^2/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(60) = 120$.
time = 0.39, size = 348, normalized size = 5.04

$$\frac{(2\,ad^3x^3+(6\,ac+b)d^2x^2+2\,(3\,ac^2+bc)dx)\cosh\left(1\right)^2+2\,(2\,ad^3x^3+(6\,ac+b)d^2x^2+2\,(3\,ac^2+bc)dx)\cosh\left(1\right)\sinh\left(1\right)+(2\,ad^3x^3+(6\,ac+b)d^2x^2+2\,(3\,ac^2+bc)dx)\sinh\left(1\right)^2+((bc^3+b)\cosh\left(1\right)^2+2\,(bc^3+b)\cosh\left(1\right)\sinh\left(1\right)+(bc^3+b)\sinh\left(1\right)^2)\log\left(dx+c+1\right)-((bc^3-b)\cosh\left(1\right)^2+2\,(bc^3-b)\cosh\left(1\right)\sinh\left(1\right)+(bc^3-b)\sinh\left(1\right)^2)\log\left(dx+c-1\right)+((bd^3x^3+3\,bcd^2x^2+3\,bc^2dx)\cosh\left(1\right)^2+2\,(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx)\cosh\left(1\right)\sinh\left(1\right)+(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx)\sinh\left(1\right)^2)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a*d^3*x^3 + (6*a*c + b)*d^2*x^2 + 2*(3*a*c^2 + b*c)*d*x)*cosh(1)^2
+ 2*(2*a*d^3*x^3 + (6*a*c + b)*d^2*x^2 + 2*(3*a*c^2 + b*c)*d*x)*cosh(1)*sin
h(1) + (2*a*d^3*x^3 + (6*a*c + b)*d^2*x^2 + 2*(3*a*c^2 + b*c)*d*x)*sinh(1)^
2 + ((b*c^3 + b)*cosh(1)^2 + 2*(b*c^3 + b)*cosh(1)*sinh(1) + (b*c^3 + b)*si
nh(1)^2)*log(d*x + c + 1) - ((b*c^3 - b)*cosh(1)^2 + 2*(b*c^3 - b)*cosh(1)*
sinh(1) + (b*c^3 - b)*sinh(1)^2)*log(d*x + c - 1) + ((b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x)*cosh(1)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)
```

```
*cosh(1)*sinh(1) + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*sinh(1)^2)*log
(-(d*x + c + 1)/(d*x + c - 1)))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(56) = 112$.
time = 1.07, size = 180, normalized size = 2.61

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\,\mathrm{atanh}\,(c+dx)}{3d} + bc^2e^2x\,\mathrm{atanh}\,(c+dx) + bcde^2x^2\,\mathrm{atanh}\,(c+dx) + \frac{bce^2x}{3} + \frac{bd^2e^2x^3\,\mathrm{atanh}\,(c+dx)}{3} + \frac{bde^2x^2}{6} + \frac{be^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{3d} - \frac{be^2\,\mathrm{atanh}\,(c+dx)}{3d} & \text{for } d \neq 0 \\ c^2e^2x(a + b\,\mathrm{atanh}\,(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*atanh(c + d*x)/(3*d) + b*c**2*e**2*x*atanh(c + d*x) + b*c*d*e**2*x**2*at
anh(c + d*x) + b*c*e**2*x/3 + b*d**2*e**2*x**3*atanh(c + d*x)/3 + b*d*e**2*
x**2/6 + b*e**2*log(c/d + x + 1/d)/(3*d) - b*e**2*atanh(c + d*x)/(3*d), Ne(
d, 0)), (c**2*e**2*x*(a + b*atanh(c)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(63) = 126$.
time = 0.44, size = 322, normalized size = 4.67

$$-\frac{1}{6}\left((c+1)d-(c-1)d\right)\left(\frac{be^2\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2} - \frac{be^2\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2} - \frac{\left(\frac{3\,(dx+c+1)^2be^2}{(dx+c-1)^2}+be^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^3d^2}{(dx+c-1)^3} - \frac{3\,(dx+c+1)^2d^2}{(dx+c-1)^2} + \frac{3\,(dx+c+1)d^2}{dx+c-1} - d^2} - \frac{2\left(\frac{3\,(dx+c+1)^2ae^2}{(dx+c-1)^2} + ae^2 + \frac{(dx+c+1)^2be^2}{(dx+c-1)^2} - \frac{(dx+c+1)be^2}{dx+c-1}\right)}{\frac{(dx+c+1)^3d^2}{(dx+c-1)^3} - \frac{3\,(dx+c+1)^2d^2}{(dx+c-1)^2} + \frac{3\,(dx+c+1)d^2}{dx+c-1} - d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*((c + 1)*d - (c - 1)*d)*(b*e^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d
^2 - b*e^2*log(-(d*x + c + 1)/(d*x + c - 1))/d^2 - (3*(d*x + c + 1)^2*b*e^2
/(d*x + c - 1)^2 + b*e^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^
3*d^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 + 3*(d*x + c
+ 1)*d^2/(d*x + c - 1) - d^2) - 2*(3*(d*x + c + 1)^2*a*e^2/(d*x + c - 1)^2
+ a*e^2 + (d*x + c + 1)^2*b*e^2/(d*x + c - 1)^2 - (d*x + c + 1)*b*e^2/(d*x
+ c - 1))/((d*x + c + 1)^3*d^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^2/(d*x
 + c - 1)^2 + 3*(d*x + c + 1)*d^2/(d*x + c - 1) - d^2))
```

Mupad [**B**]
time = 0.62, size = 237, normalized size = 3.43

$$\frac{a\,d^2\,e^2\,x^3}{3}+\frac{b\,c\,e^2\,x}{3}+\frac{b\,e^2\,\ln\left(c+d\,x-1\right)}{6\,d}+\frac{b\,e^2\,\ln\left(c+d\,x+1\right)}{6\,d}+a\,c^2\,e^2\,x+\frac{b\,d\,e^2\,x^2}{6}+a\,c\,d\,e^2\,x^2+\frac{b\,c^2\,e^2\,x\,\ln\left(c+d\,x+1\right)}{2}-\frac{b\,c^3\,e^2\,\ln\left(c+d\,x-1\right)}{6\,d}+\frac{b\,c^3\,e^2\,\ln\left(c+d\,x+1\right)}{6\,d}-\frac{b\,c^2\,e^2\,x\,\ln\left(1-d\,x-c\right)}{2}+\frac{b\,d^2\,e^2\,x^3\,\ln\left(c+d\,x+1\right)}{6}-\frac{b\,d^2\,e^2\,x^3\,\ln\left(1-d\,x-c\right)}{6}+\frac{b\,c\,d\,e^2\,x^2\,\ln\left(c+d\,x+1\right)}{2}-\frac{b\,c\,d\,e^2\,x^2\,\ln\left(1-d\,x-c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x)),x)
```

```
[Out] (a*d^2*e^2*x^3)/3 + (b*c*e^2*x)/3 + (b*e^2*log(c + d*x - 1))/(6*d) + (b*e^2
*log(c + d*x + 1))/(6*d) + a*c^2*e^2*x + (b*d*e^2*x^2)/6 + a*c*d*e^2*x^2 +
```

```
(b*c^2*e^2*x*log(c + d*x + 1))/2 - (b*c^3*e^2*log(c + d*x - 1))/(6*d) + (b*
c^3*e^2*log(c + d*x + 1))/(6*d) - (b*c^2*e^2*x*log(1 - d*x - c))/2 + (b*d^2
*e^2*x^3*log(c + d*x + 1))/6 - (b*d^2*e^2*x^3*log(1 - d*x - c))/6 + (b*c*d*
e^2*x^2*log(c + d*x + 1))/2 - (b*c*d*e^2*x^2*log(1 - d*x - c))/2
```

3.11 $\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{bex}{2}-\frac{be\tanh^{-1}(c+dx)}{2d}+\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2d}$$

```
[Out] 1/2*b*x*e-1/2*b*e*arctanh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6242, 12, 6037, 327, 212}

$$\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2d}-\frac{be\tanh^{-1}(c+dx)}{2d}+\frac{bex}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*e*x)/2 - (b*e*ArcTanh[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTanh[c
+ d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int ex\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d} \\
&= \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,c+
\end{aligned}$$

$$= \frac{bex}{2} + \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2d} - \frac{(be)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,
$$

$$= \frac{bex}{2} - \frac{be\tanh^{-1}(c+dx)}{2d} + \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2d}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 1.60

$$\frac{e\left(2bc+2ac^2+2bdx+4acdx+2ad^2x^2+2b(c+dx)^2\tanh^{-1}(c+dx)+b\log(1-c-dx)-b\log(1+c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (e*(2*b*c + 2*a*c^2 + 2*b*d*x + 4*a*c*d*x + 2*a*d^2*x^2 + 2*b*(c + d*x)^2*A
rcTanh[c + d*x] + b*Log[1 - c - d*x] - b*Log[1 + c + d*x]))/(4*d)
```

Maple [A]
time = 0.04, size = 65, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a}{2}+\frac{be(dx+c)^2\operatorname{arctanh}(dx+c)}{2}+\frac{e(dx+c)b}{2}+\frac{be\ln(dx+c-1)}{4}-\frac{be\ln(dx+c+1)}{4}}{d}$

default	$\frac{\frac{e(dx+c)^2 a}{2}+\frac{be(dx+c)^2\,\mathrm{arctanh}(dx+c)}{2}+\frac{e(dx+c)b}{2}+\frac{be\ln(dx+c-1)}{4}-\frac{be\ln(dx+c+1)}{4}}{d}$
risch	$\frac{ebx(dx+2c)\ln(dx+c+1)}{4}-\frac{edb\,x^2\ln(-dx-c+1)}{4}-\frac{ebx\ln(-dx-c+1)c}{2}+\frac{ade\,x^2}{2}+\frac{e\ln(-dx-c-1)b\,c^2}{4d}-\frac{e\ln(d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a+1/2*b*e*(d*x+c)^2*arctanh(d*x+c)+1/2*e*(d*x+c)*b+1/4
*b*e*ln(d*x+c-1)-1/4*b*e*ln(d*x+c+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(45) = 90$.
time = 0.27, size = 117, normalized size = 2.44

$$\frac{1}{2}adx^2e+\frac{1}{4}\left(2x^2\,\mathrm{artanh}\,(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bde+acxe+\frac{(2(dx+c)\,\mathrm{artanh}\,(dx+c)+\log(-(dx+c)^2+1))bce}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2*e + 1/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*d*e + a*c*x
*e + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*c*e/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.
time = 0.34, size = 121, normalized size = 2.52

$$\frac{2\left(ad^2x^2+(2\,ac+b)dx\right)\cosh(1)+\left((bd^2x^2+2\,bcdx+bc^2-b)\cosh(1)+(bd^2x^2+2\,bcdx+bc^2-b)\sinh(1)\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\left(ad^2x^2+(2\,ac+b)dx\right)\sinh(1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*d^2*x^2 + (2*a*c + b)*d*x)*cosh(1) + ((b*d^2*x^2 + 2*b*c*d*x + b*
c^2 - b)*cosh(1) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - b)*sinh(1))*log(-(d*x +
 c + 1)/(d*x + c - 1)) + 2*(a*d^2*x^2 + (2*a*c + b)*d*x)*sinh(1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.
time = 0.68, size = 95, normalized size = 1.98

$$\begin{cases} acex+\frac{adex^2}{2}+\frac{bc^2e\,\mathrm{atanh}\,(c+dx)}{2d}+bcex\,\mathrm{atanh}\,(c+dx)+\frac{bdex^2\,\mathrm{atanh}\,(c+dx)}{2}+\frac{bex}{2}-\frac{be\,\mathrm{atanh}\,(c+dx)}{2d} & \text{for } d\neq 0 \\ cex(a+b\,\mathrm{atanh}\,(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atanh(c + d*x)/(2*d) + b*c*e*x
*atanh(c + d*x) + b*d*e*x**2*atanh(c + d*x)/2 + b*e*x/2 - b*e*atanh(c + d*x
)/(2*d), Ne(d, 0)), (c*e*x*(a + b*atanh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(42) = 84.
time = 0.42, size = 180, normalized size = 3.75

$$\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{(dx+c+1)be\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\left(\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2\,(dx+c+1)d^2}{dx+c-1}+d^2\right)(dx+c-1)}+\frac{\frac{2\,(dx+c+1)ae}{dx+c-1}+\frac{(dx+c+1)be}{dx+c-1}-be}{\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2\,(dx+c+1)d^2}{dx+c-1}+d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*((d*x + c + 1)*b*e*log(-(d*x + c + 1)/(d*x + c
- 1))/(((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c
- 1) + d^2)*(d*x + c - 1)) + (2*(d*x + c + 1)*a*e/(d*x + c - 1) + (d*x + c
+ 1)*b*e/(d*x + c - 1) - b*e)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x
 + c + 1)*d^2/(d*x + c - 1) + d^2))
```

Mupad [B]
time = 1.84, size = 73, normalized size = 1.52

$$\frac{b\,e\,x}{2}+a\,c\,e\,x-\frac{b\,e\,\mathrm{atanh}(c+d\,x)}{2\,d}+\frac{a\,d\,e\,x^2}{2}+\frac{b\,c^2\,e\,\mathrm{atanh}(c+d\,x)}{2\,d}+b\,c\,e\,x\,\mathrm{atanh}(c+d\,x)+\frac{b\,d\,e\,x^2\,\mathrm{atanh}(c+d\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atanh(c + d*x)),x)
```

```
[Out] (b*e*x)/2 + a*c*e*x - (b*e*atanh(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e
*atanh(c + d*x))/(2*d) + b*c*e*x*atanh(c + d*x) + (b*d*e*x^2*atanh(c + d*x)
)/2
```

3.12 $\int \frac{a+b\tanh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=54

$$\frac{a\log(c+dx)}{de}-\frac{b\text{PolyLog}(2,-c-dx)}{2de}+\frac{b\text{PolyLog}(2,c+dx)}{2de}$$

```
[Out] a*ln(d*x+c)/d/e-1/2*b*polylog(2,-d*x-c)/d/e+1/2*b*polylog(2,d*x+c)/d/e
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6242, 12, 6031}

$$\frac{a\log(c+dx)}{de}-\frac{b\text{Li}_2(-c-dx)}{2de}+\frac{b\text{Li}_2(c+dx)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) - (b*PolyLog[2, -c - d*x])/(2*d*e) + (b*PolyLog[2, c
 + d*x])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(c+dx)}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{a\log(c+dx)}{de} - \frac{b\text{Li}_2(-c-dx)}{2de} + \frac{b\text{Li}_2(c+dx)}{2de}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 288, normalized size = 5.33

$$\frac{a\log(c+dx)}{de} - \frac{ib\left(i\tanh^{-1}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+\log\left(\frac{i(c+dx)}{\sqrt{1-(c+dx)^2}}\right)\right)+\frac{1}{2}\left(-\frac{1}{4}i\left(\pi-2i\tanh^{-1}(c+dx)\right)^2+i\tanh^{-1}(c+dx)^2+\left(\pi-2i\tanh^{-1}(c+dx)\right)\log\left(1-e^{i(\pi-2i\tanh^{-1}(c+dx))}\right)+2i\tanh^{-1}(c+dx)\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)-2i\tanh^{-1}(c+dx)\log\left(\frac{2i(c+dx)}{\sqrt{1-(c+dx)^2}}\right)-\left(\pi-2i\tanh^{-1}(c+dx)\right)\log\left(2\sin\left(\frac{1}{2}\left(\pi-2i\tanh^{-1}(c+dx)\right)\right)\right)-i\text{PolyLog}\left(2,e^{i(\pi-2i\tanh^{-1}(c+dx))}\right)-i\text{PolyLog}\left(2,e^{-2\tanh^{-1}(c+dx)}\right)\right)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) - (I*b*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x
)^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) + ((-1/4*I)*(Pi - (2*I)*A
rcTanh[c + d*x])^2 + I*ArcTanh[c + d*x]^2 + (Pi - (2*I)*ArcTanh[c + d*x])*L
og[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] + (2*I)*ArcTanh[c + d*x]*Log[1
- E^(-2*ArcTanh[c + d*x])] - (2*I)*ArcTanh[c + d*x]*Log[((2*I)*(c + d*x))/S
qrt[1 - (c + d*x)^2]] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2*Sin[(Pi - (2*I)
*ArcTanh[c + d*x])/2]] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))]
- I*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/2))/(d*e)
```

Maple [A]
time = 0.79, size = 78, normalized size = 1.44

method	result	size
risch	$\frac{b\,\text{dilog}(-dx-c+1)}{2de}+\frac{a\ln(-dx-c)}{de}-\frac{b\,\text{dilog}(dx+c+1)}{2ed}$	54
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\ln(dx+c)\,\text{arctanh}(dx+c)}{e}-\frac{b\,\text{dilog}(dx+c+1)}{2e}-\frac{b\ln(dx+c)\ln(dx+c+1)}{2e}-\frac{b\,\text{dilog}(dx+c)}{2e}}{d}$	78
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\ln(dx+c)\,\text{arctanh}(dx+c)}{e}-\frac{b\,\text{dilog}(dx+c+1)}{2e}-\frac{b\ln(dx+c)\ln(dx+c+1)}{2e}-\frac{b\,\text{dilog}(dx+c)}{2e}}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/e*ln(d*x+c)+b/e*ln(d*x+c)*arctanh(d*x+c)-1/2*b/e*dilog(d*x+c+1)-1/2*
b/e*ln(d*x+c)*ln(d*x+c+1)-1/2*b/e*dilog(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
[Out] 1/2*b*integrate((log(d*x + c + 1) - log(-d*x - c + 1))/(d*x*e + c*e), x) +
a*e^(-1)*log(d*x*e + c*e)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
[Out] integral((b*arctanh(d*x + c) + a)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx}\,dx + \int \frac{b\,\operatorname{atanh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(d*e*x+c*e),x)
[Out] (Integral(a/(c + d*x), x) + Integral(b*atanh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arctanh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atanh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(c*e + d*e*x),x)
[Out] int((a + b*atanh(c + d*x))/(c*e + d*e*x), x)
```

3.13 $\int \frac{a+b\tanh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1-(c+dx)^2\right)}{2de^2}$$

```
[Out] (-a-b*arctanh(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1-(d*x+c)^2)
/d/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6242, 12, 6037, 272, 36, 31, 29}

$$-\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1-(c+dx)^2\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) -
(b*Log[1 - (c + d*x)^2])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x(1-x^2)}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx,x,(c+dx)^2\right)}{2de^2} \\
&= -\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,(c+dx)^2\right)}{2de^2}+\frac{b\text{Subst}\left(\int \frac{1}{x}\,dx,x,\right.}{2de^2} \\
&= -\frac{a+b\tanh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1-(c+dx)^2\right)}{2de^2}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 1.10

$$-\frac{\frac{2a}{c+dx}+\frac{2b\tanh^{-1}(c+dx)}{c+dx}-2b\log(c+dx)+b\log\left(1-c^2-2cdx-d^2x^2\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -1/2*((2*a)/(c + d*x) + (2*b*ArcTanh[c + d*x])/(c + d*x) - 2*b*Log[c + d*x]
 + b*Log[1 - c^2 - 2*c*d*x - d^2*x^2])/(d*e^2)
```

Maple [A]
time = 0.59, size = 75, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}-\frac{b\,\mathrm{arctanh}(dx+c)}{e^2(dx+c)}-\frac{b\ln(dx+c+1)}{2e^2}+\frac{b\ln(dx+c)}{e^2}-\frac{b\ln(dx+c-1)}{2e^2}}{d}$
default	$\frac{-\frac{a}{e^2(dx+c)}-\frac{b\,\mathrm{arctanh}(dx+c)}{e^2(dx+c)}-\frac{b\ln(dx+c+1)}{2e^2}+\frac{b\ln(dx+c)}{e^2}-\frac{b\ln(dx+c-1)}{2e^2}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{2d(dx+c)e^2}-\frac{\ln(d^2x^2+2cdx+c^2-1)bdx-2\ln(-dx-c)bdx+\ln(d^2x^2+2cdx+c^2-1)bc-2\ln(-dx-c)bc-b\ln(-dx-c}{2e^2(dx+c)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)-b/e^2/(d*x+c)*arctanh(d*x+c)-1/2*b/e^2*ln(d*x+c+1)+b/e^
2*ln(d*x+c)-1/2*b/e^2*ln(d*x+c-1))
```

Maxima [A]
time = 0.26, size = 88, normalized size = 1.40

$$-\frac{1}{2}\left(d\left(\frac{e^{(-2)}\log\left(dx+c+1\right)}{d^2}-\frac{2\,e^{(-2)}\log\left(dx+c\right)}{d^2}+\frac{e^{(-2)}\log\left(dx+c-1\right)}{d^2}\right)+\frac{2\,\mathrm{artanh}\left(dx+c\right)}{d^2xe^2+cde^2}\right)b-\frac{a}{d^2xe^2+cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(e^(-2)*log(d*x + c + 1)/d^2 - 2*e^(-2)*log(d*x + c)/d^2 + e^(-2)*l
og(d*x + c - 1)/d^2) + 2*arctanh(d*x + c)/(d^2*x*e^2 + c*d*e^2))*b - a/(d^2
*x*e^2 + c*d*e^2)
```

Fricas [A]
time = 0.43, size = 114, normalized size = 1.81

$$-\frac{(bdx+bc)\log\left(d^2x^2+2\,cdx+c^2-1\right)-2\,(bdx+bc)\log\left(dx+c\right)+b\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,a}{2\,\left(\left(d^2x+cd\right)\cosh\left(1\right)^2+2\,\left(d^2x+cd\right)\cosh\left(1\right)\sinh\left(1\right)+\left(d^2x+cd\right)\sinh\left(1\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 - 1) - 2*(b*d*x + b*c)*log(
d*x + c) + b*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a)/((d^2*x + c*d)*cosh(1
)^2 + 2*(d^2*x + c*d)*cosh(1)*sinh(1) + (d^2*x + c*d)*sinh(1)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(53) = 106$.
time = 1.20, size = 235, normalized size = 3.73

$$\begin{cases} \frac{\tilde{\infty}a}{e^2x} & \text{for } c = 0 \wedge d = 0 \\ \frac{x(a+b\operatorname{atanh}(c))}{c^2e^2} & \text{for } d = 0 \\ \tilde{\infty}ax & \text{for } c = -dx \\ -\frac{a}{cde^2+d^2e^2x} + \frac{bc\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x} - \frac{bc\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{cde^2+d^2e^2x} + \frac{bc\operatorname{atanh}(c+dx)}{cde^2+d^2e^2x} + \frac{bdx\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x} - \frac{bdx\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{cde^2+d^2e^2x} + \frac{bdx\operatorname{atanh}(c+dx)}{cde^2+d^2e^2x} - \frac{b\operatorname{atanh}(c+dx)}{cde^2+d^2e^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] Piecewise((zoo*a/(e**2*x), Eq(c, 0) & Eq(d, 0)), (x*(a + b*atanh(c))/(c**2*
e**2), Eq(d, 0)), (zoo*a*x, Eq(c, -d*x)), (-a/(c*d*e**2 + d**2*e**2*x) + b*
c*log(c/d + x)/(c*d*e**2 + d**2*e**2*x) - b*c*log(c/d + x + 1/d)/(c*d*e**2
+ d**2*e**2*x) + b*c*atanh(c + d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/
d + x)/(c*d*e**2 + d**2*e**2*x) - b*d*x*log(c/d + x + 1/d)/(c*d*e**2 + d**2
*e**2*x) + b*d*x*atanh(c + d*x)/(c*d*e**2 + d**2*e**2*x) - b*atanh(c + d*x)
/(c*d*e**2 + d**2*e**2*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(61) = 122$.
time = 0.41, size = 152, normalized size = 2.41

$$\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)d^2e^2}{dx+c-1}+d^2e^2} + \frac{2a}{\frac{(dx+c+1)d^2e^2}{dx+c-1}+d^2e^2} + \frac{b\log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{d^2e^2} - \frac{b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c
+ 1)*d^2*e^2/(d*x + c - 1) + d^2*e^2) + 2*a/((d*x + c + 1)*d^2*e^2/(d*x + c
 - 1) + d^2*e^2) + b*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d^2*e^2) - b*lo
g(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^2))
```

Mupad [B]
time = 1.37, size = 122, normalized size = 1.94

$$\frac{b\ln(1-d\,x-c)}{2\,x\,d^2\,e^2+2\,c\,d\,e^2} - \frac{b\ln(c+d\,x+1)}{2\,\left(x\,d^2\,e^2+c\,d\,e^2\right)} - \frac{a}{x\,d^2\,e^2+c\,d\,e^2} - \frac{b\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2\,d\,e^2} + \frac{b\ln(c+d\,x)}{d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] (b*log(1 - d*x - c))/(2*d^2*e^2*x + 2*c*d*e^2) - (b*log(c + d*x + 1))/(2*(d
^2*e^2*x + c*d*e^2)) - a/(d^2*e^2*x + c*d*e^2) - (b*log(c^2 + d^2*x^2 + 2*c
*d*x - 1))/(2*d*e^2) + (b*log(c + d*x))/(d*e^2)
```

3.14 $\int \frac{a+b\tanh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=63

$$-\frac{b}{2de^3(c+dx)}+\frac{b\tanh^{-1}(c+dx)}{2de^3}-\frac{a+b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] -1/2*b/d/e^3/(d*x+c)+1/2*b*arctanh(d*x+c)/d/e^3+1/2*(-a-b*arctanh(d*x+c))/d
/e^3/(d*x+c)^2
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6242, 12, 6037, 331, 212}

$$-\frac{a+b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}+\frac{b\tanh^{-1}(c+dx)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*b/(d*e^3*(c + d*x)) + (b*ArcTanh[c + d*x])/(2*d*e^3) - (a + b*ArcTanh[
c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{a+b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{x^2(1-x^2)}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{a+b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}+\frac{b\tanh^{-1}(c+dx)}{2de^3}-\frac{a+b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 100, normalized size = 1.59

$$-\frac{a}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\tanh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\log(1-c-dx)}{4de^3}+\frac{b\log(1+c+dx)}{4de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*a/(d*e^3*(c + d*x)^2) - b/(2*d*e^3*(c + d*x)) - (b*ArcTanh[c + d*x])/(
2*d*e^3*(c + d*x)^2) - (b*Log[1 - c - d*x])/(4*d*e^3) + (b*Log[1 + c + d*x]
)/(4*d*e^3)
```

Maple [A]
time = 0.58, size = 77, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{a}{2e^{3}(dx+c)^{2}}-\frac{b\,\mathrm{arctanh}(dx+c)}{2e^{3}(dx+c)^{2}}+\frac{b\ln(dx+c+1)}{4e^{3}}-\frac{b}{2e^{3}(dx+c)}-\frac{b\ln(dx+c-1)}{4e^{3}}}{d}$
default	$\frac{-\frac{a}{2e^{3}(dx+c)^{2}}-\frac{b\,\mathrm{arctanh}(dx+c)}{2e^{3}(dx+c)^{2}}+\frac{b\ln(dx+c+1)}{4e^{3}}-\frac{b}{2e^{3}(dx+c)}-\frac{b\ln(dx+c-1)}{4e^{3}}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{4d(dx+c)^{2}e^{3}}+\frac{\ln(-dx-c-1)b\,d^{2}x^{2}-b\,d^{2}x^{2}\ln(-dx-c+1)+2\ln(-dx-c-1)bcdx-2bdx\ln(-dx-c+1)c+\ln(-dx-c}{4e^{3}(dx+c)^{2}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2-1/2*b/e^3/(d*x+c)^2*arctanh(d*x+c)+1/4*b/e^3*ln(d
*x+c+1)-1/2*b/e^3/(d*x+c)-1/4*b/e^3*ln(d*x+c-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(54) = 108$.
time = 0.26, size = 121, normalized size = 1.92

$$\frac{1}{4}\left(d\left(\frac{e^{(-3)}\log\left(dx+c+1\right)}{d^{2}}-\frac{e^{(-3)}\log\left(dx+c-1\right)}{d^{2}}-\frac{2}{d^{3}xe^{3}+cd^{2}e^{3}}\right)-\frac{2\,\mathrm{artanh}\left(dx+c\right)}{d^{3}x^{2}e^{3}+2\,cd^{2}xe^{3}+c^{2}de^{3}}\right)b-\frac{a}{2\left(d^{3}x^{2}e^{3}+2\,cd^{2}xe^{3}+c^{2}de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(d*(e^(-3)*log(d*x + c + 1)/d^2 - e^(-3)*log(d*x + c - 1)/d^2 - 2/(d^3*
x*e^3 + c*d^2*e^3)) - 2*arctanh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2
*d*e^3))*b - 1/2*a/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(54) = 108$.
time = 0.37, size = 166, normalized size = 2.63

$$-\frac{2\,bdx+2\,bc-\left(bd^{2}x^{2}+2\,bcdx+bc^{2}-b\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,a}{4\left(\left(d^{3}x^{2}+2\,cd^{2}x+c^{2}d\right)\cosh\left(1\right)^{3}+3\left(d^{3}x^{2}+2\,cd^{2}x+c^{2}d\right)\cosh\left(1\right)^{2}\sinh\left(1\right)+3\left(d^{3}x^{2}+2\,cd^{2}x+c^{2}d\right)\cosh\left(1\right)\sinh\left(1\right)^{2}+\left(d^{3}x^{2}+2\,cd^{2}x+c^{2}d\right)\sinh\left(1\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d*x + 2*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - b)*log(-(d*x + c +
 1)/(d*x + c - 1)) + 2*a)/((d^3*x^2 + 2*c*d^2*x + c^2*d)*cosh(1)^3 + 3*(d^3
*x^2 + 2*c*d^2*x + c^2*d)*cosh(1)^2*sinh(1) + 3*(d^3*x^2 + 2*c*d^2*x + c^2*
d)*cosh(1)*sinh(1)^2 + (d^3*x^2 + 2*c*d^2*x + c^2*d)*sinh(1)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(54) = 108$.
time = 1.46, size = 313, normalized size = 4.97

$$\begin{cases} -\frac{a}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}+\frac{bc^{2}\,\mathrm{atanh}\left(c+dx\right)}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}+\frac{2bcdx\,\mathrm{atanh}\left(c+dx\right)}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}-\frac{bc}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}+\frac{bd^{2}x^{2}\,\mathrm{atanh}\left(c+dx\right)}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}-\frac{bdx}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}}-\frac{b\,\mathrm{atanh}\left(c+dx\right)}{2c^{2}de^{3}+4cd^{2}e^{3}x+2d^{3}e^{3}x^{2}} & \text{for}\: d \neq 0 \\ \frac{x\left(a+b\,\mathrm{atanh}\left(c\right)\right)}{c^{3}e^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*c**2
*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*
c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
 b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*d**2*x**2*ata
nh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2
*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atanh(c + d*x)/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atanh
(c))/(c**3*e**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(57) = 114.
time = 0.41, size = 194, normalized size = 3.08

$$\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{(dx+c+1)b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\left(\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2}+\frac{2\,(dx+c+1)d^2e^3}{dx+c-1}+d^2e^3\right)(dx+c-1)}+\frac{\frac{2\,(dx+c+1)a}{dx+c-1}+\frac{(dx+c+1)b}{dx+c-1}+b}{\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2}+\frac{2\,(dx+c+1)d^2e^3}{dx+c-1}+d^2e^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*((d*x + c + 1)*b*log(-(d*x + c + 1)/(d*x + c -
1))/((((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*
x + c - 1) + d^2*e^3)*(d*x + c - 1)) + (2*(d*x + c + 1)*a/(d*x + c - 1) + (
d*x + c + 1)*b/(d*x + c - 1) + b)/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2
+ 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3))
```

Mupad [B]
time = 1.73, size = 67, normalized size = 1.06

$$\frac{b\,\mathrm{atanh}(c+d\,x)}{2\,d\,e^3}-\frac{\frac{a}{2}+\frac{b\,c}{2}+\frac{b\,\ln(c+d\,x+1)}{4}-\frac{b\,\ln(1-d\,x-c)}{4}+\frac{b\,d\,x}{2}}{d\,e^3\,{(c+d\,x)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] (b*atanh(c + d*x))/(2*d*e^3) - (a/2 + (b*c)/2 + (b*log(c + d*x + 1))/4 - (b
*log(1 - d*x - c))/4 + (b*d*x)/2)/(d*e^3*(c + d*x)^2)
```

3.15 $\int (ce+dex)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=159

$$\frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d}+\frac{b^2e^3(c+dx)\tanh^{-1}(c+dx)}{2d}+\frac{be^3(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tanh^{-1}\right.}{4d}$$

```
[Out] 1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctanh(d*x+c)/d
+1/6*b*e^3*(d*x+c)^3*(a+b*arctanh(d*x+c))/d-1/4*e^3*(a+b*arctanh(d*x+c))^2/
d+1/4*e^3*(d*x+c)^4*(a+b*arctanh(d*x+c))^2/d+1/3*b^2*e^3*ln(1-(d*x+c)^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6242, 12, 6037, 6127, 272, 45, 6021, 266, 6095}

$$\frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)^2}{4d}+\frac{be^3(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{6d}-\frac{e^3\left(a+b\tanh^{-1}(c+dx)\right)^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d}+\frac{b^2e^3\log\left(1-(c+dx)^2\right)}{3d}+\frac{b^2e^3(c+dx)\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTanh[c
 + d*x])/(2*d) + (b*e^3*(c + d*x)^3*(a + b*ArcTanh[c + d*x]))/(6*d) - (e^3*
(a + b*ArcTanh[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTanh[c + d*x
])^2)/(4*d) + (b^2*e^3*Log[1 - (c + d*x)^2])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\tanh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\tanh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\tan}{1-x}\right.}{2d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\tanh^{-1}(c+dx)\right)^2}{4d} + \frac{(be^3)\,\text{Subst}\left(\int x^2(a+b\right.}{}$$
$$= \frac{be^3(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{6d} + \frac{e^3(c+dx)^4\left(a+b\tanh\right.}{4d}$$
$$= \frac{1}{2}abe^3x + \frac{be^3(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{6d} - \frac{e^3\left(a+b\tanh\right.}{4}$$
$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\tanh^{-1}(c+dx)}{2d} + \frac{be^3(c+dx)^3\left(a+b\,\text{ta}\right.}{6d}$$
$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\tanh^{-1}(c+dx)}{2d} + \frac{be^3(}{}$$

Mathematica [A]
time = 0.06, size = 148, normalized size = 0.93

$$\frac{e^3\left(6ab(c+dx)+b^2(c+dx)^2+2ab(c+dx)^3+3a^2(c+dx)^4+2b(c+dx)\left(3b+b(c+dx)^2+3a(c+dx)^3\right)\tanh^{-1}(c+dx)+3b^2\left(-1+(c+dx)^4\right)\tanh^{-1}(c+dx)^2+b(3a+4b)\log(1-c-dx)+b(-3a+4b)\log(1+c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (e^3*(6*a*b*(c + d*x) + b^2*(c + d*x)^2 + 2*a*b*(c + d*x)^3 + 3*a^2*(c + d*
x)^4 + 2*b*(c + d*x)*(3*b + b*(c + d*x)^2 + 3*a*(c + d*x)^3)*ArcTanh[c + d*
x] + 3*b^2*(-1 + (c + d*x)^4)*ArcTanh[c + d*x]^2 + b*(3*a + 4*b)*Log[1 - c
- d*x] + b*(-3*a + 4*b)*Log[1 + c + d*x]))/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(145) = 290$.
time = 2.70, size = 357, normalized size = 2.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a^2+1/4*e^3*b^2*(d*x+c)^4*arctanh(d*x+c)^2+1/6*e^3*b
^2*(d*x+c)^3*arctanh(d*x+c)+1/2*e^3*b^2*(d*x+c)*arctanh(d*x+c)+1/4*e^3*b^2*
```

```
arctanh(d*x+c)*ln(d*x+c-1)-1/4*e^3*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/8*e^3*b
^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/16*e^3*b^2*ln(d*x+c-1)^2-1/8*e^3*b^2
*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/8*e^3*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1
/2*d*x+1/2*c+1/2)+1/16*e^3*b^2*ln(d*x+c+1)^2+1/12*e^3*b^2*(d*x+c)^2+1/3*e^3
*b^2*ln(d*x+c-1)+1/3*e^3*b^2*ln(d*x+c+1)+1/2*e^3*a*b*(d*x+c)^4*arctanh(d*x+
c)+1/6*e^3*(d*x+c)^3*a*b+1/2*e^3*a*b*(d*x+c)+1/4*e^3*a*b*ln(d*x+c-1)-1/4*e^
3*a*b*ln(d*x+c+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(138) = 276$.
time = 0.46, size = 755, normalized size = 4.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4*e^3 + a^2*c*d^2*x^3*e^3 + 3/2*a^2*c^2*d*x^2*e^3 + 3/2*(2*x^
2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c
^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctanh(d*x + c
) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 -
 (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*a*b*c*d^2*e^3 + 1/12*(6*x^4
*arctanh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c
^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2
 - 4*c + 1)*log(d*x + c - 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*x*e^3 + (2*(d*x +
c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/48*(4*b^2*d^
2*x^2*e^3 + 8*b^2*c*d*x*e^3 + 3*(b^2*d^4*x^4*e^3 + 4*b^2*c*d^3*x^3*e^3 + 6*
b^2*c^2*d^2*x^2*e^3 + 4*b^2*c^3*d*x*e^3 + (c^4 - 1)*b^2*e^3)*log(d*x + c +
1)^2 + 3*(b^2*d^4*x^4*e^3 + 4*b^2*c*d^3*x^3*e^3 + 6*b^2*c^2*d^2*x^2*e^3 + 4
*b^2*c^3*d*x*e^3 + (c^4 - 1)*b^2*e^3)*log(-d*x - c + 1)^2 + 4*(b^2*d^3*x^3*
e^3 + 3*b^2*c*d^2*x^2*e^3 + 3*(c^2*d + d)*b^2*x*e^3 + (c^3 + 3*c + 4)*b^2*e
^3)*log(d*x + c + 1) - 2*(2*b^2*d^3*x^3*e^3 + 6*b^2*c*d^2*x^2*e^3 + 6*(c^2*
d + d)*b^2*x*e^3 + 2*(c^3 + 3*c - 4)*b^2*e^3 + 3*(b^2*d^4*x^4*e^3 + 4*b^2*c
*d^3*x^3*e^3 + 6*b^2*c^2*d^2*x^2*e^3 + 4*b^2*c^3*d*x*e^3 + (c^4 - 1)*b^2*e^
3)*log(d*x + c + 1))*log(-d*x - c + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(138) = 276$.
time = 0.38, size = 1288, normalized size = 8.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(4*(3*a^2*d^4*x^4 + 2*(6*a^2*c + a*b)*d^3*x^3 + (18*a^2*c^2 + 6*a*b*c
+ b^2)*d^2*x^2 + 2*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*x)*cosh(1)^3 +
 12*(3*a^2*d^4*x^4 + 2*(6*a^2*c + a*b)*d^3*x^3 + (18*a^2*c^2 + 6*a*b*c + b^
2)*d^2*x^2 + 2*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*x)*cosh(1)^2*sinh(
1) + 12*(3*a^2*d^4*x^4 + 2*(6*a^2*c + a*b)*d^3*x^3 + (18*a^2*c^2 + 6*a*b*c
+ b^2)*d^2*x^2 + 2*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*x)*cosh(1)*sin
h(1)^2 + 4*(3*a^2*d^4*x^4 + 2*(6*a^2*c + a*b)*d^3*x^3 + (18*a^2*c^2 + 6*a*b
*c + b^2)*d^2*x^2 + 2*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*x)*sinh(1)^
3 + 3*((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4 - b^2)*cosh(1)^3 + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*cosh(1)^2*sinh(1) + 3*(b^2*d^4*x^4 +
 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*cosh(
1)*sinh(1)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4 - b^2)*sinh(1)^3)*log(-(d*x + c + 1)/(d*x + c - 1))^2 + 4*
((3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b + 4*b^2)*cosh(1)^3 + 3*(3*a*b*c^4 +
 b^2*c^3 + 3*b^2*c - 3*a*b + 4*b^2)*cosh(1)^2*sinh(1) + 3*(3*a*b*c^4 + b^2*
c^3 + 3*b^2*c - 3*a*b + 4*b^2)*cosh(1)*sinh(1)^2 + (3*a*b*c^4 + b^2*c^3 + 3
*b^2*c - 3*a*b + 4*b^2)*sinh(1)^3)*log(d*x + c + 1) - 4*((3*a*b*c^4 + b^2*c
^3 + 3*b^2*c - 3*a*b - 4*b^2)*cosh(1)^3 + 3*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c
- 3*a*b - 4*b^2)*cosh(1)^2*sinh(1) + 3*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a
*b - 4*b^2)*cosh(1)*sinh(1)^2 + (3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b - 4*
b^2)*sinh(1)^3)*log(d*x + c - 1) + 4*((3*a*b*d^4*x^4 + (12*a*b*c + b^2)*d^3
*x^3 + 3*(6*a*b*c^2 + b^2*c)*d^2*x^2 + 3*(4*a*b*c^3 + b^2*c^2 + b^2)*d*x)*c
osh(1)^3 + 3*(3*a*b*d^4*x^4 + (12*a*b*c + b^2)*d^3*x^3 + 3*(6*a*b*c^2 + b^2
*c)*d^2*x^2 + 3*(4*a*b*c^3 + b^2*c^2 + b^2)*d*x)*cosh(1)^2*sinh(1) + 3*(3*a
*b*d^4*x^4 + (12*a*b*c + b^2)*d^3*x^3 + 3*(6*a*b*c^2 + b^2*c)*d^2*x^2 + 3*(
4*a*b*c^3 + b^2*c^2 + b^2)*d*x)*cosh(1)*sinh(1)^2 + (3*a*b*d^4*x^4 + (12*a*
b*c + b^2)*d^3*x^3 + 3*(6*a*b*c^2 + b^2*c)*d^2*x^2 + 3*(4*a*b*c^3 + b^2*c^2
 + b^2)*d*x)*sinh(1)^3)*log(-(d*x + c + 1)/(d*x + c - 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(138) = 276$.
time = 2.38, size = 581, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atanh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*atanh(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atanh(c + d*x) + a*b*c**
2*e**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atanh(c + d*x) + a*b*c*d*e**3*x**2/2 +
a*b*d**3*e**3*x**4*atanh(c + d*x)/2 + a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 -
 a*b*e**3*atanh(c + d*x)/(2*d) + b**2*c**4*e**3*atanh(c + d*x)**2/(4*d) + b
```

```
**2*c**3*e**3*x*atanh(c + d*x)**2 + b**2*c**3*e**3*atanh(c + d*x)/(6*d) + 3
*b**2*c**2*d*e**3*x**2*atanh(c + d*x)**2/2 + b**2*c**2*e**3*x*atanh(c + d*x
)/2 + b**2*c*d**2*e**3*x**3*atanh(c + d*x)**2 + b**2*c*d*e**3*x**2*atanh(c
+ d*x)/2 + b**2*c*e**3*x/6 + b**2*c*e**3*atanh(c + d*x)/(2*d) + b**2*d**3*e
**3*x**4*atanh(c + d*x)**2/4 + b**2*d**2*e**3*x**3*atanh(c + d*x)/6 + b**2*
d*e**3*x**2/12 + b**2*e**3*x*atanh(c + d*x)/2 + 2*b**2*e**3*log(c/d + x + 1
/d)/(3*d) - b**2*e**3*atanh(c + d*x)**2/(4*d) - 2*b**2*e**3*atanh(c + d*x)/
(3*d), Ne(d, 0)), (c**3*e**3*x*(a + b*atanh(c))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(145) = 290.
time = 0.46, size = 733, normalized size = 4.61

$$-\frac{1}{12}\left(\frac{4\,b^2e^3\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2}-\frac{4\,b^2e^3\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2}-\frac{3\left(\frac{(dx+c+1)^3b^2e^3}{(dx+c-1)^3}+\frac{(dx+c+1)b^2e^3}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4}-\frac{4\,(dx+c+1)^3d^2}{(dx+c-1)^3}+\frac{6\,(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{4\,(dx+c+1)d^2}{dx+c-1}+d^2}-\frac{2\left(\frac{6\,(dx+c+1)^3abe^3}{(dx+c-1)^3}+\frac{6\,(dx+c+1)abe^3}{dx+c-1}+\frac{3\,(dx+c+1)^3b^2e^3}{(dx+c-1)^3}-\frac{6\,(dx+c+1)^2b^2e^3}{(dx+c-1)^2}+\frac{5\,(dx+c+1)b^2e^3}{dx+c-1}-2\,b^2e^3\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4}-\frac{4\,(dx+c+1)^3d^2}{(dx+c-1)^3}+\frac{6\,(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{4\,(dx+c+1)d^2}{dx+c-1}+d^2}-\frac{2\left(\frac{6\,(dx+c+1)^3a^2e^3}{(dx+c-1)^3}+\frac{6\,(dx+c+1)a^2e^3}{dx+c-1}+\frac{6\,(dx+c+1)^3abe^3}{(dx+c-1)^3}-\frac{12\,(dx+c+1)^2abe^3}{(dx+c-1)^2}+\frac{10\,(dx+c+1)abe^3}{dx+c-1}-4\,abe^3+\frac{(dx+c+1)^3b^2e^3}{(dx+c-1)^3}-\frac{2\,(dx+c+1)^2b^2e^3}{(dx+c-1)^2}+\frac{(dx+c+1)b^2e^3}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4}-\frac{4\,(dx+c+1)^3d^2}{(dx+c-1)^3}+\frac{6\,(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{4\,(dx+c+1)d^2}{dx+c-1}+d^2}\right)((c+1)d-(c-1)d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^2*e^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^2 - 4*b^2*e^3*log(
-(d*x + c + 1)/(d*x + c - 1))/d^2 - 3*((d*x + c + 1)^3*b^2*e^3/(d*x + c - 1
)^3 + (d*x + c + 1)*b^2*e^3/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1)
)^2/((d*x + c + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c -
 1)^3 + 6*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x +
c - 1) + d^2) - 2*(6*(d*x + c + 1)^3*a*b*e^3/(d*x + c - 1)^3 + 6*(d*x + c +
 1)*a*b*e^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b^2*e^3/(d*x + c - 1)^3 - 6*(
d*x + c + 1)^2*b^2*e^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b^2*e^3/(d*x + c -
 1) - 2*b^2*e^3)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^2/(d*
x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^
2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2) - 2*(6*(d*x +
c + 1)^3*a^2*e^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a^2*e^3/(d*x + c - 1) +
6*(d*x + c + 1)^3*a*b*e^3/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2*a*b*e^3/(d*x
 + c - 1)^2 + 10*(d*x + c + 1)*a*b*e^3/(d*x + c - 1) - 4*a*b*e^3 + (d*x + c
 + 1)^3*b^2*e^3/(d*x + c - 1)^3 - 2*(d*x + c + 1)^2*b^2*e^3/(d*x + c - 1)^2
 + (d*x + c + 1)*b^2*e^3/(d*x + c - 1))/((d*x + c + 1)^4*d^2/(d*x + c - 1)^
4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2/(d*x + c
- 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2))*((c + 1)*d - (c - 1)*d)
```

Mupad [B]
time = 2.12, size = 1730, normalized size = 10.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] log(1 - d*x - c)^2*((b^2*c^3*e^3*x)/4 - (b^2*e^3 - b^2*c^4*e^3)/(16*d) + (b
^2*d^3*e^3*x^4)/16 + (3*b^2*c^2*d*e^3*x^2)/8 + (b^2*c*d^2*e^3*x^3)/4) + x*(
(c*e^3*(b^2 - 6*a^2 + 20*a^2*c^2 + 6*a*b*c))/2 + ((6*c^2 - 6)*(2*a^2*c*d^2*
e^3 - (a*d^2*e^3*(b + 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e^3 -
(a*d^2*e^3*(b + 10*a*c))/2))/d + (d*e^3*(b^2 - 6*a^2 + 60*a^2*c^2 + 12*a*b*
c))/6 - (a^2*d*e^3*(6*c^2 - 6))/6))/d) - log(1 - d*x - c)*(log(c + d*x + 1)
*((b^2*c^3*e^3*x)/2 - ((b^2*e^3)/2 - (b^2*c^4*e^3)/2)/(4*d) + (b^2*d^3*e^3*
x^4)/8 + (3*b^2*c^2*d*e^3*x^2)/4 + (b^2*c*d^2*e^3*x^3)/2) + (x^2*(((d*(c -
1) + d*(c + 1))*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c - 1) + d*(c + 1)) +
 8*b^2*d^4*e^3*(c - 1)))/d^2 - 48*b^2*c^2*d^3*e^3 + 8*b^2*d^3*e^3*(c - 1)*(
c + 1) - 32*b^2*c*d^3*e^3*(c - 1)))/(128*d^2) - (x^2*(((d*(c - 1) + d*(c +
1))*(32*b*d^4*e^3*(8*a*c - 2*a + b*c) - 8*b*d^3*e^3*(d*(c - 1) + d*(c + 1))
*(8*a + b) + 8*b*d^4*e^3*(8*a + b)*(c + 1)))/d^2 - 48*b*c*d^3*e^3*(8*a*c -
4*a + b*c) - 32*b*d^3*e^3*(c + 1)*(8*a*c - 2*a + b*c) + 8*b*d^3*e^3*(8*a +
b)*(c - 1)*(c + 1)))/(128*d^2) + (x^3*(32*b*d^4*e^3*(8*a*c - 2*a + b*c) - 8
*b*d^3*e^3*(d*(c - 1) + d*(c + 1))*(8*a + b) + 8*b*d^4*e^3*(8*a + b)*(c + 1
)))/(192*d^2) - (x^3*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c - 1) + d*(c +
1)) + 8*b^2*d^4*e^3*(c - 1)))/(192*d^2) + (x*(((d*(c - 1) + d*(c + 1))*(((d
*(c - 1) + d*(c + 1))*(32*b*d^4*e^3*(8*a*c - 2*a + b*c) - 8*b*d^3*e^3*(d*(c
 - 1) + d*(c + 1))*(8*a + b) + 8*b*d^4*e^3*(8*a + b)*(c + 1)))/d^2 - 48*b*c
*d^3*e^3*(8*a*c - 4*a + b*c) - 32*b*d^3*e^3*(c + 1)*(8*a*c - 2*a + b*c) + 8
*b*d^3*e^3*(8*a + b)*(c - 1)*(c + 1)))/d^2 - ((c - 1)*(c + 1)*(32*b*d^4*e^3
*(8*a*c - 2*a + b*c) - 8*b*d^3*e^3*(d*(c - 1) + d*(c + 1))*(8*a + b) + 8*b*
d^4*e^3*(8*a + b)*(c + 1)))/d^2 + 32*b*c^2*d^2*e^3*(8*a*c - 6*a + b*c) + 48
*b*c*d^2*e^3*(c + 1)*(8*a*c - 4*a + b*c)))/(64*d^2) - (x*(((d*(c - 1) + d*(
c + 1))*(((d*(c - 1) + d*(c + 1))*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c -
 1) + d*(c + 1)) + 8*b^2*d^4*e^3*(c - 1)))/d^2 - 48*b^2*c^2*d^3*e^3 + 8*b^2
*d^3*e^3*(c - 1)*(c + 1) - 32*b^2*c*d^3*e^3*(c - 1)))/d^2 + 32*b^2*c^3*d^2*
e^3 - ((c - 1)*(c + 1)*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c - 1) + d*(c
+ 1)) + 8*b^2*d^4*e^3*(c - 1)))/d^2 + 48*b^2*c^2*d^2*e^3*(c - 1)))/(64*d^2)
 - (b^2*d^3*e^3*x^4)/32 + (b*d^3*e^3*x^4*(8*a + b))/32) + x^2*((c*(2*a^2*c*
d^2*e^3 - (a*d^2*e^3*(b + 10*a*c))/2))/d + (d*e^3*(b^2 - 6*a^2 + 60*a^2*c^2
 + 12*a*b*c))/12 - (a^2*d*e^3*(6*c^2 - 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 -
 (a*d^2*e^3*(b + 10*a*c))/6) + log(c + d*x + 1)^2*((b^2*c^3*e^3*x)/4 - (b^2
*e^3 - b^2*c^4*e^3)/(16*d) + (b^2*d^3*e^3*x^4)/16 + (3*b^2*c^2*d*e^3*x^2)/8
 + (b^2*c*d^2*e^3*x^3)/4) - (log(c + d*x - 1)*(3*b^2*c*e^3 - 4*b^2*e^3 + b^
2*c^3*e^3 - 3*a*b*e^3 + 3*a*b*c^4*e^3))/(12*d) + (log(c + d*x + 1)*(4*b^2*e
^3 + 3*b^2*c*e^3 + b^2*c^3*e^3 - 3*a*b*e^3 + 3*a*b*c^4*e^3))/(12*d) + d*log
(c + d*x + 1)*(x^2*((b^2*c*e^3)/4 + (3*a*b*c^2*e^3)/2) + x^3*((b^2*d*e^3)/1
2 + a*b*c*d*e^3) + (x*(b^2*e^3 + b^2*c^2*e^3 + 4*a*b*c^3*e^3))/(4*d) + (a*b
*d^2*e^3*x^4)/4) + (a^2*d^3*e^3*x^4)/4
```

3.16 $\int (ce+dex)^2 \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=179

$$\frac{1}{3}b^2e^2x-\frac{b^2e^2\tanh^{-1}(c+dx)}{3d}+\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{3d}+\frac{e^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{3d}+\frac{e^2(c+dx)^3}{}$$

```
[Out] 1/3*b^2*e^2*x-1/3*b^2*e^2*arctanh(d*x+c)/d+1/3*b*e^2*(d*x+c)^2*(a+b*arctanh
(d*x+c))/d+1/3*e^2*(a+b*arctanh(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctanh(
d*x+c))^2/d-2/3*b*e^2*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-1/3*b^2*e^2*p
olylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

Rubi [A]
time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6242, 12, 6037, 6127, 327, 212, 6131, 6055, 2449, 2352}

$$\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)^2}{3d}+\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{3d}+\frac{e^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{3d}-\frac{b^2e^2\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{3d}-\frac{b^2e^2\tanh^{-1}(c+dx)}{3d}+\frac{1}{3}b^2e^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTanh[c + d*x])/(3*d) + (b*e^2*(c + d*x)^2*(a +
b*ArcTanh[c + d*x]))/(3*d) + (e^2*(a + b*ArcTanh[c + d*x])^2)/(3*d) + (e^2*
(c + d*x)^3*(a + b*ArcTanh[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTanh[c +
 d*x])*Log[2/(1 - c - d*x)])/(3*d) - (b^2*e^2*PolyLog[2, -((1 + c + d*x)/(1
 - c - d*x))])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\tanh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\tanh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\tanh}{1-x^2}\right.}{3d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2}{3d} + \frac{(2be^2)\,\text{Subst}\left(\int x\left(a+b\,\tanh\right.\right.}{3}$$

$$= \frac{be^2(c+dx)^2 \left(a+b\tanh^{-1}(c+dx)\right)}{3d} + \frac{e^2\left(a+b\tanh^{-1}(c+dx)\right)}{3d}$$

$$= \frac{1}{3}b^2e^2x + \frac{be^2(c+dx)^2 \left(a+b\tanh^{-1}(c+dx)\right)}{3d} + \frac{e^2\left(a+b\tanh^{-}\right.}{3d}$$

$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tanh^{-1}(c+dx)}{3d} + \frac{be^2(c+dx)^2 \left(a+b\tanh^{-1}(c+\right.}{3d}$$

$$= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tanh^{-1}(c+dx)}{3d} + \frac{be^2(c+dx)^2 \left(a+b\tanh^{-1}(c+\right.}{3d}$$

Mathematica [**A**]
time = 0.29, size = 150, normalized size = 0.84

$$\frac{e^2\left(a^2(c+dx)^3+ab\left((c+dx)^2+2(c+dx)^3\tanh^{-1}(c+dx)+\log\left(-1+(c+dx)^2\right)\right)+b^2\left(c+dx-\tanh^{-1}(c+dx)+(c+dx)^2\tanh^{-1}(c+dx)-\tanh^{-1}(c+dx)^2+(c+dx)^3\tanh^{-1}(c+dx)^2-2\tanh^{-1}(c+dx)\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (e^2*(a^2*(c + d*x)^3 + a*b*((c + d*x)^2 + 2*(c + d*x)^3*ArcTanh[c + d*x] +
 Log[-1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTanh[c + d*x] + (c + d*x)^2*Arc
Tanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*ArcTanh[c + d*x]^2 - 2*Arc
Tanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])] + PolyLog[2, -E^(-2*ArcTanh[
c + d*x])])))/(3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(167) = 334$.
time = 4.12, size = 342, normalized size = 1.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a^2+1/3*b^2*e^2*(d*x+c)^3*arctanh(d*x+c)^2+1/3*b^2*e
^2*(d*x+c)^2*arctanh(d*x+c)+1/3*b^2*e^2*arctanh(d*x+c)*ln(d*x+c-1)+1/3*b^2*
e^2*arctanh(d*x+c)*ln(d*x+c+1)+1/3*b^2*e^2*(d*x+c)+1/6*b^2*e^2*ln(d*x+c-1)-
1/6*b^2*e^2*ln(d*x+c+1)-1/3*b^2*e^2*dilog(1/2*d*x+1/2*c+1/2)-1/6*b^2*e^2*ln
(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/12*b^2*e^2*ln(d*x+c-1)^2+1/6*b^2*e^2*ln(-
1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/6*b^2*e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*
x+1/2*c+1/2)-1/12*b^2*e^2*ln(d*x+c+1)^2+2/3*a*b*e^2*(d*x+c)^3*arctanh(d*x+c
)+1/3*e^2*(d*x+c)^2*a*b+1/3*a*b*e^2*ln(d*x+c-1)+1/3*a*b*e^2*ln(d*x+c+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(150) = 300$.
time = 0.45, size = 572, normalized size = 3.20

$$\frac{1}{3}a^2d^2x^3e^2 + a^2cdx^2e^2 + \left(2x^2\operatorname{artanh}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)abcde^2 + \frac{1}{3}\left(2x^3\operatorname{artanh}(dx+c) + d\left(\frac{dx^2-4cx}{d^3} + \frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3c^2+3c-1)\log(dx+c-1)}{d^4}\right)\right)abd^2e^2 + a^2c^2xe^2 + \frac{\left(2(dx+c)\operatorname{artanh}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)abc^2e^2}{d} + \frac{(c^2-1)b^2e^2\log(dx+c+1)}{6d} - \frac{(c^2-1)b^2e^2\log(dx+c-1)}{6d} + \frac{\left(\log(dx+c+1)\log\left(-\frac{1}{2}dx-\frac{1}{2}c+\frac{1}{2}\right) + \mathrm{Li}_2\left(\frac{1}{2}dx+\frac{1}{2}c+\frac{1}{2}\right)\right)b^2e^2}{3d} + \frac{4b^2dxe^2 + (b^2d^3x^3e^2+3b^2cd^2x^2e^2+3b^2c^2dxe^2+(c^3+1)b^2e^2)\log(dx+c+1)^2 + (b^2d^3x^3e^2+3b^2cd^2x^2e^2+3b^2c^2dxe^2+(c^3-1)b^2e^2)\log(-dx-c+1)^2 + 2(b^2d^2x^2e^2+2b^2cdxe^2)\log(dx+c+1) - 2\left(b^2d^2x^2e^2+2b^2cdxe^2+(b^2d^3x^3e^2+3b^2cd^2x^2e^2+3b^2c^2dxe^2+(c^3+1)b^2e^2)\log(dx+c+1)\right)\log(-dx-c+1)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3*e^2 + a^2*c*d*x^2*e^2 + (2*x^2*arctanh(d*x + c) + d*(2*x/d^
2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)
/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 +
 (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log
(d*x + c - 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*x*e^2 + (2*(d*x + c)*arctanh(d*x
+ c) + log(-(d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/6*(c^2 - 1)*b^2*e^2*log(d*x
 + c + 1)/d - 1/6*(c^2 - 1)*b^2*e^2*log(d*x + c - 1)/d + 1/3*(log(d*x + c +
 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2))*b^2*e^2/d +
 1/12*(4*b^2*d*x*e^2 + (b^2*d^3*x^3*e^2 + 3*b^2*c*d^2*x^2*e^2 + 3*b^2*c^2*d
*x*e^2 + (c^3 + 1)*b^2*e^2)*log(d*x + c + 1)^2 + (b^2*d^3*x^3*e^2 + 3*b^2*c
*d^2*x^2*e^2 + 3*b^2*c^2*d*x*e^2 + (c^3 - 1)*b^2*e^2)*log(-d*x - c + 1)^2 +
 2*(b^2*d^2*x^2*e^2 + 2*b^2*c*d*x*e^2)*log(d*x + c + 1) - 2*(b^2*d^2*x^2*e^
2 + 2*b^2*c*d*x*e^2 + (b^2*d^3*x^3*e^2 + 3*b^2*c*d^2*x^2*e^2 + 3*b^2*c^2*d*
x*e^2 + (c^3 + 1)*b^2*e^2)*log(d*x + c + 1))*log(-d*x - c + 1))/d
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*arctanh(d*x + c)^2*e^2 + 2*(
a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*arctanh(d*x + c)*e^2 + (a^2*d^2*x^2 +
2*a^2*c*d*x + a^2*c^2)*e^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int a^{2}c^{2}\,dx+\int a^{2}d^{2}x^{2}\,dx+\int b^{2}c^{2}\operatorname{atanh}^{2}\left(c+dx\right)dx+\int 2abc^{2}\operatorname{atanh}\left(c+dx\right)dx+\int 2a^{2}cdx\,dx+\int b^{2}d^{2}x^{2}\operatorname{atanh}^{2}\left(c+dx\right)dx+\int 2abd^{2}x^{2}\operatorname{atanh}\left(c+dx\right)dx+\int 2b^{2}cdx\operatorname{atanh}^{2}\left(c+dx\right)dx+\int 4abcdx\operatorname{atanh}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*
c**2*atanh(c + d*x)**2, x) + Integral(2*a*b*c**2*atanh(c + d*x), x) + Integ
ral(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atanh(c + d*x)**2, x) + Inte
gral(2*a*b*d**2*x**2*atanh(c + d*x), x) + Integral(2*b**2*c*d*x*atanh(c + d
*x)**2, x) + Integral(4*a*b*c*d*x*atanh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arctanh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^2\,{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2, x)
```

3.17 $\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=95

$$abex+\frac{b^2e(c+dx)\tanh^{-1}(c+dx)}{d}-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{b^2e\log$$

```
[Out] a*b*e*x+b^2*e*(d*x+c)*arctanh(d*x+c)/d-1/2*e*(a+b*arctanh(d*x+c))^2/d+1/2*e
*(d*x+c)^2*(a+b*arctanh(d*x+c))^2/d+1/2*b^2*e*ln(1-(d*x+c)^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6242, 12, 6037, 6127, 6021, 266, 6095}

$$\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+abex+\frac{b^2e\log\left(1-(c+dx)^2\right)}{2d}+\frac{b^2e(c+dx)\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] a*b*e*x + (b^2*e*(c + d*x)*ArcTanh[c + d*x])/d - (e*(a + b*ArcTanh[c + d*x]
)^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 - (c + d*x)^2])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\tanh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\tanh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2(a+b\tanh^{-1}(x)}{1-x^2}\right.}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d} + \frac{(be)\text{Subst}\left(\int \left(a+b\tanh^{-1}\right.\right.}{d}$$

$$= abex - \frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d} + \frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+\right.}{2d}$$

$$= abex + \frac{b^2e(c+dx)\tanh^{-1}(c+dx)}{d} - \frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}$$

$$= abex + \frac{b^2e(c+dx)\tanh^{-1}(c+dx)}{d} - \frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}$$

Mathematica [A]
time = 0.04, size = 134, normalized size = 1.41

$$e\left(\frac{ab(c+dx)}{d}+\frac{a^2(c+dx)^2}{2d}+\frac{b(c+dx)(b+a(c+dx))\tanh^{-1}(c+dx)}{d}+\frac{(-b^2+b^2(c+dx)^2)\tanh^{-1}(c+dx)^2}{2d}+\frac{(ab+b^2)\log(1-c-dx)}{2d}+\frac{(-ab+b^2)\log(1+c+dx)}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] e*((a*b*(c + d*x))/d + (a^2*(c + d*x)^2)/(2*d) + (b*(c + d*x)*(b + a*(c + d
*x))*ArcTanh[c + d*x])/d + ((-b^2 + b^2*(c + d*x)^2)*ArcTanh[c + d*x]^2)/(2
*d) + ((a*b + b^2)*Log[1 - c - d*x])/(2*d) + ((-(a*b) + b^2)*Log[1 + c + d*
x])/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(89) = 178.
time = 0.08, size = 272, normalized size = 2.86

method	result
derivativedivides	$\frac{e(dx+c)^2a^2}{2}+\frac{e\,b^2(dx+c)^2\operatorname{arctanh}(dx+c)^2}{2}+e\,b^2(dx+c)\operatorname{arctanh}(dx+c)+\frac{e\,b^2\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}-\frac{e\,b^2\operatorname{arctanh}(dx+c)\ln}{2}$
default	$\frac{e(dx+c)^2a^2}{2}+\frac{e\,b^2(dx+c)^2\operatorname{arctanh}(dx+c)^2}{2}+e\,b^2(dx+c)\operatorname{arctanh}(dx+c)+\frac{e\,b^2\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}-\frac{e\,b^2\operatorname{arctanh}(dx+c)\ln}{2}$
risch	$\frac{e\,b^2(d^2x^2+2cdx+c^2-1)\ln(dx+c+1)^2}{8d}+\frac{be(-b\,d^2x^2\ln(-dx-c+1)+2a\,d^2x^2-2bdx\ln(-dx-c+1)c+4adxc-\ln(-dx-}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^2+1/2*e*b^2*(d*x+c)^2*arctanh(d*x+c)^2+e*b^2*(d*x+c)
*arctanh(d*x+c)+1/2*e*b^2*arctanh(d*x+c)*ln(d*x+c-1)-1/2*e*b^2*arctanh(d*x+
c)*ln(d*x+c+1)-1/4*e*b^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/8*e*b^2*ln(d*x
+c-1)^2+1/2*e*b^2*ln(d*x+c-1)+1/2*e*b^2*ln(d*x+c+1)-1/4*e*b^2*ln(-1/2*d*x-1
/2*c+1/2)*ln(d*x+c+1)+1/4*e*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2
)+1/8*e*b^2*ln(d*x+c+1)^2+b*a*e*(d*x+c)^2*arctanh(d*x+c)+e*(d*x+c)*a*b+1/2*
b*a*e*ln(d*x+c-1)-1/2*b*a*e*ln(d*x+c+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(94) = 188.
time = 0.44, size = 320, normalized size = 3.37

$$\frac{1}{2}a^2dx^2e+\frac{1}{2}\left(2x^2\operatorname{artanh}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)abde+a^2cxe+\frac{(2(dx+c)\operatorname{artanh}(dx+c)+\log(-(dx+c)^2+1))abce}{d}+\frac{(b^2d^2x^2e+2b^2cdxe+(c^2-1)b^2e)\log(dx+c+1)^2+(b^2d^2x^2e+2b^2cdxe+(c^2-1)b^2e)\log(-dx-c+1)^2+4(b^2dxe+b^2(c+1)e)\log(dx+c+1)-2(2b^2dxe+2b^2(c-1)e+(b^2d^2x^2e+2b^2cdxe+(c^2-1)b^2e)\log(dx+c+1))\log(-dx-c+1)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2*e + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1
)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*d*e + a
^2*c*x*e + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*c*e/d
 + 1/8*((b^2*d^2*x^2*e + 2*b^2*c*d*x*e + (c^2 - 1)*b^2*e)*log(d*x + c + 1)^
2 + (b^2*d^2*x^2*e + 2*b^2*c*d*x*e + (c^2 - 1)*b^2*e)*log(-d*x - c + 1)^2 +
 4*(b^2*d*x*e + b^2*(c + 1)*e)*log(d*x + c + 1) - 2*(2*b^2*d*x*e + 2*b^2*(c
 - 1)*e + (b^2*d^2*x^2*e + 2*b^2*c*d*x*e + (c^2 - 1)*b^2*e)*log(d*x + c + 1
))*log(-d*x - c + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(94) = 188$.
time = 0.40, size = 333, normalized size = 3.51

$$\frac{\left(\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - b^2\right)\cosh\left(1\right) + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - b^2\right)\sinh\left(1\right)\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2 + 4\left(a^2d^2x^2 + 2\left(a^2c + ab\right)dx\right)\cosh\left(1\right) + 4\left(\left(abc^2 + b^2c - ab + b^2\right)\cosh\left(1\right) + \left(abc^2 + b^2c - ab + b^2\right)\sinh\left(1\right)\right)\log\left(dx + c + 1\right) - 4\left(\left(abc^2 + b^2c - ab - b^2\right)\cosh\left(1\right) + \left(abc^2 + b^2c - ab - b^2\right)\sinh\left(1\right)\right)\log\left(dx + c - 1\right) + 4\left(\left(abd^2x^2 + \left(2\,abc + b^2\right)dx\right)\cosh\left(1\right) + \left(abd^2x^2 + \left(2\,abc + b^2\right)dx\right)\sinh\left(1\right)\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right) + 4\left(a^2d^2x^2 + 2\left(a^2c + ab\right)dx\right)\sinh\left(1\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - b^2)*cosh(1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2 - b^2)*sinh(1))*log(-(d*x + c + 1)/(d*x + c - 1))^2 +
 4*(a^2*d^2*x^2 + 2*(a^2*c + a*b)*d*x)*cosh(1) + 4*((a*b*c^2 + b^2*c - a*b
+ b^2)*cosh(1) + (a*b*c^2 + b^2*c - a*b + b^2)*sinh(1))*log(d*x + c + 1) -
4*((a*b*c^2 + b^2*c - a*b - b^2)*cosh(1) + (a*b*c^2 + b^2*c - a*b - b^2)*si
nh(1))*log(d*x + c - 1) + 4*((a*b*d^2*x^2 + (2*a*b*c + b^2)*d*x)*cosh(1) +
(a*b*d^2*x^2 + (2*a*b*c + b^2)*d*x)*sinh(1))*log(-(d*x + c + 1)/(d*x + c -
1)) + 4*(a^2*d^2*x^2 + 2*(a^2*c + a*b)*d*x)*sinh(1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(83) = 166$.
time = 1.10, size = 238, normalized size = 2.51

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\operatorname{atanh}\left(c+dx\right)}{d} + 2abcex\operatorname{atanh}\left(c + dx\right) + abdex^2\operatorname{atanh}\left(c + dx\right) + abex - \frac{abe\operatorname{atanh}\left(c+dx\right)}{d} + \frac{b^2c^2e\operatorname{atanh}^2\left(c+dx\right)}{2d} + b^2cex\operatorname{atanh}^2\left(c + dx\right) + \frac{b^2ce\operatorname{atanh}\left(c+dx\right)}{d} + \frac{b^2dex^2\operatorname{atanh}^2\left(c+dx\right)}{2} + b^2ex\operatorname{atanh}\left(c + dx\right) + \frac{b^2e\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{b^2e\operatorname{atanh}^2\left(c+dx\right)}{2d} - \frac{b^2e\operatorname{atanh}\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ cex\left(a + b\operatorname{atanh}\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atanh(c + d*x)/d + 2*a
*b*c*e*x*atanh(c + d*x) + a*b*d*e*x**2*atanh(c + d*x) + a*b*e*x - a*b*e*ata
nh(c + d*x)/d + b**2*c**2*e*atanh(c + d*x)**2/(2*d) + b**2*c*e*x*atanh(c +
d*x)**2 + b**2*c*e*atanh(c + d*x)/d + b**2*d*e*x**2*atanh(c + d*x)**2/2 + b
**2*e*x*atanh(c + d*x) + b**2*e*log(c/d + x + 1/d)/d - b**2*e*atanh(c + d*x
)**2/(2*d) - b**2*e*atanh(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atanh(c))**2
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(89) = 178$.

time = 0.42, size = 351, normalized size = 3.69

$$\frac{1}{4}\left(\frac{(dx+c+1)b^2e\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\left(\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2(dx+c+1)d^2}{dx+c-1}+d^2\right)(dx+c-1)}-\frac{2b^2e\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2}+\frac{2b^2e\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2}+\frac{2\left(\frac{2(dx+c+1)abe}{dx+c-1}+\frac{(dx+c+1)b^2e}{dx+c-1}-b^2e\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2(dx+c+1)d^2}{dx+c-1}+d^2}+\frac{4\left(\frac{(dx+c+1)a^2e}{dx+c-1}+\frac{(dx+c+1)abe}{dx+c-1}-abe\right)}{\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2(dx+c+1)d^2}{dx+c-1}+d^2}\right)((c+1)d-(c-1)d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*((d*x + c + 1)*b^2*e*log(-(d*x + c + 1)/(d*x + c - 1))^2/((((d*x + c + 1
)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2)*(d*x + c
 - 1)) - 2*b^2*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^2 + 2*b^2*e*log(-(
d*x + c + 1)/(d*x + c - 1))/d^2 + 2*(2*(d*x + c + 1)*a*b*e/(d*x + c - 1) +
(d*x + c + 1)*b^2*e/(d*x + c - 1) - b^2*e)*log(-(d*x + c + 1)/(d*x + c - 1)
)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c - 1)
+ d^2) + 4*((d*x + c + 1)*a^2*e/(d*x + c - 1) + (d*x + c + 1)*a*b*e/(d*x +
c - 1) - a*b*e)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/
(d*x + c - 1) + d^2))*((c + 1)*d - (c - 1)*d)
```

Mupad [B]
time = 1.56, size = 432, normalized size = 4.55

$$x\,(a\,e\,(b+3\,a\,c)-2\,a^2\,c\,e)+\ln(1-d\,x-c)^2\left(\frac{b^2\,c\,e\,x}{4}-\frac{b^2\,e-b^2\,c^2\,e}{8\,d}+\frac{b^2\,d\,e\,x^2}{8}\right)-\ln(1-d\,x-c)\left(\ln(c+d\,x+1)\left(\frac{b^2\,c\,e\,x}{2}-\frac{\frac{b^2\,e}{2}-\frac{b^2\,c^2\,e}{2}}{2\,d}+\frac{b^2\,d\,e\,x^2}{4}\right)-\frac{x\,(4\,b^2\,d^2\,e\,(c-1)-4\,b^2\,d\,e\,(d\,(c-1)+d\,(c+1))+8\,b^2\,c\,d^2\,e)}{16\,d^2}+\frac{x\,(8\,b\,d^2\,e\,(4\,a\,c-2\,a+b\,c)+4\,b\,d^2\,e\,(4\,a+b)\,(c+1)-4\,b\,d\,e\,(d\,(c-1)+d\,(c+1))\,(4\,a+b))}{16\,d^2}-\frac{b^2\,d\,e\,x^2}{8}+\frac{b\,d\,e\,x^2\,(4\,a+b)}{8}\right)+\ln(c+d\,x+1)^2\left(\frac{b^2\,c\,e\,x}{4}-\frac{b^2\,e-b^2\,c^2\,e}{8\,d}+\frac{b^2\,d\,e\,x^2}{8}\right)+\frac{\ln(c+d\,x+1)\,(e\,b^2\,c+e\,b^2+a\,e\,b\,c^2-a\,e\,b)}{2\,d}+\frac{\ln(c+d\,x-1)\,(-e\,b^2\,c+e\,b^2-a\,e\,b\,c^2+a\,e\,b)}{2\,d}+d\,\ln(c+d\,x+1)\left(\frac{x\,(e\,b^2+2\,a\,c\,e\,b)}{2\,d}+\frac{a\,b\,e\,x^2}{2}\right)+\frac{a^2\,d\,e\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] x*(a*e*(b + 3*a*c) - 2*a^2*c*e) + log(1 - d*x - c)^2*((b^2*c*e*x)/4 - (b^2*
e - b^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) - log(1 - d*x - c)*(log(c + d*x + 1
)*((b^2*c*e*x)/2 - ((b^2*e)/2 - (b^2*c^2*e)/2)/(2*d) + (b^2*d*e*x^2)/4) - (
x*(4*b^2*d^2*e*(c - 1) - 4*b^2*d*e*(d*(c - 1) + d*(c + 1)) + 8*b^2*c*d^2*e)
)/(16*d^2) + (x*(8*b*d^2*e*(4*a*c - 2*a + b*c) + 4*b*d^2*e*(4*a + b)*(c + 1
) - 4*b*d*e*(d*(c - 1) + d*(c + 1))*(4*a + b)))/(16*d^2) - (b^2*d*e*x^2)/8
+ (b*d*e*x^2*(4*a + b))/8) + log(c + d*x + 1)^2*((b^2*c*e*x)/4 - (b^2*e - b
^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) + (log(c + d*x + 1)*(b^2*e - a*b*e + b^2
*c*e + a*b*c^2*e))/(2*d) + (log(c + d*x - 1)*(b^2*e + a*b*e - b^2*c*e - a*b
*c^2*e))/(2*d) + d*log(c + d*x + 1)*((x*(b^2*e + 2*a*b*c*e))/(2*d) + (a*b*e
*x^2)/2) + (a^2*d*e*x^2)/2
```

3.18 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=168

$$\frac{2\left(a+b\tanh^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{de}+\frac{b\left(a+b\tanh\right.}{}$$

```
[Out] -2*(a+b*arctanh(d*x+c))^2*arctanh(-1+2/(-d*x-c+1))/d/e-b*(a+b*arctanh(d*x+c
))*polylog(2,1-2/(-d*x-c+1))/d/e+b*(a+b*arctanh(d*x+c))*polylog(2,-1+2/(-d*
x-c+1))/d/e+1/2*b^2*polylog(3,1-2/(-d*x-c+1))/d/e-1/2*b^2*polylog(3,-1+2/(-
d*x-c+1))/d/e
```

Rubi [A]
time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6242, 12, 6033, 6199, 6095, 6205, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{de}+\frac{b\text{Li}_2\left(\frac{2}{-c-dx+1}-1\right)\left(a+b\tanh^{-1}(c+dx)\right)}{de}+\frac{2\tanh^{-1}\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{de}+\frac{b^2\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2de}-\frac{b^2\text{Li}_3\left(\frac{2}{-c-dx+1}-1\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTanh[c + d*x])^2*ArcTanh[1 - 2/(1 - c - d*x)])/(d*e) - (b*(a +
 b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/(d*e) + (b*(a + b*Arc
Tanh[c + d*x])*PolyLog[2, -1 + 2/(1 - c - d*x)])/(d*e) + (b^2*PolyLog[3, 1
- 2/(1 - c - d*x)])/(2*d*e) - (b^2*PolyLog[3, -1 + 2/(1 - c - d*x)])/(2*d*e
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{x}\,dx,x,c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{(4b)\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right.}{1\text{-}}\right.}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{(2b)\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x)}{1-x}\right.}{d}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)\text{L}}{de}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)\text{L}}{de}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 424, normalized size = 2.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a^2*Log[c + d*x] + 2*a*b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] +
 Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) - (a*b*(Pi^2 - (4*I)*Pi*ArcTanh[
c + d*x] - 8*ArcTanh[c + d*x]^2 - 8*ArcTanh[c + d*x]*Log[1 - E^(-2*ArcTanh[
c + d*x])] + (4*I)*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] + 8*ArcTanh[c + d*x]*
Log[1 + E^(2*ArcTanh[c + d*x])] - (4*I)*Pi*Log[2/Sqrt[1 - (c + d*x)^2]] - 8
*ArcTanh[c + d*x]*Log[2/Sqrt[1 - (c + d*x)^2]] + 8*ArcTanh[c + d*x]*Log[((2
*I)*(c + d*x))/Sqrt[1 - (c + d*x)^2]] + 4*PolyLog[2, E^(-2*ArcTanh[c + d*x]
)] + 4*PolyLog[2, -E^(2*ArcTanh[c + d*x])]))/4 + b^2*((I/24)*Pi^3 - (2*ArcT
anh[c + d*x]^3)/3 - ArcTanh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] + A
rcTanh[c + d*x]^2*Log[1 - E^(2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]*PolyLo
g[2, -E^(-2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c
 + d*x])] + PolyLog[3, -E^(-2*ArcTanh[c + d*x])]/2 - PolyLog[3, E^(2*ArcTan
h[c + d*x])]/2))/(d*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.78, size = 840, normalized size = 5.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)+b^2/e*ln(d*x+c)*arctanh(d*x+c)^2-b^2/e*arctanh(d*x+c)*
polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/2*b^2/e*polylog(3,-(d*x+c+1)^2/(1-(
d*x+c)^2))-b^2/e*arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)+b^2/e*arc
tanh(d*x+c)^2*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*b^2/e*arctanh(d*x+c)*po
lylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*b^2/e*polylog(3,-(d*x+c+1)/(1-(d*
x+c)^2)^(1/2))+b^2/e*arctanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2
*b^2/e*arctanh(d*x+c)*polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*b^2/e*poly
log(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+1/2*I*b^2/e*Pi*csgn(I*((d*x+c+1)^2/(1-
(d*x+c)^2)-1))*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)^2/(1
-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*arctanh(d*x+c)^2-1/2*I*b^2/e*
Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-
1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2+1/2*I*b^2/e*Pi*csgn(I*
((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*arctanh(d*x+
c)^2-1/2*I*b^2/e*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)
^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2+2*a*b
/e*ln(d*x+c)*arctanh(d*x+c)-a*b/e*dilog(d*x+c+1)-a*b/e*ln(d*x+c)*ln(d*x+c+1
)-a*b/e*dilog(d*x+c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(d*x*e + c*e)/d + integrate(1/4*b^2*(log(d*x + c + 1) - log(-
d*x - c + 1))^2/(d*x*e + c*e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/
(d*x*e + c*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*e^(-1)/(d*
x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2\operatorname{atanh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab\operatorname{atanh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atanh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*atanh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x), x)
```

3.19 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=104

$$\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{2b\left(a+b\tanh^{-1}(c+dx)\right)\log\left(2-\frac{2}{1+c+dx}\right)}{de^2}-\frac{b^2\text{PolyLog}}{}$$

```
[Out] (a+b*arctanh(d*x+c))^2/d/e^2-(a+b*arctanh(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b*
arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^2-b^2*polylog(2,-1+2/(d*x+c+1))/d/e^2
```

Rubi [A]
time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6242, 12, 6037, 6135, 6079, 2497}

$$-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}+\frac{2b\log\left(2-\frac{2}{c+dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{de^2}-\frac{b^2\text{Li}_2\left(\frac{2}{c+dx+1}-1\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (a + b*ArcTanh[c + d*x])^2/(d*e^2) - (a + b*ArcTanh[c + d*x])^2/(d*e^2*(c +
 d*x)) + (2*b*(a + b*ArcTanh[c + d*x])*Log[2 - 2/(1 + c + d*x)])/(d*e^2) -
(b^2*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{e^2x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^2}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{(2b)\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x(1-x^2)}\,dx,x,c+dx\right)}{de^2}\\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{(2b)\text{Subst}\left(\int \frac{a+b\tanh}{x(1+}\right.}{de}\\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{2b\left(a+b\tanh^{-1}(c+\right.}{d}\\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{2b\left(a+b\tanh^{-1}(c+\right.}{d}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 126, normalized size = 1.21

$$\frac{b^2(-1+c+dx)\tanh^{-1}(c+dx)^2+2b\tanh^{-1}(c+dx)\left(-a+b(c+dx)\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)\right)+a\left(-a+2b(c+dx)\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)\right)-b^2(c+dx)\text{PolyLog}\left(2,e^{-2\tanh^{-1}(c+dx)}\right)}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(-a + b*(c +
d*x)*Log[1 - E^(-2*ArcTanh[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c + d*x
)/Sqrt[1 - (c + d*x)^2]]) - b^2*(c + d*x)*PolyLog[2, E^(-2*ArcTanh[c + d*x]
)])/(d*e^2*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(104) = 208$.
time = 1.79, size = 346, normalized size = 3.33

method	result
derivativedivides	$-\frac{a^2}{e^2(dx+c)} - \frac{b^2 \operatorname{arctanh}(dx+c)^2}{e^2(dx+c)} - \frac{b^2 \operatorname{arctanh}(dx+c)\ln(dx+c+1)}{e^2} + \frac{2b^2 \ln(dx+c)\operatorname{arctanh}(dx+c)}{e^2} - \frac{b^2 \operatorname{arctanh}(dx+c)\ln(dx+c-1)}{e^2} +$
default	$-\frac{a^2}{e^2(dx+c)} - \frac{b^2 \operatorname{arctanh}(dx+c)^2}{e^2(dx+c)} - \frac{b^2 \operatorname{arctanh}(dx+c)\ln(dx+c+1)}{e^2} + \frac{2b^2 \ln(dx+c)\operatorname{arctanh}(dx+c)}{e^2} - \frac{b^2 \operatorname{arctanh}(dx+c)\ln(dx+c-1)}{e^2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)-b^2/e^2/(d*x+c)*arctanh(d*x+c)^2-b^2/e^2*arctanh(d*x+
c)*ln(d*x+c+1)+2*b^2/e^2*ln(d*x+c)*arctanh(d*x+c)-b^2/e^2*arctanh(d*x+c)*ln
(d*x+c-1)+b^2/e^2*dilog(1/2*d*x+1/2*c+1/2)+1/2*b^2/e^2*ln(d*x+c-1)*ln(1/2*d
*x+1/2*c+1/2)-1/4*b^2/e^2*ln(d*x+c-1)^2-1/2*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*
ln(d*x+c+1)+1/2*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+1/4*b^
2/e^2*ln(d*x+c+1)^2-b^2/e^2*dilog(d*x+c+1)-b^2/e^2*ln(d*x+c)*ln(d*x+c+1)-b^
2/e^2*dilog(d*x+c)-2*a*b/e^2/(d*x+c)*arctanh(d*x+c)-a*b/e^2*ln(d*x+c+1)+2*a
*b/e^2*ln(d*x+c)-a*b/e^2*ln(d*x+c-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(e^(-2)*log(d*x + c + 1)/d^2 - 2*e^(-2)*log(d*x + c)/d^2 + e^(-2)*log(d
*x + c - 1)/d^2) + 2*arctanh(d*x + c)/(d^2*x*e^2 + c*d*e^2))*a*b - 1/4*b^2*
(log(-d*x - c + 1)^2/(d^2*x*e^2 + c*d*e^2) + integrate(-((d*x + c - 1)*log(
d*x + c + 1)^2 + 2*(d*x - (d*x + c - 1)*log(d*x + c + 1) + c)*log(-d*x - c
+ 1))/(d^3*x^3*e^2 + (3*c*d^2 - d^2)*x^2*e^2 + (3*c^2*d - 2*c*d)*x*e^2 + (c
^3 - c^2)*e^2), x)) - a^2/(d^2*x*e^2 + c*d*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*e^(-2)/(d^
2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atanh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atanh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.20 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=119

$$-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{de^3(c+dx)}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log(1-}{2d}$$

```
[Out] -b*(a+b*arctanh(d*x+c))/d/e^3/(d*x+c)+1/2*(a+b*arctanh(d*x+c))^2/d/e^3-1/2*
(a+b*arctanh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1-(d*
x+c)^2)/d/e^3
```

Rubi [A]

time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6242, 12, 6037, 6129, 272, 36, 31, 29, 6095}

$$-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left(1-(c+dx)^2\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*(a + b*ArcTanh[c + d*x]))/(d*e^3*(c + d*x))) + (a + b*ArcTanh[c + d*x]
)^2/(2*d*e^3) - (a + b*ArcTanh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log
[c + d*x])/(d*e^3) - (b^2*Log[1 - (c + d*x)^2])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tanh^{-1}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{e^3x^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{x^3}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^2(1-x^2)}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^3} + \frac{b\text{Sub}}{}$$
$$= -\frac{b(a+b\tanh^{-1}(c+dx))}{de^3(c+dx)} + \frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tanh^{-1}(c-}{2de^3(c+dx}$$
$$= -\frac{b(a+b\tanh^{-1}(c+dx))}{de^3(c+dx)} + \frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tanh^{-1}(c-}{2de^3(c+dx}$$
$$= -\frac{b(a+b\tanh^{-1}(c+dx))}{de^3(c+dx)} + \frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tanh^{-1}(c-}{2de^3(c+dx}$$
$$= -\frac{b(a+b\tanh^{-1}(c+dx))}{de^3(c+dx)} + \frac{(a+b\tanh^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tanh^{-1}(c-}{2de^3(c+dx}$$

Mathematica [A]
time = 0.12, size = 136, normalized size = 1.14

$$\frac{-\frac{a^2}{(c+dx)^2} - \frac{2ab}{c+dx} - \frac{2b(a+b(c+dx))\tanh^{-1}(c+dx)}{(c+dx)^2} + \frac{b^2(-1+c^2+2cdx+d^2x^2)\tanh^{-1}(c+dx)^2}{(c+dx)^2} - b(a+b)\log(1-c-dx) + 2b^2\log(c+dx) + (a-b)b\log(1+c+dx)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^2/(c + d*x)^2) - (2*a*b)/(c + d*x) - (2*b*(a + b*(c + d*x))*ArcTanh[c
 + d*x])/(c + d*x)^2 + (b^2*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^
2)/(c + d*x)^2 - b*(a + b)*Log[1 - c - d*x] + 2*b^2*Log[c + d*x] + (a - b)*
b*Log[1 + c + d*x])/(2*d*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(113) = 226.
time = 0.77, size = 324, normalized size = 2.72

method	result

derivativedivides	$-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2\,\mathrm{arctanh}(dx+c)^2}{2e^3(dx+c)^2}+\frac{b^2\,\mathrm{arctanh}(dx+c)\ln(dx+c+1)}{2e^3}-\frac{b^2\,\mathrm{arctanh}(dx+c)}{e^3(dx+c)}-\frac{b^2\,\mathrm{arctanh}(dx+c)\ln(dx+c-1)}{2e^3}+\frac{b^2\ln(dx+c-}{}$
default	$-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2\,\mathrm{arctanh}(dx+c)^2}{2e^3(dx+c)^2}+\frac{b^2\,\mathrm{arctanh}(dx+c)\ln(dx+c+1)}{2e^3}-\frac{b^2\,\mathrm{arctanh}(dx+c)}{e^3(dx+c)}-\frac{b^2\,\mathrm{arctanh}(dx+c)\ln(dx+c-1)}{2e^3}+\frac{b^2\ln(dx+c-}{}$
risch	$\frac{b^2(d^2x^2+2cdx+c^2-1)\ln(dx+c+1)^2}{8e^3(dx+c)^2d}-\frac{b(b\,d^2x^2\ln(-dx-c+1)+2bdx\ln(-dx-c+1)c+\ln(-dx-c+1)b\,c^2+2bdx+2bc-b\,\mathrm{l}}{4e^3(dx+c)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2-1/2*b^2/e^3/(d*x+c)^2*arctanh(d*x+c)^2+1/2*b^2/
e^3*arctanh(d*x+c)*ln(d*x+c+1)-b^2/e^3/(d*x+c)*arctanh(d*x+c)-1/2*b^2/e^3*a
rctanh(d*x+c)*ln(d*x+c-1)+1/4*b^2/e^3*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/8
*b^2/e^3*ln(d*x+c-1)^2-1/2*b^2/e^3*ln(d*x+c+1)+b^2/e^3*ln(d*x+c)-1/2*b^2/e^
3*ln(d*x+c-1)+1/4*b^2/e^3*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/4*b^2/e^3*ln
(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)-1/8*b^2/e^3*ln(d*x+c+1)^2-a*b/e^
3/(d*x+c)^2*arctanh(d*x+c)+1/2*a*b/e^3*ln(d*x+c+1)-a*b/e^3/(d*x+c)-1/2*a*b/
e^3*ln(d*x+c-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(108) = 216$.
time = 0.28, size = 309, normalized size = 2.60

$$\frac{1}{2}\left(d\left(\frac{e^{(-3)}\log(dx+c+1)}{d^2}-\frac{e^{(-3)}\log(dx+c-1)}{d^2}-\frac{2}{d^3xe^3+cd^2e^3}\right)-\frac{2\,\mathrm{artanh}(dx+c)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)ab-\frac{1}{8}\left(d^2\left(\frac{(\log(dx+c+1)^2-2\,\log(dx+c+1)\log(dx+c-1)+\log(dx+c-1)^2+4\,\log(dx+c-1))e^{(-3)}}{d^3}+\frac{4\,e^{(-3)}\log(dx+c+1)}{d^3}-\frac{8\,e^{(-3)}\log(dx+c)}{d^3}\right)-4\,d\left(\frac{e^{(-3)}\log(dx+c+1)}{d^2}-\frac{e^{(-3)}\log(dx+c-1)}{d^2}-\frac{2}{d^3xe^3+cd^2e^3}\right)\mathrm{artanh}(dx+c)\right)b^2-\frac{b^2\,\mathrm{artanh}(dx+c)^2}{2\,(d^3x^2e^3+2\,cd^2xe^3+c^2de^3)}-\frac{a^2}{2\,(d^3x^2e^3+2\,cd^2xe^3+c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(e^(-3)*log(d*x + c + 1)/d^2 - e^(-3)*log(d*x + c - 1)/d^2 - 2/(d^3*
x*e^3 + c*d^2*e^3)) - 2*arctanh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2
*d*e^3))*a*b - 1/8*(d^2*((log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d*x +
 c - 1) + log(d*x + c - 1)^2 + 4*log(d*x + c - 1))*e^(-3)/d^3 + 4*e^(-3)*lo
g(d*x + c + 1)/d^3 - 8*e^(-3)*log(d*x + c)/d^3) - 4*d*(e^(-3)*log(d*x + c +
 1)/d^2 - e^(-3)*log(d*x + c - 1)/d^2 - 2/(d^3*x*e^3 + c*d^2*e^3))*arctanh(
d*x + c))*b^2 - 1/2*b^2*arctanh(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c
^2*d*e^3) - 1/2*a^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(108) = 216$.
time = 0.38, size = 348, normalized size = 2.92

$$-\frac{8\,abdx+8\,abc-(b^2d^2x^2+2\,b^2cdx+b^2c^2-b^2)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2+4\,a^2-4\,((ab-b^2)d^2x^2+2\,(ab-b^2)cdx+(ab-b^2)c^2)\log(dx+c+1)-8\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(dx+c)+4\,((ab+b^2)d^2x^2+2\,(ab+b^2)cdx+(ab+b^2)c^2)\log(dx+c-1)+4\,(b^2dx+b^2c+ab)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{8\,((d^3x^2+2\,cd^2x+c^2d)\cosh(1)^3+3\,(d^3x^2+2\,cd^2x+c^2d)\cosh(1)^2\sinh(1)+3\,(d^3x^2+2\,cd^2x+c^2d)\cosh(1)\sinh(1)^2+(d^3x^2+2\,cd^2x+c^2d)\sinh(1)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b*d*x + 8*a*b*c - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - b^2)*log
(-(d*x + c + 1)/(d*x + c - 1))^2 + 4*a^2 - 4*((a*b - b^2)*d^2*x^2 + 2*(a*b
- b^2)*c*d*x + (a*b - b^2)*c^2)*log(d*x + c + 1) - 8*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*log(d*x + c) + 4*((a*b + b^2)*d^2*x^2 + 2*(a*b + b^2)*c*d*x
 + (a*b + b^2)*c^2)*log(d*x + c - 1) + 4*(b^2*d*x + b^2*c + a*b)*log(-(d*x
+ c + 1)/(d*x + c - 1)))/((d^3*x^2 + 2*c*d^2*x + c^2*d)*cosh(1)^3 + 3*(d^3*
x^2 + 2*c*d^2*x + c^2*d)*cosh(1)^2*sinh(1) + 3*(d^3*x^2 + 2*c*d^2*x + c^2*d
)*cosh(1)*sinh(1)^2 + (d^3*x^2 + 2*c*d^2*x + c^2*d)*sinh(1)^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. $2(100) = 200$.
time = 1.59, size = 1102, normalized size = 9.26

$$\begin{cases} -\frac{a^2}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abc^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{4abcdx\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abd^2x^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abdx}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2ab\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2b^2c^2\log\left(\frac{c}{d}+x\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2b^2c^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{b^2c^2\operatorname{atanh}^2(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2b^2c^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{4b^2cdx\log\left(\frac{c}{d}+x\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{4b^2cdx\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2b^2cdx\operatorname{atanh}^2(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{4b^2cdx\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2b^2c\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2b^2d^2x^2\log\left(\frac{c}{d}+x\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2b^2d^2x^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{b^2d^2x^2\operatorname{atanh}^2(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2b^2d^2x^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2b^2dx\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{b^2\operatorname{atanh}^2(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} & \text{for}\ d \neq 0 \\ \frac{x(a+b\operatorname{atanh}(c))^2}{c^3e^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*a
*b*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
 + 4*a*b*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**
3*x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*
a*b*d**2*x**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3
*x**2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2
*a*b*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) +
2*b**2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**
2) - 2*b**2*c**2*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d*
*3*e**3*x**2) + b**2*c**2*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*
x + 2*d**3*e**3*x**2) + 2*b**2*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**
2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d*e**3 + 4
*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x + 1/d)/(2*c**
2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c*d*x*atanh(c + d*x
)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*at
anh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*
c*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b
**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x
**2) - 2*b**2*d**2*x**2*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x
 + 2*d**3*e**3*x**2) + b**2*d**2*x**2*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*
c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*d**2*x**2*atanh(c + d*x)/(2*c**2
*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*d*x*atanh(c + d*x)/(
2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*atanh(c + d*x)**
2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a +
b*atanh(c))**2/(c**3*e**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(113) = 226.
time = 0.42, size = 375, normalized size = 3.15

$$\frac{1}{4}\left(\frac{(dx+c+1)b^2\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\left(\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2}+\frac{2(dx+c+1)d^2e^3}{dx+c-1}+d^2e^3\right)(dx+c-1)}+\frac{2\left(\frac{2(dx+c+1)ab}{dx+c-1}+\frac{(dx+c+1)b^2}{dx+c-1}+b^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2}+\frac{2(dx+c+1)d^2e^3}{dx+c-1}+d^2e^3}+\frac{4\left(\frac{(dx+c+1)a^2}{dx+c-1}+\frac{(dx+c+1)ab}{dx+c-1}+ab\right)}{\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2}+\frac{2(dx+c+1)d^2e^3}{dx+c-1}+d^2e^3}+\frac{2b^2\log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{d^2e^3}-\frac{2b^2\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2e^3}\right)((c+1)d-(c-1)d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] 1/4*((d*x + c + 1)*b^2*log(-(d*x + c + 1)/(d*x + c - 1))^2/(((d*x + c + 1)^
2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3
)*(d*x + c - 1)) + 2*(2*(d*x + c + 1)*a*b/(d*x + c - 1) + (d*x + c + 1)*b^2
/(d*x + c - 1) + b^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^2*d^
2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3) +
4*((d*x + c + 1)*a^2/(d*x + c - 1) + (d*x + c + 1)*a*b/(d*x + c - 1) + a*b)
/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x +
c - 1) + d^2*e^3) + 2*b^2*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d^2*e^3) -
 2*b^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^3))*((c + 1)*d - (c - 1)*d)
```

Mupad [B]
time = 2.42, size = 776, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] log(1 - d*x - c)^2*(b^2/(8*d*e^3) - b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8
*c*d*e^3*x))) + log(c + d*x + 1)^2*(b^2/(8*d*e^3) - b^2/(8*d^2*e^3*(2*c*x +
 d*x^2 + c^2/d))) + log(1 - d*x - c)*(log(c + d*x + 1)*(b^2/(2*d*(2*c^2*e^3
 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x)) - (b^2*(c^2 + d^2*x^2 + 2*c*d*x))/(2*d*(2*
c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x))) + b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*
x^2 + 8*c*d*e^3*x)) + (b*(4*a - b))/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d
*e^3*x)) - (b^2*(x*(4*c*d - d + d*(2*c - 1)) - c + c^2 + c*(2*c - 1) + 3*d^
2*x^2 + 1))/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d*e^3*x)) + (b^2*(x*(2*d*
e^3 + d*(4*c*e^3 + 2*e^3) + 8*c*d*e^3) + 2*c*e^3 + 2*e^3 + c*(4*c*e^3 + 2*e
^3) + 2*c^2*e^3 + 6*d^2*e^3*x^2))/(4*d*e^3*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c
*d*e^3*x))) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^2*e^3*x^2 + 2*c*
d*e^3*x) - (log(c + d*x + 1)*(x*((2*b^2*c + b^2)/(4*d*e^3) + (b^2*c)/(4*d*e
^3) - (b^2*(3*c - 1))/(4*d*e^3)) + (4*a*b + b^2*c + b^2 + b^2*c^2)/(8*d^2*e
^3) - (b^2*((c^2 - c + 1)/(2*d) + (c*(2*c - 1))/(2*d)))/(4*d*e^3) + (c*(2*b
^2*c + b^2))/(8*d^2*e^3)))/(2*c*x + d*x^2 + c^2/d) + (b^2*log(c + d*x))/(d*
e^3) - (log(c + d*x - 1)*(a*b + b^2))/(2*d*e^3) + (log(c + d*x + 1)*(a*b -
b^2))/(2*d*e^3)
```

3.21 $\int \frac{(a+b\tanh^{-1}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=180

$$-\frac{b^2}{3de^4(c+dx)}+\frac{b^2\tanh^{-1}(c+dx)}{3de^4}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4}-\frac{(a+b\tanh^{-1}(c+}{3de^4(c+dx}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)+1/3*b^2*arctanh(d*x+c)/d/e^4-1/3*b*(a+b*arctanh(d*x+
c))/d/e^4/(d*x+c)^2+1/3*(a+b*arctanh(d*x+c))^2/d/e^4-1/3*(a+b*arctanh(d*x+c
))^2/d/e^4/(d*x+c)^3+2/3*b*(a+b*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^4-1/3
*b^2*polylog(2,-1+2/(d*x+c+1))/d/e^4
```

Rubi [A]
time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6242, 12, 6037, 6129, 331, 212, 6135, 6079, 2497}

$$-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4}+\frac{2b\log\left(2-\frac{2}{c+dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4}-\frac{b^2\text{Li}_2\left(\frac{2}{c+dx+1}-1\right)}{3de^4}-\frac{b^2}{3de^4(c+dx)}+\frac{b^2\tanh^{-1}(c+dx)}{3de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*b^2/(d*e^4*(c + d*x)) + (b^2*ArcTanh[c + d*x])/(3*d*e^4) - (b*(a + b*A
rcTanh[c + d*x]))/(3*d*e^4*(c + d*x)^2) + (a + b*ArcTanh[c + d*x])^2/(3*d*e
^4) - (a + b*ArcTanh[c + d*x])^2/(3*d*e^4*(c + d*x)^3) + (2*b*(a + b*ArcTan
h[c + d*x])*Log[2 - 2/(1 + c + d*x)])/(3*d*e^4) - (b^2*PolyLog[2, -1 + 2/(1
 + c + d*x)])/(3*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^3(1-x^2)}\,dx, x, c+dx\right)}{3de^4} \\
&= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{3de^4} + \frac{(2}{} \\
&= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4} - \frac{\left(a+b\tanh^{-1}(c-\right.}{3de^4(c+dx} \\
&= -\frac{b^2}{3de^4(c+dx)} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{3de^4} - \frac{(}{} \\
&= -\frac{b^2}{3de^4(c+dx)} + \frac{b^2\tanh^{-1}(c+dx)}{3de^4} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} + \frac{\left(a+b\,\mathrm{t}\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 218, normalized size = 1.21

$$-\frac{a^2-ab\left(-2\tanh^{-1}(c+dx)+(c+dx)\left(-1+c^2+2cdx+d^2x^2+2(c+dx)^2\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)\right)\right)+b^2\left((c+dx)^2+(c+dx)^2\tanh^{-1}(c+dx)^2+(1-(c+dx)^2)\tanh^{-1}(c+dx)^2+(c+dx)\tanh^{-1}(c+dx)\left(1-(c+dx)^2-(c+dx)^2\tanh^{-1}(c+dx)-2(c+dx)^2\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)\right)+(c+dx)^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(c+dx)}\right)\right)}{3de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*(a^2 - a*b*(-2*ArcTanh[c + d*x] + (c + d*x)*(-1 + c^2 + 2*c*d*x + d^2*
x^2 + 2*(c + d*x)^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]])) + b^2*((c + d*x)
^2 + (c + d*x)^2*ArcTanh[c + d*x]^2 + (1 - (c + d*x)^2)*ArcTanh[c + d*x]^2
+ (c + d*x)*ArcTanh[c + d*x]*(1 - (c + d*x)^2 - (c + d*x)^2*ArcTanh[c + d*x
] - 2*(c + d*x)^2*Log[1 - E^(-2*ArcTanh[c + d*x])]) + (c + d*x)^3*PolyLog[2
, E^(-2*ArcTanh[c + d*x])]))/(d*e^4*(c + d*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(166) = 332$.
time = 1.75, size = 427, normalized size = 2.37

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arctanh}(dx+c)^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{3e^4}-\frac{b^2\operatorname{arctanh}(dx+c)}{3e^4(dx+c)^2}+\frac{2b^2\ln(dx+c)\operatorname{arctanh}(dx+c)}{3e^4}-\frac{b^2\operatorname{arctanh}(dx}{3}$
default	$-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arctanh}(dx+c)^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{3e^4}-\frac{b^2\operatorname{arctanh}(dx+c)}{3e^4(dx+c)^2}+\frac{2b^2\ln(dx+c)\operatorname{arctanh}(dx+c)}{3e^4}-\frac{b^2\operatorname{arctanh}(dx}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3-1/3*b^2/e^4/(d*x+c)^3*arctanh(d*x+c)^2-1/3*b^2/
e^4*arctanh(d*x+c)*ln(d*x+c+1)-1/3*b^2/e^4/(d*x+c)^2*arctanh(d*x+c)+2/3*b^2
/e^4*ln(d*x+c)*arctanh(d*x+c)-1/3*b^2/e^4*arctanh(d*x+c)*ln(d*x+c-1)+1/6*b^
2/e^4*ln(d*x+c+1)-1/3*b^2/e^4/(d*x+c)-1/6*b^2/e^4*ln(d*x+c-1)+1/3*b^2/e^4*d
ilog(1/2*d*x+1/2*c+1/2)+1/6*b^2/e^4*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/12*
b^2/e^4*ln(d*x+c-1)^2-1/6*b^2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/6*b^
2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+1/12*b^2/e^4*ln(d*x+c+1)
^2-1/3*b^2/e^4*dilog(d*x+c+1)-1/3*b^2/e^4*ln(d*x+c)*ln(d*x+c+1)-1/3*b^2/e^4
*dilog(d*x+c)-2/3*a*b/e^4/(d*x+c)^3*arctanh(d*x+c)-1/3*a*b/e^4*ln(d*x+c+1)-
1/3*a*b/e^4/(d*x+c)^2+2/3*a*b/e^4*ln(d*x+c)-1/3*a*b/e^4*ln(d*x+c-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(d*(e^(-4)*log(d*x + c + 1)/d^2 - 2*e^(-4)*log(d*x + c)/d^2 + e^(-4)*l
og(d*x + c - 1)/d^2 + 1/(d^4*x^2*e^4 + 2*c*d^3*x*e^4 + c^2*d^2*e^4)) + 2*ar
ctanh(d*x + c)/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4
))*a*b - 1/12*b^2*(log(-d*x - c + 1)^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c
^2*d^2*x*e^4 + c^3*d*e^4) + 3*integrate(-1/3*(3*(d*x + c - 1)*log(d*x + c +
 1)^2 + 2*(d*x - 3*(d*x + c - 1)*log(d*x + c + 1) + c)*log(-d*x - c + 1))/(
d^5*x^5*e^4 + (5*c*d^4 - d^4)*x^4*e^4 + 2*(5*c^2*d^3 - 2*c*d^3)*x^3*e^4 + 2
*(5*c^3*d^2 - 3*c^2*d^2)*x^2*e^4 + (5*c^4*d - 4*c^3*d)*x*e^4 + (c^5 - c^4)*
e^4), x)) - 1/3*a^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*
d*e^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*e^(-4)/(d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^2\operatorname{atanh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2ab\operatorname{atanh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*atanh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atanh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.22 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^5}\,dx$

Optimal. Leaf size=172

$$-\frac{b^2}{12de^5(c+dx)^2}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5}-\frac{\left(a+b\tan\right.}{4de^5(}$$

```
[Out] -1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctanh(d*x+c))/d/e^5/(d*x+c)^3-1/2*b*
(a+b*arctanh(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctanh(d*x+c))^2/d/e^5-1/4*(a+
b*arctanh(d*x+c))^2/d/e^5/(d*x+c)^4+2/3*b^2*ln(d*x+c)/d/e^5-1/3*b^2*ln(1-(d
*x+c)^2)/d/e^5
```

Rubi [A]
time = 0.19, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6242, 12, 6037, 6129, 272, 46, 36, 31, 29, 6095}

$$-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5}-\frac{b^2}{12de^5(c+dx)^2}+\frac{2b^2\log(c+dx)}{3de^5}-\frac{b^2\log\left(1-(c+dx)^2\right)}{3de^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*b^2/(d*e^5*(c + d*x)^2) - (b*(a + b*ArcTanh[c + d*x]))/(6*d*e^5*(c +
d*x)^3) - (b*(a + b*ArcTanh[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTanh
[c + d*x])^2/(4*d*e^5) - (a + b*ArcTanh[c + d*x])^2/(4*d*e^5*(c + d*x)^4) +
 (2*b^2*Log[c + d*x])/(3*d*e^5) - (b^2*Log[1 - (c + d*x)^2])/(3*d*e^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{e^5x^5}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^5}\,dx, x, c+dx\right)}{de^5}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^4(1-x^2)}\,dx, x, c+dx\right)}{2de^5}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{2de^5} + \frac{b\text{Subst}}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^-}{x^2}\right.}{2d}$$

$$= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tanh^{-1}(c+\right.}{4de^5}$$

$$= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)} + \frac{\left(a+b\tanh^{-1}(c+\right.}{4de^5}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)} +$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{6de^5(c+dx)^3} - \frac{b\left(a+b\tanh^{-1}(c+dx)\right)}{2de^5(c+dx)} +$$

Mathematica [A]
time = 0.16, size = 218, normalized size = 1.27

$$-\frac{\frac{3a^2}{(c+dx)^4} + \frac{2ab}{(c+dx)^3} + \frac{b^2}{(c+dx)^2} + \frac{6ab}{c+dx} + \frac{2b(3a+b(c+3c^3+dx+9c^2dx+9cd^2x^2+3d^3x^3))\tanh^{-1}(c+dx)}{(c+dx)^4} - \frac{3b^2(-1+c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4)\tanh^{-1}(c+dx)^2}{(c+dx)^4} + b(3a+4b)\log(1-c-dx) - 8b^2\log(c+dx) - (3a-4b)b\log(1+c+dx)}{12de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*((3*a^2)/(c + d*x)^4 + (2*a*b)/(c + d*x)^3 + b^2/(c + d*x)^2 + (6*a*b
)/(c + d*x) + (2*b*(3*a + b*(c + 3*c^3 + d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*
d^3*x^3))*ArcTanh[c + d*x])/(c + d*x)^4 - (3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*
c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTanh[c + d*x]^2)/(c + d*x)^4 + b*(3
*a + 4*b)*Log[1 - c - d*x] - 8*b^2*Log[c + d*x] - (3*a - 4*b)*b*Log[1 + c +
 d*x])/(d*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(158) = 316.
time = 0.79, size = 375, normalized size = 2.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a^2/e^5/(d*x+c)^4-1/4*b^2/e^5/(d*x+c)^4*arctanh(d*x+c)^2+1/4*b^2/
e^5*arctanh(d*x+c)*ln(d*x+c+1)-1/6*b^2/e^5/(d*x+c)^3*arctanh(d*x+c)-1/2*b^2
/e^5/(d*x+c)*arctanh(d*x+c)-1/4*b^2/e^5*arctanh(d*x+c)*ln(d*x+c-1)+1/8*b^2/
e^5*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/16*b^2/e^5*ln(d*x+c-1)^2+1/8*b^2/e^
5*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/8*b^2/e^5*ln(-1/2*d*x-1/2*c+1/2)*ln(
1/2*d*x+1/2*c+1/2)-1/16*b^2/e^5*ln(d*x+c+1)^2-1/3*b^2/e^5*ln(d*x+c+1)-1/12*
b^2/e^5/(d*x+c)^2+2/3*b^2/e^5*ln(d*x+c)-1/3*b^2/e^5*ln(d*x+c-1)-1/2*a*b/e^5
/(d*x+c)^4*arctanh(d*x+c)+1/4*a*b/e^5*ln(d*x+c+1)-1/6*a*b/e^5/(d*x+c)^3-1/2
*a*b/e^5/(d*x+c)-1/4*a*b/e^5*ln(d*x+c-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(151) = 302$.
time = 0.31, size = 582, normalized size = 3.38

$$-\frac{1}{12}\left(d\left(\frac{2\left(3d^2x^2+6cdx+3c^2+1\right)}{d^5x^3e^5+3cd^4x^2e^5+3c^2d^3xe^5+c^3d^2e^5}-\frac{3e^{(-5)}\log\left(dx+c+1\right)}{d^2}+\frac{3e^{(-5)}\log\left(dx+c-1\right)}{d^2}\right)+\frac{6\,\mathrm{artanh}\left(dx+c\right)}{d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5}\right)ab-\frac{1}{48}\left(d^2\left(\frac{3\left(d^2x^2+2cdx+c^2\right)\log\left(dx+c+1\right)^2+3\left(d^2x^2+2cdx+c^2\right)\log\left(dx+c-1\right)^2+2\left(8d^2x^2+16cdx+8c^2-3\left(d^2x^2+2cdx+c^2\right)\log\left(dx+c-1\right)\right)\log\left(dx+c+1\right)+16\left(d^2x^2+2cdx+c^2\right)\log\left(dx+c-1\right)+4}{d^5x^2e^5+2cd^4xe^5+c^2d^3e^5}-\frac{32e^{(-5)}\log\left(dx+c\right)}{d^3}\right)+4d\left(\frac{2\left(3d^2x^2+6cdx+3c^2+1\right)}{d^5x^3e^5+3cd^4x^2e^5+3c^2d^3xe^5+c^3d^2e^5}-\frac{3e^{(-5)}\log\left(dx+c+1\right)}{d^2}+\frac{3e^{(-5)}\log\left(dx+c-1\right)}{d^2}\right)\mathrm{artanh}\left(dx+c\right)\right)b^2-\frac{b^2\,\mathrm{artanh}\left(dx+c\right)^2}{4\left(d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5\right)}-\frac{a^2}{4\left(d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*x^3*e^5 + 3*c*d^4*x^2*e^
5 + 3*c^2*d^3*x*e^5 + c^3*d^2*e^5) - 3*e^(-5)*log(d*x + c + 1)/d^2 + 3*e^(-
5)*log(d*x + c - 1)/d^2) + 6*arctanh(d*x + c)/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^
5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c^4*d*e^5))*a*b - 1/48*(d^2*((3*(
d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c + 1)^2 + 3*(d^2*x^2 + 2*c*d*x + c^2)*l
og(d*x + c - 1)^2 + 2*(8*d^2*x^2 + 16*c*d*x + 8*c^2 - 3*(d^2*x^2 + 2*c*d*x
+ c^2)*log(d*x + c - 1))*log(d*x + c + 1) + 16*(d^2*x^2 + 2*c*d*x + c^2)*lo
g(d*x + c - 1) + 4)/(d^5*x^2*e^5 + 2*c*d^4*x*e^5 + c^2*d^3*e^5) - 32*e^(-5)
*log(d*x + c)/d^3) + 4*d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*x^3*e^5
+ 3*c*d^4*x^2*e^5 + 3*c^2*d^3*x*e^5 + c^3*d^2*e^5) - 3*e^(-5)*log(d*x + c +
 1)/d^2 + 3*e^(-5)*log(d*x + c - 1)/d^2)*arctanh(d*x + c))*b^2 - 1/4*b^2*ar
ctanh(d*x + c)^2/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3
*d^2*x*e^5 + c^4*d*e^5) - 1/4*a^2/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^
3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c^4*d*e^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(151) = 302$.
time = 0.39, size = 789, normalized size = 4.59

$$-\frac{24abd^3x^3+24abc^3+4\left(18abc+b^2\right)d^2x^2+4b^2c^2+8abc+8\left(9abc^2+b^2c+ab\right)dx-3\left(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4-b^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2+12a^2-4\left(\left(3ab-4b^2\right)d^4x^4+4\left(3ab-4b^2\right)cd^3x^3+6\left(3ab-4b^2\right)c^2d^2x^2+4\left(3ab-4b^2\right)c^3dx+\left(3ab-4b^2\right)c^4\right)\log\left(dx+c+1\right)-32\left(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4\right)\log\left(dx+c\right)+4\left(\left(3ab+4b^2\right)d^4x^4+4\left(3ab+4b^2\right)cd^3x^3+6\left(3ab+4b^2\right)c^2d^2x^2+4\left(3ab+4b^2\right)c^3dx+\left(3ab+4b^2\right)c^4\right)\log\left(dx+c-1\right)+4\left(3b^2d^3x^3+9b^2cd^2x^2+3b^2c^3+b^2c+\left(9b^2c^2+b^2\right)dx+3ab\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{48\left(\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\cosh\left(1\right)^5+5\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\cosh\left(1\right)^4\sinh\left(1\right)+10\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\cosh\left(1\right)^3\sinh\left(1\right)^2+10\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\cosh\left(1\right)^2\sinh\left(1\right)^3+5\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\cosh\left(1\right)\sinh\left(1\right)^4+\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)\sinh\left(1\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*a*b*d^3*x^3 + 24*a*b*c^3 + 4*(18*a*b*c + b^2)*d^2*x^2 + 4*b^2*c^2
 + 8*a*b*c + 8*(9*a*b*c^2 + b^2*c + a*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3
*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*log(-(d*x + c + 1
)/(d*x + c - 1))^2 + 12*a^2 - 4*((3*a*b - 4*b^2)*d^4*x^4 + 4*(3*a*b - 4*b^2
)*c*d^3*x^3 + 6*(3*a*b - 4*b^2)*c^2*d^2*x^2 + 4*(3*a*b - 4*b^2)*c^3*d*x + (
3*a*b - 4*b^2)*c^4)*log(d*x + c + 1) - 32*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 +
6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d*x + c) + 4*((3*a*b + 4*b
^2)*d^4*x^4 + 4*(3*a*b + 4*b^2)*c*d^3*x^3 + 6*(3*a*b + 4*b^2)*c^2*d^2*x^2 +
 4*(3*a*b + 4*b^2)*c^3*d*x + (3*a*b + 4*b^2)*c^4)*log(d*x + c - 1) + 4*(3*b
^2*d^3*x^3 + 9*b^2*c*d^2*x^2 + 3*b^2*c^3 + b^2*c + (9*b^2*c^2 + b^2)*d*x +
3*a*b)*log(-(d*x + c + 1)/(d*x + c - 1)))/((d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*d
^3*x^2 + 4*c^3*d^2*x + c^4*d)*cosh(1)^5 + 5*(d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*
d^3*x^2 + 4*c^3*d^2*x + c^4*d)*cosh(1)^4*sinh(1) + 10*(d^5*x^4 + 4*c*d^4*x^
3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)*cosh(1)^3*sinh(1)^2 + 10*(d^5*x^4
+ 4*c*d^4*x^3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)*cosh(1)^2*sinh(1)^3 +
5*(d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)*cosh(1)*sin
h(1)^4 + (d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)*sinh
(1)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3516 vs. $2(148) = 296$.
time = 5.39, size = 3516, normalized size = 20.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**5,x)
```

```
[Out] Piecewise((-3*a**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**
5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 6*a*b*c**4*atanh(c + d*
x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d*
*4*e**5*x**3 + 12*d**5*e**5*x**4) + 24*a*b*c**3*d*x*atanh(c + d*x)/(12*c**4
*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**
3 + 12*d**5*e**5*x**4) - 6*a*b*c**3/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x +
 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 36*a*b
*c**2*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c
**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 18*a*b*c**2
*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*
d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 24*a*b*c*d**3*x**3*atanh(c + d*x)/(12
*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**
5*x**3 + 12*d**5*e**5*x**4) - 18*a*b*c*d**2*x**2/(12*c**4*d*e**5 + 48*c**3*
d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x
**4) - 2*a*b*c/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x*
*2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 6*a*b*d**4*x**4*atanh(c + d
*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d
```

```
**4*e**5*x**3 + 12*d**5*e**5*x**4) - 6*a*b*d**3*x**3/(12*c**4*d*e**5 + 48*c
**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e*
*5*x**4) - 2*a*b*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e
**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 6*a*b*atanh(c + d*x)/
(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*
e**5*x**3 + 12*d**5*e**5*x**4) + 8*b**2*c**4*log(c/d + x)/(12*c**4*d*e**5 +
 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d*
*5*e**5*x**4) - 8*b**2*c**4*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48*c**3*d*
*2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**
4) + 3*b**2*c**4*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x +
72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 8*b**2*
c**4*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e*
*5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 32*b**2*c**3*d*x*log(c
/d + x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48
*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 32*b**2*c**3*d*x*log(c/d + x + 1/d
)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**
4*e**5*x**3 + 12*d**5*e**5*x**4) + 12*b**2*c**3*d*x*atanh(c + d*x)**2/(12*c
**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*
x**3 + 12*d**5*e**5*x**4) + 32*b**2*c**3*d*x*atanh(c + d*x)/(12*c**4*d*e**5
 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*
d**5*e**5*x**4) - 6*b**2*c**3*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2
*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4)
 + 48*b**2*c**2*d**2*x**2*log(c/d + x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*
x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 48*
b**2*c**2*d**2*x**2*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*
x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 18*
b**2*c**2*d**2*x**2*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x
 + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 48*b
**2*c**2*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 7
2*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 18*b**2*
c**2*d*x*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**
3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - b**2*c**2/(12*c**4
*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**
3 + 12*d**5*e**5*x**4) + 32*b**2*c*d**3*x**3*log(c/d + x)/(12*c**4*d*e**5 +
 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d*
*5*e**5*x**4) - 32*b**2*c*d**3*x**3*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48
*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*
e**5*x**4) + 12*b**2*c*d**3*x**3*atanh(c + d*x)**2/(12*c**4*d*e**5 + 48*c**
3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5
*x**4) + 32*b**2*c*d**3*x**3*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*
e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4)
- 18*b**2*c*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x
+ 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 2*b**
2*c*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48
*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 2*b**2*c*atanh(c + d*x)/(12*c**4*d
```

```
*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3
+ 12*d**5*e**5*x**4) + 8*b**2*d**4*x**4*log(c/d + x)/(12*c**4*d*e**5 + 48*c
**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e*
*5*x**4) - 8*b**2*d**4*x**4*log(c/d + x + 1/d)/(12*c**4*d*e**5 + 48*c**3*d*
*2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(158) = 316$.
time = 0.43, size = 730, normalized size = 4.24

$$\frac{1}{12}\left((c+1)d-(c-1)d\right)\left(\frac{3\left(\frac{(dx+c+1)^3b^2}{(dx+c-1)^3}+\frac{(dx+c+1)b^2}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\frac{(dx+c+1)^4d^2e^5}{(dx+c-1)^4}+\frac{4(dx+c+1)^3d^2e^5}{(dx+c-1)^3}+\frac{6(dx+c+1)^2d^2e^5}{(dx+c-1)^2}+\frac{4(dx+c+1)d^2e^5}{dx+c-1}+d^2e^5}+\frac{2\left(\frac{6(dx+c+1)^3ab}{(dx+c-1)^3}+\frac{6(dx+c+1)ab}{dx+c-1}+\frac{3(dx+c+1)^3b^2}{(dx+c-1)^3}+\frac{6(dx+c+1)^2b^2}{(dx+c-1)^2}+\frac{5(dx+c+1)b^2}{dx+c-1}+2b^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2e^5}{(dx+c-1)^4}+\frac{4(dx+c+1)^3d^2e^5}{(dx+c-1)^3}+\frac{6(dx+c+1)^2d^2e^5}{(dx+c-1)^2}+\frac{4(dx+c+1)d^2e^5}{dx+c-1}+d^2e^5}+\frac{2\left(\frac{6(dx+c+1)^3a^2}{(dx+c-1)^3}+\frac{6(dx+c+1)a^2}{dx+c-1}+\frac{6(dx+c+1)^3ab}{(dx+c-1)^3}+\frac{12(dx+c+1)^2ab}{(dx+c-1)^2}+\frac{10(dx+c+1)ab}{dx+c-1}+4ab+\frac{(dx+c+1)^3b^2}{(dx+c-1)^3}+\frac{2(dx+c+1)^2b^2}{(dx+c-1)^2}+\frac{(dx+c+1)b^2}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2e^5}{(dx+c-1)^4}+\frac{4(dx+c+1)^3d^2e^5}{(dx+c-1)^3}+\frac{6(dx+c+1)^2d^2e^5}{(dx+c-1)^2}+\frac{4(dx+c+1)d^2e^5}{dx+c-1}+d^2e^5}+\frac{4b^2\log\left(-\frac{dx+c+1}{dx+c-1}-1\right)}{d^2e^5}-\frac{4b^2\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2e^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] 1/12*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b^2/(d*x + c - 1)^3 + (d*x
 + c + 1)*b^2/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))^2/((d*x + c
+ 1)^4*d^2*e^5/(d*x + c - 1)^4 + 4*(d*x + c + 1)^3*d^2*e^5/(d*x + c - 1)^3
+ 6*(d*x + c + 1)^2*d^2*e^5/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^5/(d*x
+ c - 1) + d^2*e^5) + 2*(6*(d*x + c + 1)^3*a*b/(d*x + c - 1)^3 + 6*(d*x + c
 + 1)*a*b/(d*x + c - 1) + 3*(d*x + c + 1)^3*b^2/(d*x + c - 1)^3 + 6*(d*x +
c + 1)^2*b^2/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b^2/(d*x + c - 1) + 2*b^2)*l
og(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^2*e^5/(d*x + c - 1)^4 +
 4*(d*x + c + 1)^3*d^2*e^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2*e^5/(d*x
 + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^5/(d*x + c - 1) + d^2*e^5) + 2*(6*(d*x
+ c + 1)^3*a^2/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a^2/(d*x + c - 1) + 6*(d*x
 + c + 1)^3*a*b/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*a*b/(d*x + c - 1)^2 +
10*(d*x + c + 1)*a*b/(d*x + c - 1) + 4*a*b + (d*x + c + 1)^3*b^2/(d*x + c -
 1)^3 + 2*(d*x + c + 1)^2*b^2/(d*x + c - 1)^2 + (d*x + c + 1)*b^2/(d*x + c
- 1))/((d*x + c + 1)^4*d^2*e^5/(d*x + c - 1)^4 + 4*(d*x + c + 1)^3*d^2*e^5/
(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2*e^5/(d*x + c - 1)^2 + 4*(d*x + c +
1)*d^2*e^5/(d*x + c - 1) + d^2*e^5) + 4*b^2*log(-(d*x + c + 1)/(d*x + c - 1
) - 1)/(d^2*e^5) - 4*b^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^5))
```

Mupad [B]
time = 3.42, size = 2746, normalized size = 15.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^5,x)
```

```
[Out] log(1 - d*x - c)^2*(b^2/(16*d*e^5) - b^2/(4*d*(4*c^4*e^5 + 4*d^4*e^5*x^4 +
16*c*d^3*e^5*x^3 + 24*c^2*d^2*e^5*x^2 + 16*c^3*d*e^5*x))) + log(c + d*x + 1
)^2*(b^2/(16*d*e^5) - b^2/(16*d^2*e^5*(4*c^3*x + c^4/d + d^3*x^4 + 6*c^2*d*
x^2 + 4*c*d^2*x^3))) + log(1 - d*x - c)*(log(c + d*x + 1)*(b^2/(4*d*(2*c^4*
```

```
e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*c^3*d*e^5*x)
) - (b^2*(c^4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x))/(4*d*(2
*c^4*e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*c^3*d*e
^5*x))) + (3*b^2)/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 14
4*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) + (3*b*(8*a - b))/(4*d*(24*c^4*e^5 + 2
4*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) -
 (b^2*(c*(2*c - 3*c^2 + 4*c^3 + c*(6*c^2 - 3*c + c*(12*c - 3) + 1) - 1) - 3
*c + x^2*(d*(2*d - 6*c*d + 12*c^2*d + d*(6*c^2 - 3*c + c*(12*c - 3) + 1) +
c*(24*c*d - 3*d + d*(12*c - 3))) - 9*c*d^2 + c*(30*c*d^2 - 3*d^2 + d*(24*c*
d - 3*d + d*(12*c - 3))) + 3*d^2 + 18*c^2*d^2) + x*(d*(2*c - 3*c^2 + 4*c^3
+ c*(6*c^2 - 3*c + c*(12*c - 3) + 1) - 1) - 3*d + 6*c*d + c*(2*d - 6*c*d +
12*c^2*d + d*(6*c^2 - 3*c + c*(12*c - 3) + 1) + c*(24*c*d - 3*d + d*(12*c -
 3))) - 9*c^2*d + 12*c^3*d) + 3*c^2 - 3*c^3 + 3*c^4 + 25*d^4*x^4 + x^3*(34*
c*d^3 + d*(30*c*d^2 - 3*d^2 + d*(24*c*d - 3*d + d*(12*c - 3))) - 3*d^3) + 3
))/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x
^2 + 96*c^3*d*e^5*x)) + (b^2*(c*(c*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5)
+ 12*c^2*e^5) + 4*c*e^5 + 2*e^5 + 6*c^2*e^5 + 8*c^3*e^5) + x*(d*(c*(6*c*e^5
 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*c*e^5 + 2*e^5 + 6*c^2*e^5
 + 8*c^3*e^5) + 6*d*e^5 + c*(c*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5
) + d*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*d*e^5 + 24*
c^2*d*e^5 + 12*c*d*e^5) + 18*c^2*d*e^5 + 24*c^3*d*e^5 + 12*c*d*e^5) + x^3*(
d*(d*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5) + 6*d^2*e^5 + 60*c*d^2*e
^5) + 6*d^3*e^5 + 68*c*d^3*e^5) + 6*c*e^5 + 6*e^5 + 6*c^2*e^5 + 6*c^3*e^5 +
 6*c^4*e^5 + x^2*(c*(d*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5) + 6*d^
2*e^5 + 60*c*d^2*e^5) + d*(c*(6*d*e^5 + d*(24*c*e^5 + 6*e^5) + 48*c*d*e^5)
+ d*(6*c*e^5 + 2*e^5 + c*(24*c*e^5 + 6*e^5) + 12*c^2*e^5) + 4*d*e^5 + 24*c^
2*d*e^5 + 12*c*d*e^5) + 6*d^2*e^5 + 18*c*d^2*e^5 + 36*c^2*d^2*e^5) + 50*d^4
*e^5*x^4))/(8*d*e^5*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c
^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x))) - (x^2*((b^2*d)/2 + 9*a*b*c*d) + x*(a*b
+ b^2*c + 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c + 6*a*b*c^3)/(2*d) + 3*a*
b*d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5*x
^2 + 24*c^3*d*e^5*x) - (log(c + d*x + 1)*(x^2*((3*b^2*c + b^2 + 6*b^2*c^2)/
(32*e^5) + d*(d*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b
^2))/(32*d^2*e^5)) + c*((4*b^2*c + b^2)/(16*d*e^5) + (b^2*c)/(8*d*e^5)) + (
3*b^2*c + b^2 + 6*b^2*c^2)/(48*d*e^5)) + c*(d*((4*b^2*c + b^2)/(16*d*e^5) +
 (b^2*c)/(8*d*e^5)) + (4*b^2*c + b^2)/(32*e^5) + (3*b^2*c)/(16*e^5)) - (b^2
*(d/4 - (3*c*d)/4 + (3*c^2*d)/2 + d*(d*((6*c^2 - 3*c + 1)/(12*d) + (c*(4*c
- 1))/(4*d)) - c/2 + c^2 + c*(3*c - 1/2) + 1/6) + c*((5*c*d)/2 - d/4 + d*(3
*c - 1/2))))/(8*d*e^5)) + x*(c*(d*((3*b^2*c + b^2 + 6*b^2*c^2)/(96*d^2*e^5)
 + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + c*((4*b^2*c + b^2)/(16*d*e^5) + (b^2
*c)/(8*d*e^5)) + (3*b^2*c + b^2 + 6*b^2*c^2)/(48*d*e^5)) + d*(c*((3*b^2*c +
 b^2 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + (2*b^2
*c + b^2 + 3*b^2*c^2 + 4*b^2*c^3)/(96*d^2*e^5)) + (2*b^2*c + b^2 + 3*b^2*c^
2 + 4*b^2*c^3)/(32*d*e^5) - (b^2*(c/2 + c*(d*((6*c^2 - 3*c + 1)/(12*d) + (c
*(4*c - 1))/(4*d)) - c/2 + c^2 + c*(3*c - 1/2) + 1/6) - (3*c^2)/4 + c^3 + d
```

```
*(c*((6*c^2 - 3*c + 1)/(12*d) + (c*(4*c - 1))/(4*d)) + (2*c - 3*c^2 + 4*c^3
 - 1)/(12*d)) - 1/4))/(8*d*e^5)) + x^3*(d*(d*((4*b^2*c + b^2)/(16*d*e^5) +
(b^2*c)/(8*d*e^5)) + (4*b^2*c + b^2)/(32*e^5) + (3*b^2*c)/(16*e^5)) + (b^2*
d + 4*b^2*c*d)/(32*e^5) - (b^2*((17*c*d^2)/6 - d^2/4 + d*((5*c*d)/2 - d/4 +
 d*(3*c - 1/2))))/(8*d*e^5) + (11*b^2*c*d)/(48*e^5)) + c*(c*((3*b^2*c + b^2
 + 6*b^2*c^2)/(96*d^2*e^5) + (c*(4*b^2*c + b^2))/(32*d^2*e^5)) + (2*b^2*c +
 b^2 + 3*b^2*c^2 + 4*b^2*c^3)/(96*d^2*e^5)) + (8*a*b + b^2*c + b^2 + b^2*c^
2 + b^2*c^3 + b^2*c^4)/(32*d^2*e^5) - (b^2*(c*(c*((6*c^2 - 3*c + 1)/(12*d)
+ (c*(4*c - 1))/(4*d)) + (2*c - 3*c^2 + 4*c^3 - 1)/(12*d)) + (c^2 - c - c^3
 + c^4 + 1)/(4*d)))/(8*d*e^5)))/(4*c^3*x + c^4/d + d^3*x^4 + 6*c^2*d*x^2 +
4*c*d^2*x^3) + (2*b^2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1)*(3*a*b +
4*b^2))/(12*d*e^5) + (log(c + d*x + 1)*(3*a*b - 4*b^2))/(12*d*e^5)
```

3.23 $\int (ce+dex)^2 \left(a+b\tanh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=263

$$ab^2e^2x+\frac{b^3e^2(c+dx)\tanh^{-1}(c+dx)}{d}-\frac{be^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+$$

```
[Out] a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctanh(d*x+c)/d-1/2*b*e^2*(a+b*arctanh(d*x+c))
^2/d+1/2*b*e^2*(d*x+c)^2*(a+b*arctanh(d*x+c))^2/d+1/3*e^2*(a+b*arctanh(d*x+
c))^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctanh(d*x+c))^3/d-b*e^2*(a+b*arctanh(d*x+c
))^2*ln(2/(-d*x-c+1))/d+1/2*b^3*e^2*ln(1-(d*x+c)^2)/d-b^2*e^2*(a+b*arctanh(
d*x+c))*polylog(2,1-2/(-d*x-c+1))/d+1/2*b^3*e^2*polylog(3,1-2/(-d*x-c+1))/d
```

Rubi [A]
time = 0.34, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6242, 12, 6037, 6127, 6021, 266, 6095, 6131, 6055, 6205, 6745}

$$-\frac{b^2e^2\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d}+ab^2e^2x-\frac{be^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d}+\frac{e^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d}-\frac{be^2\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}+\frac{b^3e^2\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d}+\frac{b^3e^2\log\left(1-(c+dx)^2\right)}{2d}+\frac{b^3e^2(c+dx)\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTanh[c + d*x])/d - (b*e^2*(a + b*ArcTan
h[c + d*x])^2)/(2*d) + (b*e^2*(c + d*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*d)
 + (e^2*(a + b*ArcTanh[c + d*x])^3)/(3*d) + (e^2*(c + d*x)^3*(a + b*ArcTanh
[c + d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)
])/d + (b^3*e^2*Log[1 - (c + d*x)^2])/(2*d) - (b^2*e^2*(a + b*ArcTanh[c + d
*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d + (b^3*e^2*PolyLog[3, 1 - 2/(1 - c
- d*x)])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\tanh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\tanh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2\text{Subst}\left(\int x^2\left(a+b\tanh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d}-\frac{(be^2)\,\text{Subst}\left(\int\frac{x^3(a+b\tan}{1-x}\right.}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d}+\frac{(be^2)\,\text{Subst}\left(\int x\left(a+b\,\mathrm{t}\right.\right.}{}$$

$$=\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{e^2\left(a+b\tanh^{-1}(c+d\right.}{3d}$$

$$=\frac{be^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{e^2\left(a+b\tanh^{-1}(c+d\right.}{3d}$$

$$=ab^2e^2x-\frac{be^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{be^2(c+dx)^2\left(a+b\tan\right.}{2d}$$

$$=ab^2e^2x+\frac{b^3e^2(c+dx)\tanh^{-1}(c+dx)}{d}-\frac{be^2\left(a+b\tanh^{-1}(c+\right.}{2d}$$

$$=ab^2e^2x+\frac{b^3e^2(c+dx)\tanh^{-1}(c+dx)}{d}-\frac{be^2\left(a+b\tanh^{-1}(c+\right.}{2d}$$

Mathematica [A]
time = 0.44, size = 336, normalized size = 1.28

$$\frac{e^2\left(3a^2b(c+dx)^2+2a^3(c+dx)^3+6a^2b(c+dx)^3\tanh^{-1}(c+dx)+3a^2b\log\left(1-(c+dx)^2\right)+6ab^2\left(c+dx-\tanh^{-1}(c+dx)+(c+dx)^2\tanh^{-1}(c+dx)-\tanh^{-1}(c+dx)^2+(c+dx)^3\tanh^{-1}(c+dx)^2-2\tanh^{-1}(c+dx)\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)\right)+b^3\left(6(c+dx)\tanh^{-1}(c+dx)-3\left(1-(c+dx)^2\right)\tanh^{-1}(c+dx)^2-2\tanh^{-1}(c+dx)^3+2(c+dx)\tanh^{-1}(c+dx)^3-2(c+dx)\left(1-(c+dx)^2\right)\tanh^{-1}(c+dx)^3-6\tanh^{-1}(c+dx)^2\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)-6\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+6\tanh^{-1}(c+dx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)+3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(c+dx)}\right)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (e^2*(3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTanh
[c + d*x] + 3*a^2*b*Log[1 - (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTanh[c + d
*x] + (c + d*x)^2*ArcTanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*ArcTa
nh[c + d*x]^2 - 2*ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])] + PolyL
og[2, -E^(-2*ArcTanh[c + d*x])]) + b^3*(6*(c + d*x)*ArcTanh[c + d*x] - 3*(1
 - (c + d*x)^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]^3 + 2*(c + d*x)*Arc
Tanh[c + d*x]^3 - 2*(c + d*x)*(1 - (c + d*x)^2)*ArcTanh[c + d*x]^3 - 6*ArcT
anh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*Log[1/Sqrt[1 - (c + d*x
)^2]] + 6*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + 3*PolyLog
[3, -E^(-2*ArcTanh[c + d*x])])))/(6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.65, size = 1345, normalized size = 5.11

method	result	size
derivativedivides	Expression too large to display	1345
default	Expression too large to display	1345

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*I*e^2*b^3*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)
))^3+1/3*e^2*(d*x+c)^3*a^3+1/4*I*e^2*b^3*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x
+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-
(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))-1/4*I*e^2*b^3*Pi*arctanh(d*x
+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3-1/4
*I*e^2*b^3*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^3+1/2*I*e^
2*b^3*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+e^2*a*b^2
*(d*x+c)^3*arctanh(d*x+c)^2+e^2*a*b^2*(d*x+c)^2*arctanh(d*x+c)+e^2*a*b^2*ar
ctanh(d*x+c)*ln(d*x+c-1)+e^2*a*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/2*e^2*a*b^2
*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/2*e^2*a*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(
d*x+c+1)-1/2*e^2*a*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+e^2*a^2
*b*(d*x+c)^3*arctanh(d*x+c)-1/2*I*e^2*b^3*Pi*arctanh(d*x+c)^2+1/2*e^2*a^2*b
*ln(d*x+c-1)+1/2*e^2*a^2*b*ln(d*x+c+1)+e^2*a*b^2*(d*x+c)+1/2*e^2*a*b^2*ln(d
*x+c-1)-1/2*e^2*a*b^2*ln(d*x+c+1)-e^2*a*b^2*dilog(1/2*d*x+1/2*c+1/2)+1/4*e^
2*a*b^2*ln(d*x+c-1)^2-1/4*e^2*a*b^2*ln(d*x+c+1)^2+1/3*e^2*b^3*(d*x+c)^3*arc
tanh(d*x+c)^3+1/2*e^2*b^3*(d*x+c)^2*arctanh(d*x+c)^2+1/2*e^2*b^3*arctanh(d*
x+c)^2*ln(d*x+c-1)+1/2*e^2*b^3*arctanh(d*x+c)^2*ln(d*x+c+1)-e^2*b^3*arctanh
(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-e^2*b^3*arctanh(d*x+c)*polylog(
2,-(d*x+c+1)^2/(1-(d*x+c)^2))-e^2*b^3*ln(2)*arctanh(d*x+c)^2+e^2*b^3*(d*x+c
)*arctanh(d*x+c)+1/2*e^2*(d*x+c)^2*a^2*b+1/2*e^2*b^3*polylog(3,-(d*x+c+1)^2
```

```
/(1-(d*x+c)^2))+1/3*e^2*b^3*arctanh(d*x+c)^3-1/2*e^2*b^3*arctanh(d*x+c)^2+e
^2*b^3*arctanh(d*x+c)-e^2*b^3*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))-1/2*I*e^2*b^3
*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2*csgn(I*(d*x+c+1)/(
1-(d*x+c)^2)^(1/2))-1/4*I*e^2*b^3*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((
d*x+c)^2-1))*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2-1/4*I*e^2*b^3*Pi*arcta
nh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+
c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+1/4*I*e^2*b^3*Pi*arctanh(d*x+c)^2*
csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(d
*x+c+1)^2/((d*x+c)^2-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*x^3*e^2 + a^3*c*d*x^2*e^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*(2*
x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c
- 1)/d^3))*a^2*b*c*d*e^2 + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)
/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c -
1)*log(d*x + c - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*x*e^2 + 3/2*(2*(d*x + c)*
arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*c^2*e^2/d - 1/24*((b^3*d^3*
x^3*e^2 + 3*b^3*c*d^2*x^2*e^2 + 3*b^3*c^2*d*x*e^2 + (c^3 - 1)*b^3*e^2)*log(
-d*x - c + 1)^3 - 3*(2*a*b^2*d^3*x^3*e^2 + (6*a*b^2*c*d^2 + b^3*d^2)*x^2*e^
2 + 2*(3*a*b^2*c^2*d + b^3*c*d)*x*e^2 + (b^3*d^3*x^3*e^2 + 3*b^3*c*d^2*x^2*
e^2 + 3*b^3*c^2*d*x*e^2 + (c^3 + 1)*b^3*e^2)*log(d*x + c + 1))*log(-d*x - c
 + 1)^2)/d - integrate(-1/8*((b^3*d^3*x^3*e^2 + (3*c*d^2 - d^2)*b^3*x^2*e^2
 + (3*c^2*d - 2*c*d)*b^3*x*e^2 + (c^3 - c^2)*b^3*e^2)*log(d*x + c + 1)^3 +
6*(a*b^2*d^3*x^3*e^2 + (3*c*d^2 - d^2)*a*b^2*x^2*e^2 + (3*c^2*d - 2*c*d)*a*
b^2*x*e^2 + (c^3 - c^2)*a*b^2*e^2)*log(d*x + c + 1)^2 - (4*a*b^2*d^3*x^3*e^
2 + 2*(6*a*b^2*c*d^2 + b^3*d^2)*x^2*e^2 + 4*(3*a*b^2*c^2*d + b^3*c*d)*x*e^2
 + 3*(b^3*d^3*x^3*e^2 + (3*c*d^2 - d^2)*b^3*x^2*e^2 + (3*c^2*d - 2*c*d)*b^3
*x*e^2 + (c^3 - c^2)*b^3*e^2)*log(d*x + c + 1)^2 + 2*((6*a*b^2*d^3 + b^3*d^
3)*x^3*e^2 + 3*(b^3*c*d^2 + 2*(3*c*d^2 - d^2)*a*b^2)*x^2*e^2 + 3*(b^3*c^2*d
 + 2*(3*c^2*d - 2*c*d)*a*b^2)*x*e^2 + (6*(c^3 - c^2)*a*b^2 + (c^3 + 1)*b^3)
*e^2)*log(d*x + c + 1))*log(-d*x - c + 1))/(d*x + c - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctanh(d*x + c)^3*e^2 + 3*(
a*b^2*d^2*x^2 + 2*a*b^2*c*d*x + a*b^2*c^2)*arctanh(d*x + c)^2*e^2 + 3*(a^2*
b*d^2*x^2 + 2*a^2*b*c*d*x + a^2*b*c^2)*arctanh(d*x + c)*e^2 + (a^3*d^2*x^2
+ 2*a^3*c*d*x + a^3*c^2)*e^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int a^3c^2\,dx + \int a^3d^2x^2\,dx + \int b^3c^2\operatorname{atanh}^3(c+dx)\,dx + \int 3ab^2c^2\operatorname{atanh}^2(c+dx)\,dx + \int 3a^2bc^2\operatorname{atanh}(c+dx)\,dx + \int 2a^3cdx\,dx + \int b^3d^2x^2\operatorname{atanh}^3(c+dx)\,dx + \int 3ab^2d^2x^2\operatorname{atanh}^2(c+dx)\,dx + \int 3a^2bd^2x^2\operatorname{atanh}(c+dx)\,dx + \int 2b^3cdx\operatorname{atanh}^3(c+dx)\,dx + \int 6ab^2cdx\operatorname{atanh}^2(c+dx)\,dx + \int 6a^2bcdx\operatorname{atanh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*
c**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atanh(c + d*x)**2, x) +
 Integral(3*a**2*b*c**2*atanh(c + d*x), x) + Integral(2*a**3*c*d*x, x) + In
tegral(b**3*d**2*x**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*a
tanh(c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atanh(c + d*x), x) + Int
egral(2*b**3*c*d*x*atanh(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atanh(c
+ d*x)**2, x) + Integral(6*a**2*b*c*d*x*atanh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arctanh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^2\,{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3, x)
```

3.24 $\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=160

$$\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}+\frac{e(c+dx)^2\,(\ldots}{}$$

```
[Out] 3/2*b*e*(a+b*arctanh(d*x+c))^2/d+3/2*b*e*(d*x+c)*(a+b*arctanh(d*x+c))^2/d-1
/2*e*(a+b*arctanh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))^3/d-3*b^
2*e*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-3/2*b^3*e*polylog(2,(-d*x-c-1)/
(-d*x-c+1))/d
```

Rubi [A]
time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6242, 12, 6037, 6127, 6021, 6131, 6055, 2449, 2352, 6095}

$$-\frac{3b^2e\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d}+\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}-\frac{3b^3e\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (3*b*e*(a + b*ArcTanh[c + d*x])^2)/(2*d) + (3*b*e*(c + d*x)*(a + b*ArcTanh[
c + d*x])^2)/(2*d) - (e*(a + b*ArcTanh[c + d*x])^3)/(2*d) + (e*(c + d*x)^2*
(a + b*ArcTanh[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTanh[c + d*x])*Log[2
/(1 - c - d*x)])/d - (3*b^3*e*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
```

```
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\tanh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\text{Subst}\left(\int x\left(a+b\tanh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}-\frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\tanh^{-}}{1-x^2}\right.}{2d}$$

$$=\frac{e(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}+\frac{(3be)\text{Subst}\left(\int (a+b\tan\right.}{2d}$$

$$=\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx\right.}{2d}$$

$$=\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx\right.}{2d}$$

$$=\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx\right.}{2d}$$

$$=\frac{3be\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{3be(c+dx)\left(a+b\tanh^{-1}(c+dx\right.}{2d}$$

Mathematica [A]
time = 0.76, size = 213, normalized size = 1.33

$$\frac{e\left(6b^2(-1+c+dx)(b+a(1+c+dx))\tanh^{-1}(c+dx)^2+2b^3(-1+c^2+2cdx+d^2x^2)\tanh^{-1}(c+dx)^3+6b\tanh^{-1}(c+dx)\left(a(2b(c+dx)+adx(2c+dx))-2b^2\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+a\left(2adx(3b+2ac+adx)-3ab(-1+c^2)\log(1-c-dx)+3ab(-1+c^2)\log(1+c+dx)-12b^2\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)\right)+6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (e*(6*b^2*(-1 + c + d*x)*(b + a*(1 + c + d*x))*ArcTanh[c + d*x]^2 + 2*b^3*(
-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^3 + 6*b*ArcTanh[c + d*x]*(a*
(2*b*(c + d*x) + a*d*x*(2*c + d*x)) - 2*b^2*Log[1 + E^(-2*ArcTanh[c + d*x])
]) + a*(2*a*d*x*(3*b + 2*a*c + a*d*x) - 3*a*b*(-1 + c^2)*Log[1 - c - d*x] +
```

```
3*a*b*(-1 + c^2)*Log[1 + c + d*x] - 12*b^2*Log[1/Sqrt[1 - (c + d*x)^2]]) +
6*b^3*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/(4*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.89, size = 6414, normalized size = 40.09

method	result	size
risch	Expression too large to display	1297
derivativedivides	Expression too large to display	6414
default	Expression too large to display	6414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(148) = 296.
time = 0.46, size = 628, normalized size = 3.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2*e + 3/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1
)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*d*e +
 a^3*c*x*e + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2
*b*c*e/d + 3/2*a*b^2*(c + 1)*e*log(d*x + c + 1)/d - 3/2*a*b^2*(c - 1)*e*log
(d*x + c - 1)/d + 3/2*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog
(1/2*d*x + 1/2*c + 1/2))*b^3*e/d + 1/16*(24*a*b^2*d*x*e*log(d*x + c + 1) +
(b^3*d^2*x^2*e + 2*b^3*c*d*x*e + (c^2 - 1)*b^3*e)*log(d*x + c + 1)^3 - (b^3
*d^2*x^2*e + 2*b^3*c*d*x*e + (c^2 - 1)*b^3*e)*log(-d*x - c + 1)^3 + 6*(a*b^
2*d^2*x^2*e + (2*a*b^2*c*d + b^3*d)*x*e + ((c^2 - 1)*a*b^2 + b^3*(c + 1))*e
)*log(d*x + c + 1)^2 + 3*(2*a*b^2*d^2*x^2*e + 2*(2*a*b^2*c*d + b^3*d)*x*e +
 2*((c^2 - 1)*a*b^2 + b^3*(c - 1))*e + (b^3*d^2*x^2*e + 2*b^3*c*d*x*e + (c^
2 - 1)*b^3*e)*log(d*x + c + 1))*log(-d*x - c + 1)^2 - 3*(8*a*b^2*d*x*e + (b
^3*d^2*x^2*e + 2*b^3*c*d*x*e + (c^2 - 1)*b^3*e)*log(d*x + c + 1)^2 + 4*(a*b
^2*d^2*x^2*e + (2*a*b^2*c*d + b^3*d)*x*e + ((c^2 - 1)*a*b^2 + b^3*(c + 1))*
e)*log(d*x + c + 1))*log(-d*x - c + 1))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d*x + b^3*c)*arctanh(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c)*a
rctanh(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arctanh(d*x + c)*e + (a^3*d*x
 + a^3*c)*e, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^{3}c\,dx + \int a^{3}dx\,dx + \int b^{3}c\operatorname{atanh}^{3}\left(c+dx\right)dx + \int 3ab^{2}c\operatorname{atanh}^{2}\left(c+dx\right)dx + \int 3a^{2}bc\operatorname{atanh}\left(c+dx\right)dx + \int b^{3}dx\operatorname{atanh}^{3}\left(c+dx\right)dx + \int 3ab^{2}dx\operatorname{atanh}^{2}\left(c+dx\right)dx + \int 3a^{2}bdx\operatorname{atanh}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atanh(c +
d*x)**3, x) + Integral(3*a*b**2*c*atanh(c + d*x)**2, x) + Integral(3*a**2*b
*c*atanh(c + d*x), x) + Integral(b**3*d*x*atanh(c + d*x)**3, x) + Integral(
3*a*b**2*d*x*atanh(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atanh(c + d*x),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arctanh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{atanh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3, x)
```

3.25 $\int \frac{(a+b\tanh^{-1}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=257

$$\frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de}-\frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{2de}+\frac{3b\left(a+b\right.}{}$$

```
[Out] -2*(a+b*arctanh(d*x+c))^3*arctanh(-1+2/(-d*x-c+1))/d/e-3/2*b*(a+b*arctanh(d
*x+c))^2*polylog(2,1-2/(-d*x-c+1))/d/e+3/2*b*(a+b*arctanh(d*x+c))^2*polylog
(2,-1+2/(-d*x-c+1))/d/e+3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2/(-d*x-c+
1))/d/e-3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,-1+2/(-d*x-c+1))/d/e-3/4*b^3
*polylog(4,1-2/(-d*x-c+1))/d/e+3/4*b^3*polylog(4,-1+2/(-d*x-c+1))/d/e
```

Rubi [A]
time = 0.35, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6242, 12, 6033, 6199, 6095, 6205, 6209, 6745}

$$\frac{3b^2\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{2de}-\frac{3b^2\text{Li}_3\left(\frac{2}{-c-dx+1}-1\right)\left(a+b\tanh^{-1}(c+dx)\right)}{2de}-\frac{3b\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de}+\frac{3b\text{Li}_2\left(\frac{2}{-c-dx+1}-1\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de}+\frac{2\tanh^{-1}\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^3}{de}-\frac{3b^3\text{Li}_4\left(1-\frac{2}{-c-dx+1}\right)}{4de}+\frac{3b^3\text{Li}_4\left(\frac{2}{-c-dx+1}-1\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTanh[c + d*x])^3*ArcTanh[1 - 2/(1 - c - d*x)])/(d*e) - (3*b*(a
 + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - 2/(1 - c - d*x)])/(2*d*e) + (3*b*(a
 + b*ArcTanh[c + d*x])^2*PolyLog[2, -1 + 2/(1 - c - d*x)])/(2*d*e) + (3*b^2
*(a + b*ArcTanh[c + d*x])*PolyLog[3, 1 - 2/(1 - c - d*x)])/(2*d*e) - (3*b^2
*(a + b*ArcTanh[c + d*x])*PolyLog[3, -1 + 2/(1 - c - d*x)])/(2*d*e) - (3*b^
3*PolyLog[4, 1 - 2/(1 - c - d*x)])/(4*d*e) + (3*b^3*PolyLog[4, -1 + 2/(1 -
c - d*x)])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{(6b)\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right.}{1\ldots}\right.}{\ldots}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right.}{1-\ldots}\right.}{d\ldots}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\ldots}{2de}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\ldots}{2de}$$

$$= \frac{2\left(a+b\tanh^{-1}(c+dx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\ldots}{2de}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 599, normalized size = 2.33

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (64*a^3*Log[c + d*x] + 192*a^2*b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x
)^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) - (96*I)*a^2*b*((-1/4*I)*
(Pi - (2*I)*ArcTanh[c + d*x])^2 + I*ArcTanh[c + d*x]^2 + (2*I)*ArcTanh[c +
d*x]*Log[1 - E^(-2*ArcTanh[c + d*x])] + (Pi - (2*I)*ArcTanh[c + d*x])*Log[1
 + E^(2*ArcTanh[c + d*x])] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2/Sqrt[1 - (
c + d*x)^2]] - (2*I)*ArcTanh[c + d*x]*Log[((2*I)*(c + d*x))/Sqrt[1 - (c + d
*x)^2]] - I*PolyLog[2, E^(-2*ArcTanh[c + d*x])] - I*PolyLog[2, -E^(2*ArcTan
h[c + d*x])]) + 8*a*b^2*(I*Pi^3 - 16*ArcTanh[c + d*x]^3 - 24*ArcTanh[c + d*
x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] + 24*ArcTanh[c + d*x]^2*Log[1 - E^(2*
ArcTanh[c + d*x])] + 24*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x]
)] + 24*ArcTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c + d*x])] + 12*PolyLog[3,
 -E^(-2*ArcTanh[c + d*x])] - 12*PolyLog[3, E^(2*ArcTanh[c + d*x])]) + b^3*(
```

```
Pi^4 - 32*ArcTanh[c + d*x]^4 - 64*ArcTanh[c + d*x]^3*Log[1 + E^(-2*ArcTanh[
c + d*x])] + 64*ArcTanh[c + d*x]^3*Log[1 - E^(2*ArcTanh[c + d*x])] + 96*Arc
Tanh[c + d*x]^2*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + 96*ArcTanh[c + d*x]^
2*PolyLog[2, E^(2*ArcTanh[c + d*x])] + 96*ArcTanh[c + d*x]*PolyLog[3, -E^(-
2*ArcTanh[c + d*x])] - 96*ArcTanh[c + d*x]*PolyLog[3, E^(2*ArcTanh[c + d*x]
)] + 48*PolyLog[4, -E^(-2*ArcTanh[c + d*x])] + 48*PolyLog[4, E^(2*ArcTanh[c
 + d*x])]))/(64*d*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.53, size = 1738, normalized size = 6.76

method	result	size
derivativedivides	Expression too large to display	1738
default	Expression too large to display	1738

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*I*b^3/e*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I/(1+(d*x+c+
1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1
-(d*x+c)^2)))*arctanh(d*x+c)^3-3/2*I*a*b^2/e*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x
+c)^2)-1))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2
)))^2*arctanh(d*x+c)^2-3/2*I*a*b^2/e*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)
))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*ar
ctanh(d*x+c)^2+1/2*I*b^3/e*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+
c+1)^2/(1-(d*x+c)^2)))^3*arctanh(d*x+c)^3-1/2*I*b^3/e*Pi*csgn(I*((d*x+c+1)^
2/(1-(d*x+c)^2)-1))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-
(d*x+c)^2)))^2*arctanh(d*x+c)^3-1/2*I*b^3/e*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*
x+c)^2)))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)
))^2*arctanh(d*x+c)^3+3/2*I*a*b^2/e*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)
/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*arctanh(d*x+c)^2+3/2*I*a*b^2/e*Pi*csgn(I*
((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I
*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*arctanh(d*x+c
)^2+3*a^2*b/e*ln(d*x+c)*arctanh(d*x+c)-3/2*a^2*b/e*ln(d*x+c)*ln(d*x+c+1)+3*
a*b^2/e*ln(d*x+c)*arctanh(d*x+c)^2-3*a*b^2/e*arctanh(d*x+c)*polylog(2,-(d*x
+c+1)^2/(1-(d*x+c)^2))-3*a*b^2/e*arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)
^2)-1)+3*a*b^2/e*arctanh(d*x+c)^2*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+6*a*b
^2/e*arctanh(d*x+c)*polylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3*a*b^2/e*arc
tanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+6*a*b^2/e*arctanh(d*x+c)*
polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+a^3/e*ln(d*x+c)-3/4*b^3/e*polylog(
4,-(d*x+c+1)^2/(1-(d*x+c)^2))+6*b^3/e*polylog(4,-(d*x+c+1)/(1-(d*x+c)^2)^(1
/2))+6*b^3/e*polylog(4,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*b^3/e*arctanh(d*x+c
)*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-3/2*a^2*b/e*dilog(d*x+c+1)-3/2*a
^2*b/e*dilog(d*x+c)+3/2*a*b^2/e*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))-6*a*b
^2/e*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*a*b^2/e*polylog(3,(d*x+c+1
```

```
)/(1-(d*x+c)^2)^(1/2))+b^3/e*ln(d*x+c)*arctanh(d*x+c)^3-b^3/e*arctanh(d*x+c
)^3*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)-3/2*b^3/e*arctanh(d*x+c)^2*polylog(2,-(
d*x+c+1)^2/(1-(d*x+c)^2))+3/2*b^3/e*arctanh(d*x+c)*polylog(3,-(d*x+c+1)^2/(
1-(d*x+c)^2))+b^3/e*arctanh(d*x+c)^3*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3*
b^3/e*arctanh(d*x+c)^2*polylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*b^3/e*ar
ctanh(d*x+c)*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+b^3/e*arctanh(d*x+c)
^3*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3*b^3/e*arctanh(d*x+c)^2*polylog(2,(
d*x+c+1)/(1-(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*e^(-1)*log(d*x*e + c*e)/d + integrate(1/8*b^3*(log(d*x + c + 1) - log(-
d*x - c + 1))^3/(d*x*e + c*e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c
+ 1))^2/(d*x*e + c*e) + 3/2*a^2*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(d
*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3\operatorname{atanh}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2\operatorname{atanh}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2b\operatorname{atanh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*atanh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*atanh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
tanh(c + d*x)/(c + d*x), x))/e
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atanh}(c + d\,x))^3}{c\,e + d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x), x)
```

3.26 $\int \frac{(a+b\tanh^{-1}(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=143

$$\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)}+\frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\log\left(2-\frac{2}{1+c+dx}\right)}{de^2}-\frac{3b^2(a+b\mathrm{t}}{}$$

```
[Out] (a+b*arctanh(d*x+c))^3/d/e^2-(a+b*arctanh(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b*
arctanh(d*x+c))^2*ln(2-2/(d*x+c+1))/d/e^2-3*b^2*(a+b*arctanh(d*x+c))*polylo
g(2,-1+2/(d*x+c+1))/d/e^2-3/2*b^3*polylog(3,-1+2/(d*x+c+1))/d/e^2
```

Rubi [A]
time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6242, 12, 6037, 6135, 6079, 6095, 6203, 6745}

$$-\frac{3b^2\mathrm{Li}_2\left(\frac{2}{c+dx+1}-1\right)\left(a+b\tanh^{-1}(c+dx)\right)}{de^2}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2}+\frac{3b\log\left(2-\frac{2}{c+dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^2}-\frac{3b^3\mathrm{Li}_3\left(\frac{2}{c+dx+1}-1\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] (a + b*ArcTanh[c + d*x])^3/(d*e^2) - (a + b*ArcTanh[c + d*x])^3/(d*e^2*(c +
 d*x)) + (3*b*(a + b*ArcTanh[c + d*x])^2*Log[2 - 2/(1 + c + d*x)])/(d*e^2)
- (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^2)
 - (3*b^3*PolyLog[3, -1 + 2/(1 + c + d*x)])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x(1-x^2)}\,dx, x, c+dx\right)}{de^2} \\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tanh}{x(1-}\right.}{d} \\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tanh^{-1}(c+\right.}{c} \\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tanh^{-1}(c+\right.}{c} \\
&= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{3b\left(a+b\tanh^{-1}(c+\right.}{c}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.63, size = 248, normalized size = 1.73

$$
\frac{-\frac{2a^3}{c+dx} - \frac{6a^2b\tanh^{-1}(c+dx)}{c+dx} + 6a^2b\log(c+dx) - 3a^2b\log\left(1-c^2-2cdx-d^2x^2\right) + 6ab^2\left(\tanh^{-1}(c+dx)\left(\left(1-\frac{1}{c+dx}\right)\tanh^{-1}(c+dx) + 2\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)\right) - \text{PolyLog}\left(2, e^{-2\tanh^{-1}(c+dx)}\right)\right) + 2b^3\left(\frac{i\pi^3}{8} - \tanh^{-1}(c+dx)^3 - \frac{\tanh^{-1}(c+dx)^3}{c+dx} + 3\tanh^{-1}(c+dx)^2\log\left(1-e^{2\tanh^{-1}(c+dx)}\right) + 3\tanh^{-1}(c+dx)\text{PolyLog}\left(2, e^{2\tanh^{-1}(c+dx)}\right) - \frac{3}{2}\text{PolyLog}\left(3, e^{2\tanh^{-1}(c+dx)}\right)\right)}{2de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTanh[c + d*x])/(c + d*x) + 6*a^2*b*Log[c
+ d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*x^2] + 6*a*b^2*(ArcTanh[c + d*
x]*((1 - (c + d*x)^(-1))*ArcTanh[c + d*x] + 2*Log[1 - E^(-2*ArcTanh[c + d*x
])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x])]) + 2*b^3*((I/8)*Pi^3 - ArcTanh[c
 + d*x]^3 - ArcTanh[c + d*x]^3/(c + d*x) + 3*ArcTanh[c + d*x]^2*Log[1 - E^(
2*ArcTanh[c + d*x])] + 3*ArcTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c + d*x])
] - (3*PolyLog[3, E^(2*ArcTanh[c + d*x])])/2))/(2*d*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.88, size = 1867, normalized size = 13.06

method	result	size

derivativedivides	Expression too large to display	1867
default	Expression too large to display	1867

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/e^2/(d*x+c)+3/2*I*b^3/e^2*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)
)*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1
)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*arctanh(d*x+c)^2-3/4*I*b^3/e^2*Pi*arctanh(
d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^
2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))+3/2*I
*b^3/e^2*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^
2)))^3*arctanh(d*x+c)^2-3/2*a^2*b/e^2*ln(d*x+c+1)+3*a^2*b/e^2*ln(d*x+c)-3/2
*a^2*b/e^2*ln(d*x+c-1)+3*b^3/e^2*ln(2)*arctanh(d*x+c)^2+3*b^3/e^2*arctanh(d
*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-b^3/e^2/(d*x+c)*arctanh(d*x+c)^3+
3*a*b^2/e^2*dilog(1/2*d*x+1/2*c+1/2)-3/4*a*b^2/e^2*ln(d*x+c-1)^2+3/4*a*b^2/
e^2*ln(d*x+c+1)^2-3*a*b^2/e^2*dilog(d*x+c+1)-3*a*b^2/e^2*dilog(d*x+c)+3*b^3
/e^2*ln(d*x+c)*arctanh(d*x+c)^2+6*b^3/e^2*arctanh(d*x+c)*polylog(2,(d*x+c+1
)/(1-(d*x+c)^2)^(1/2))+3*b^3/e^2*arctanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)
^2)^(1/2))+3*b^3/e^2*arctanh(d*x+c)^2*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+6
*b^3/e^2*arctanh(d*x+c)*polylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-3*b^3/e^2
*arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)-3/2*b^3/e^2*arctanh(d*x+c
)^2*ln(d*x+c-1)-3/2*b^3/e^2*arctanh(d*x+c)^2*ln(d*x+c+1)+3/2*a*b^2/e^2*ln(d
*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-3/2*a*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c
+1)+3/2*a*b^2/e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)-3*a*b^2/e^2*
ln(d*x+c)*ln(d*x+c+1)-3*a^2*b/e^2/(d*x+c)*arctanh(d*x+c)-3*a*b^2/e^2/(d*x+c
)*arctanh(d*x+c)^2-3*a*b^2/e^2*arctanh(d*x+c)*ln(d*x+c+1)+6*a*b^2/e^2*ln(d*
x+c)*arctanh(d*x+c)-3*a*b^2/e^2*arctanh(d*x+c)*ln(d*x+c-1)+3/2*I*b^3/e^2*Pi
*arctanh(d*x+c)^2-6*b^3/e^2*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*b^3/
e^2*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-b^3/e^2*arctanh(d*x+c)^3+3/2*
I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3+3/4*I
*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^
2/(1-(d*x+c)^2)))^3+3/4*I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((
d*x+c)^2-1))^3-3/2*I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(
d*x+c)^2)))^2-3/4*I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)
^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))+3/
2*I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2*csgn(I*
(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3/4*I*b^3/e^2*Pi*arctanh(d*x+c)^2*csgn(I*(d*
x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2-3/2*I*b^3/e
^2*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^
2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2-3/2*I*b^3/e^2*Pi*cs
gn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1
+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2+3/4*I*b^3/e^2*Pi*arctanh(d*
```

```
x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-
1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(e^(-2)*log(d*x + c + 1)/d^2 - 2*e^(-2)*log(d*x + c)/d^2 + e^(-2)*l
og(d*x + c - 1)/d^2) + 2*arctanh(d*x + c)/(d^2*x*e^2 + c*d*e^2))*a^2*b - a^
3/(d^2*x*e^2 + c*d*e^2) - 1/8*((b^3*d*x + b^3*(c - 1))*log(-d*x - c + 1)^3
+ 3*(2*a*b^2 + (b^3*d*x + b^3*(c + 1))*log(d*x + c + 1))*log(-d*x - c + 1)^
2)/(d^2*x*e^2 + c*d*e^2) - integrate(-1/8*((b^3*d*x + b^3*(c - 1))*log(d*x
+ c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c - 1))*log(d*x + c + 1)^2 + 3*(4*a*b^2*
d*x + 4*a*b^2*c - (b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^2 + 2*(b^3*d^2*x
^2 + (c^2 + c)*b^3 - 2*a*b^2*(c - 1) + ((2*c*d + d)*b^3 - 2*a*b^2*d)*x)*log
(d*x + c + 1))*log(-d*x - c + 1))/(d^3*x^3*e^2 + (3*c*d^2 - d^2)*x^2*e^2 +
(3*c^2*d - 2*c*d)*x*e^2 + (c^3 - c^2)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)*e^(-2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{atanh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{atanh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{atanh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atanh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atanh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atanh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atanh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2,x)
[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.27 $\int \frac{(a+b\tanh^{-1}(c+dx))^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=166

$$\frac{3b(a+b\tanh^{-1}(c+dx))^2}{2de^3}-\frac{3b(a+b\tanh^{-1}(c+dx))^2}{2de^3(c+dx)}+\frac{(a+b\tanh^{-1}(c+dx))^3}{2de^3}-\frac{(a+b\tanh^{-1}(c+dx))^3}{2de^3(c+dx)^2}-$$

```
[Out] 3/2*b*(a+b*arctanh(d*x+c))^2/d/e^3-3/2*b*(a+b*arctanh(d*x+c))^2/d/e^3/(d*x+
c)+1/2*(a+b*arctanh(d*x+c))^3/d/e^3-1/2*(a+b*arctanh(d*x+c))^3/d/e^3/(d*x+c
)^2+3*b^2*(a+b*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^3-3/2*b^3*polylog(2,-1
+2/(d*x+c+1))/d/e^3
```

Rubi [A]
time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6242, 12, 6037, 6129, 6135, 6079, 2497, 6095}

$$\frac{3b^2\log\left(2-\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))}{de^3}-\frac{3b(a+b\tanh^{-1}(c+dx))^2}{2de^3(c+dx)}+\frac{3b(a+b\tanh^{-1}(c+dx))^2}{2de^3}-\frac{(a+b\tanh^{-1}(c+dx))^3}{2de^3(c+dx)^2}+\frac{(a+b\tanh^{-1}(c+dx))^3}{2de^3}-\frac{3b^3\text{Li}_2\left(\frac{2}{c+dx+1}-1\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c + d*x])^2)/(2*d*e^3) - (3*b*(a + b*ArcTanh[c + d*x])^
2)/(2*d*e^3*(c + d*x)) + (a + b*ArcTanh[c + d*x])^3/(2*d*e^3) - (a + b*ArcT
anh[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTanh[c + d*x])*Log
[2 - 2/(1 + c + d*x)])/(d*e^3) - (3*b^3*PolyLog[2, -1 + 2/(1 + c + d*x)])/(
2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
```

```
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^2(1-x^2)}\,dx, x, c+dx\right)}{2de^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{2de^3} + \cdots$$

$$= -\frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{2de^3} - \frac{\left(a+b\tanh^{-1}(c \cdots\right.}{2de^3(c+d \cdots}$$

$$= \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} + \frac{\left(a+b\tanh^{-1}(c \cdots\right.}{2de^3}$$

$$= \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} + \frac{\left(a+b\tanh^{-1}(c \cdots\right.}{2de^3}$$

$$= \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} + \frac{\left(a+b\tanh^{-1}(c \cdots\right.}{2de^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.83, size = 335, normalized size = 2.02

$$\frac{-4a^3-12a^2bc+ib^3c^3\pi^3-12a^2bdx+2ib^3c^2d\pi^3x+ib^3cd^2\pi^3x^2+12b^2(-1+c+dx)(b(c+dx)+a(1+c+dx))\tanh^{-1}(c+dx)^2+4b^3(-1+c^2+2cdx+d^2x^2)\tanh^{-1}(c+dx)^3+12b\tanh^{-1}(c+dx)\left(a(-2b(c+dx)+a(-1+c^2+2cdx+d^2x^2))+2b^2(c+dx)^2\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)\right)+24ab^2c^2\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)+48ab^2cdx\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)+24ab^2d^2x^2\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)-12b^3(c+dx)^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(c+dx)}\right)}{8de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-4*a^3 - 12*a^2*b*c + I*b^3*c^3*Pi^3 - 12*a^2*b*d*x + (2*I)*b^3*c^2*d*Pi^3
*x + I*b^3*c*d^2*Pi^3*x^2 + 12*b^2*(-1 + c + d*x)*(b*(c + d*x) + a*(1 + c +
 d*x))*ArcTanh[c + d*x]^2 + 4*b^3*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c
+ d*x]^3 + 12*b*ArcTanh[c + d*x]*(a*(-2*b*(c + d*x) + a*(-1 + c^2 + 2*c*d*x
 + d^2*x^2)) + 2*b^2*(c + d*x)^2*Log[1 - E^(-2*ArcTanh[c + d*x])]) + 24*a*b
^2*c^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] + 48*a*b^2*c*d*x*Log[(c + d*x)/
Sqrt[1 - (c + d*x)^2]] + 24*a*b^2*d^2*x^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^
2]] - 12*b^3*(c + d*x)^2*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/(8*d*e^3*(c +
 d*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.13, size = 5530, normalized size = 33.31

method	result	size
derivativedivides	Expression too large to display	5530
default	Expression too large to display	5530

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 3/4*(d*(e^(-3)*log(d*x + c + 1)/d^2 - e^(-3)*log(d*x + c - 1)/d^2 - 2/(d^3*
x*e^3 + c*d^2*e^3)) - 2*arctanh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2
*d*e^3))*a^2*b - 3/8*(d^2*((log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d*x
 + c - 1) + log(d*x + c - 1)^2 + 4*log(d*x + c - 1))*e^(-3)/d^3 + 4*e^(-3)*
log(d*x + c + 1)/d^3 - 8*e^(-3)*log(d*x + c)/d^3) - 4*d*(e^(-3)*log(d*x + c
 + 1)/d^2 - e^(-3)*log(d*x + c - 1)/d^2 - 2/(d^3*x*e^3 + c*d^2*e^3))*arctan
h(d*x + c))*a*b^2 - 1/16*b^3*(((d^2*x^2 + 2*c*d*x + c^2 - 1)*log(-d*x - c +
 1)^3 + 3*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + 2*c)*lo
g(-d*x - c + 1)^2)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) + 2*integrate(
-((d*x + c - 1)*log(d*x + c + 1)^3 + 3*(2*d^2*x^2 + 4*c*d*x - (d*x + c - 1)
*log(d*x + c + 1)^2 + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d - d)*
x - c)*log(d*x + c + 1))*log(-d*x - c + 1))/(d^4*x^4*e^3 + (4*c*d^3 - d^3)*
x^3*e^3 + 3*(2*c^2*d^2 - c*d^2)*x^2*e^3 + (4*c^3*d - 3*c^2*d)*x*e^3 + (c^4
- c^3)*e^3), x)) - 3/2*a*b^2*arctanh(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^
3 + c^2*d*e^3) - 1/2*a^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)*e^(-3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{atanh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{atanh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{atanh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atanh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*atanh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.28 $\int \frac{(a+b\tanh^{-1}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=269

$$-\frac{b^2(a+b\tanh^{-1}(c+dx))}{de^4(c+dx)}+\frac{b(a+b\tanh^{-1}(c+dx))^2}{2de^4}-\frac{b(a+b\tanh^{-1}(c+dx))^2}{2de^4(c+dx)^2}+\frac{(a+b\tanh^{-1}(c+dx))}{3de^4}$$

```
[Out] -b^2*(a+b*arctanh(d*x+c))/d/e^4/(d*x+c)+1/2*b*(a+b*arctanh(d*x+c))^2/d/e^4-
1/2*b*(a+b*arctanh(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*(a+b*arctanh(d*x+c))^3/d/e
^4-1/3*(a+b*arctanh(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^3*l
n(1-(d*x+c)^2)/d/e^4+b*(a+b*arctanh(d*x+c))^2*ln(2-2/(d*x+c+1))/d/e^4-b^2*(
a+b*arctanh(d*x+c))*polylog(2,-1+2/(d*x+c+1))/d/e^4-1/2*b^3*polylog(3,-1+2/
(d*x+c+1))/d/e^4
```

Rubi [A]
time = 0.36, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6242, 12, 6037, 6129, 272, 36, 31, 29, 6095, 6135, 6079, 6203, 6745}

$$-\frac{b^2\text{Li}_2\left(\frac{2}{c+dx+1}-1\right)\left(a+b\tanh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\left(a+b\tanh^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}+\frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4}+\frac{b\log\left(2-\frac{2}{c+dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{de^4}-\frac{b^3\text{Li}_3\left(\frac{2}{c+dx+1}-1\right)}{2de^4}+\frac{b^3\log(c+dx)}{de^4}-\frac{b^3\log\left(1-(c+dx)^2\right)}{2de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcTanh[c + d*x]))/(d*e^4*(c + d*x))) + (b*(a + b*ArcTanh[c +
 d*x])^2)/(2*d*e^4) - (b*(a + b*ArcTanh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2)
+ (a + b*ArcTanh[c + d*x])^3/(3*d*e^4) - (a + b*ArcTanh[c + d*x])^3/(3*d*e^
4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 - (c + d*x)^2])/(2
*d*e^4) + (b*(a + b*ArcTanh[c + d*x])^2*Log[2 - 2/(1 + c + d*x)])/(d*e^4) -
 (b^2*(a + b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d*x)])/(d*e^4) -
(b^3*PolyLog[3, -1 + 2/(1 + c + d*x)])/(2*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6079

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6129

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6135

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^3(1-x^2)}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^4} + \frac{b\text{Sul}}{}$$

$$= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tanh^{-1}(c+\right.}{3de^4(c+dx}$$

$$= -\frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{3de^4} - \frac{\left(a+b\tanh^{-1}(c+\right.}{3de^4(c+dx}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tanh^{-1}(c\right.}{2de^4(c+d}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tanh^{-1}(c\right.}{2de^4(c+d}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tanh^{-1}(c\right.}{2de^4(c+d}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\left(a+b\tanh^{-1}(c+dx)\right)^2}{2de^4} - \frac{b\left(a+b\tanh^{-1}(c\right.}{2de^4(c+d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 393, normalized size = 1.46

$$\frac{-\frac{2a^3}{(c+dx)^3} - \frac{3a^2b}{(c+dx)^2} - \frac{6a^2b\tanh^{-1}(c+dx)}{(c+dx)^3} + 6a^2b\log(c+dx) - 3a^2b\log\left(1-c^2-2cdx-d^2x^2\right) + 6ab^2\left(-\frac{(c+dx)^2+\tanh^{-1}(c+dx)^2}{(c+dx)^3} + \tanh^{-1}(c+dx)\left(-\frac{1-(c+dx)^2}{(c+dx)^2} + \tanh^{-1}(c+dx) + 2\log\left(1-e^{-2\tanh^{-1}(c+dx)}\right)\right) - \text{PolyLog}\left(2, e^{-2\tanh^{-1}(c+dx)}\right)\right) + 6b^3\left(\frac{i\pi^3}{24} - \frac{\tanh^{-1}(c+dx)}{c+dx} - \frac{\left(1-(c+dx)^2\right)\tanh^{-1}(c+dx)^2}{2(c+dx)^2} - \frac{1}{3}\tanh^{-1}(c+dx)^3 - \frac{\tanh^{-1}(c+dx)^3}{3(c+dx)} - \frac{\left(1-(c+dx)^2\right)\tanh^{-1}(c+dx)^3}{3(c+dx)^3} + \tanh^{-1}(c+dx)^2\log\left(1-e^{2\tanh^{-1}(c+dx)}\right) + \log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right) + \tanh^{-1}(c+dx)\text{PolyLog}\left(2, e^{2\tanh^{-1}(c+dx)}\right) - \frac{1}{2}\text{PolyLog}\left(3, e^{2\tanh^{-1}(c+dx)}\right)\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*a^3)/(c + d*x)^3 - (3*a^2*b)/(c + d*x)^2 - (6*a^2*b*ArcTanh[c + d*x])/
(c + d*x)^3 + 6*a^2*b*Log[c + d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*x^
2] + 6*a*b^2*(-(((c + d*x)^2 + ArcTanh[c + d*x]^2)/(c + d*x)^3) + ArcTanh[c
 + d*x]*(-((1 - (c + d*x)^2)/(c + d*x)^2) + ArcTanh[c + d*x] + 2*Log[1 - E^
```

```
(-2*ArcTanh[c + d*x])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x])]) + 6*b^3*((I/
24)*Pi^3 - ArcTanh[c + d*x]/(c + d*x) - ((1 - (c + d*x)^2)*ArcTanh[c + d*x]
^2)/(2*(c + d*x)^2) - ArcTanh[c + d*x]^3/3 - ArcTanh[c + d*x]^3/(3*(c + d*x
)) - ((1 - (c + d*x)^2)*ArcTanh[c + d*x]^3)/(3*(c + d*x)^3) + ArcTanh[c + d
*x]^2*Log[1 - E^(2*ArcTanh[c + d*x])] + Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]
] + ArcTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c + d*x])] - PolyLog[3, E^(2*A
rcTanh[c + d*x])]/2))/(6*d*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.38, size = 2080, normalized size = 7.73

method	result	size
derivativedivides	Expression too large to display	2080
default	Expression too large to display	2080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/e^4/(d*x+c)^3+1/2*I*b^3/e^4*Pi*csgn(I*((d*x+c+1)^2/(1-(d*x+c)
^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3*arctanh(d*x+c)^2+1/2*I*b^3/e^4*Pi*c
sgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*
csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*arctanh
(d*x+c)^2-1/4*I*b^3/e^4*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c
)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn
(I*(d*x+c+1)^2/((d*x+c)^2-1))+1/4*I*b^3/e^4*Pi*arctanh(d*x+c)^2*csgn(I*(d*x
+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2-1/2*I*b^3/e^
4*Pi*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2
)-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2-1/2*I*b^3/e^4*Pi*csg
n(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1))*csgn(I*((d*x+c+1)^2/(1-(d*x+c)^2)-1)/(1+
(d*x+c+1)^2/(1-(d*x+c)^2)))^2*arctanh(d*x+c)^2+1/4*I*b^3/e^4*Pi*arctanh(d*x
+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1
)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2-1/4*I*b^3/e^4*Pi*arctanh(d*x+c)^2*csgn(I
*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(d*x+c+1
)^2/((d*x+c)^2-1))+1/2*I*b^3/e^4*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d
*x+c)^2-1))^2*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/2*I*b^3/e^4*Pi*arctan
h(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2+1/2*I*b^3/e^4*Pi*arctanh
(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3+1/4*I*b^3/e^4*Pi*arctanh(
d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3+
1/4*I*b^3/e^4*Pi*arctanh(d*x+c)^2*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^3-a^2*b
/e^4/(d*x+c)^3*arctanh(d*x+c)-a*b^2/e^4/(d*x+c)^3*arctanh(d*x+c)^2-a*b^2/e^
4*arctanh(d*x+c)*ln(d*x+c+1)-a*b^2/e^4/(d*x+c)^2*arctanh(d*x+c)+2*a*b^2/e^4
*ln(d*x+c)*arctanh(d*x+c)-a*b^2/e^4*arctanh(d*x+c)*ln(d*x+c-1)+1/2*a*b^2/e^
4*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/2*a*b^2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln
(d*x+c+1)+1/2*a*b^2/e^4*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)-a*b^2/
e^4*ln(d*x+c)*ln(d*x+c+1)+1/2*I*b^3/e^4*Pi*arctanh(d*x+c)^2-a*b^2/e^4/(d*x+
```

```
c)+1/2*a*b^2/e^4*ln(d*x+c+1)-1/2*a*b^2/e^4*ln(d*x+c-1)+a*b^2/e^4*dilog(1/2*
d*x+1/2*c+1/2)-1/4*a*b^2/e^4*ln(d*x+c-1)^2+1/4*a*b^2/e^4*ln(d*x+c+1)^2-a*b^
2/e^4*dilog(d*x+c+1)-a*b^2/e^4*dilog(d*x+c)-1/2*a^2*b/e^4/(d*x+c)^2-1/2*a^2
*b/e^4*ln(d*x+c+1)+a^2*b/e^4*ln(d*x+c)-1/2*a^2*b/e^4*ln(d*x+c-1)-b^3/e^4/(d
*x+c)*arctanh(d*x+c)+b^3/e^4*ln(d*x+c)*arctanh(d*x+c)^2+2*b^3/e^4*arctanh(d
*x+c)*polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+b^3/e^4*arctanh(d*x+c)^2*ln(
1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+b^3/e^4*arctanh(d*x+c)^2*ln(1+(d*x+c+1)/(1
-(d*x+c)^2)^(1/2))+2*b^3/e^4*arctanh(d*x+c)*polylog(2,-(d*x+c+1)/(1-(d*x+c)
^2)^(1/2))-b^3/e^4*arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)-1/2*b^3
/e^4/(d*x+c)^2*arctanh(d*x+c)^2-1/2*b^3/e^4*arctanh(d*x+c)^2*ln(d*x+c-1)-1/
2*b^3/e^4*arctanh(d*x+c)^2*ln(d*x+c+1)+b^3/e^4*ln(2)*arctanh(d*x+c)^2+b^3/e
^4*arctanh(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/3*b^3/e^4/(d*x+c)^3
*arctanh(d*x+c)^3-b^3/e^4*arctanh(d*x+c)+1/2*b^3/e^4*arctanh(d*x+c)^2-2*b^3
/e^4*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*b^3/e^4*polylog(3,-(d*x+c+1
)/(1-(d*x+c)^2)^(1/2))+b^3/e^4*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-1/3*b^3/
e^4*arctanh(d*x+c)^3+b^3/e^4*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(e^(-4)*log(d*x + c + 1)/d^2 - 2*e^(-4)*log(d*x + c)/d^2 + e^(-4)*l
og(d*x + c - 1)/d^2 + 1/(d^4*x^2*e^4 + 2*c*d^3*x*e^4 + c^2*d^2*e^4)) + 2*ar
ctanh(d*x + c)/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4
))*a^2*b - 1/3*a^3/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d
*e^4) - 1/24*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 - 1)*b^
3)*log(-d*x - c + 1)^3 + 3*(b^3*d*x + b^3*c + 2*a*b^2 + (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 + 1)*b^3)*log(d*x + c + 1))*log(-d*x - c
 + 1)^2)/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - in
tegrate(-1/8*((b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^3 + 6*(a*b^2*d*x + a
*b^2*(c - 1))*log(d*x + c + 1)^2 + (2*b^3*d^2*x^2 + 2*b^3*c^2 + 4*a*b^2*c -
 3*(b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^2 + 4*(b^3*c*d + a*b^2*d)*x + 2
*(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + (c^4 + c)*b^3 - 6*a*b
^2*(c - 1) + ((4*c^3*d + d)*b^3 - 6*a*b^2*d)*x)*log(d*x + c + 1))*log(-d*x
- c + 1))/(d^5*x^5*e^4 + (5*c*d^4 - d^4)*x^4*e^4 + 2*(5*c^2*d^3 - 2*c*d^3)*
x^3*e^4 + 2*(5*c^3*d^2 - 3*c^2*d^2)*x^2*e^4 + (5*c^4*d - 4*c^3*d)*x*e^4 + (
c^5 - c^4)*e^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)*e^(-4)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*
d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^3\operatorname{atanh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3ab^2\operatorname{atanh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3a^2b\operatorname{atanh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*atanh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atanh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*atanh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.29 $\int \frac{\tanh^{-1}(1+x)}{2+2x}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{4}\text{PolyLog}(2,-1-x)+\frac{1}{4}\text{PolyLog}(2,1+x)$$

```
[Out] -1/4*polylog(2,-1-x)+1/4*polylog(2,1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6242, 12, 6031}

$$\frac{\text{Li}_2(x+1)}{4}-\frac{\text{Li}_2(-x-1)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + x]/(2 + 2*x),x]
```

```
[Out] -1/4*PolyLog[2, -1 - x] + PolyLog[2, 1 + x]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+x)}{2+2x}\,dx = \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{2x}\,dx, x, 1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\,dx, x, 1+x\right)$$
$$= -\frac{1}{4}\text{Li}_2(-1-x) + \frac{\text{Li}_2(1+x)}{4}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.48

$$-\frac{1}{4}\text{PolyLog}\left(2, \frac{1}{2}(-2-2x)\right) + \frac{1}{4}\text{PolyLog}\left(2, \frac{1}{2}(2+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + x]/(2 + 2*x),x]
[Out] -1/4*PolyLog[2, (-2 - 2*x)/2] + PolyLog[2, (2 + 2*x)/2]/4
```

Maple [A]
time = 0.93, size = 34, normalized size = 1.62

method	result	size
risch	$\frac{\text{dilog}(-x)}{4} - \frac{\text{dilog}(x+2)}{4}$	14
derivativedivides	$\frac{\ln(1+x)\,\text{arctanh}(1+x)}{2} - \frac{\text{dilog}(x+2)}{4} - \frac{\ln(1+x)\ln(x+2)}{4} - \frac{\text{dilog}(1+x)}{4}$	34
default	$\frac{\ln(1+x)\,\text{arctanh}(1+x)}{2} - \frac{\text{dilog}(x+2)}{4} - \frac{\ln(1+x)\ln(x+2)}{4} - \frac{\text{dilog}(1+x)}{4}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1+x)*arctanh(1+x)-1/4*dilog(x+2)-1/4*ln(1+x)*ln(x+2)-1/4*dilog(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(15) = 30.
time = 0.25, size = 58, normalized size = 2.76

$$-\frac{1}{4}\left(\log(x+2) - \log(x)\right)\log(x+1) + \frac{1}{2}\,\text{artanh}(x+1)\log(x+1) - \frac{1}{4}\log(x+1)\log(x) + \frac{1}{4}\log(x+2)\log(-x-1) - \frac{1}{4}\text{Li}_2(-x) + \frac{1}{4}\text{Li}_2(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arctanh(x + 1)*log(x + 1) - 1/4
*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dilog
(x + 2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arctanh(x + 1)/(x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atanh}(x+1)}{x+1}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+x)/(2+2*x),x)
```

```
[Out] Integral(atanh(x + 1)/(x + 1), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] integrate(1/2*arctanh(x + 1)/(x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(x+1)}{2\,x+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x + 1)/(2*x + 2),x)
```

```
[Out] int(atanh(x + 1)/(2*x + 2), x)
```

3.30 $\int \frac{\tanh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=32

$$-\frac{\text{PolyLog}(2,-a-bx)}{2d}+\frac{\text{PolyLog}(2,a+bx)}{2d}$$

```
[Out] -1/2*polylog(2,-b*x-a)/d+1/2*polylog(2,b*x+a)/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6242, 12, 6031}

$$\frac{\text{Li}_2(a+bx)}{2d}-\frac{\text{Li}_2(-a-bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -1/2*PolyLog[2, -a - b*x]/d + PolyLog[2, a + b*x]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6242

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\tanh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= -\frac{\text{Li}_2(-a-bx)}{2d} + \frac{\text{Li}_2(a+bx)}{2d}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.62

$$b\left(-\frac{\text{PolyLog}\left(2,-\frac{ad+bdx}{d}\right)}{2bd}+\frac{\text{PolyLog}\left(2,\frac{ad+bdx}{d}\right)}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/((a*d)/b + d*x),x]
[Out] b*(-1/2*PolyLog[2, -((a*d + b*d*x)/d)]/(b*d) + PolyLog[2, (a*d + b*d*x)/d]/
(2*b*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.
time = 0.87, size = 62, normalized size = 1.94

method	result	size
risch	$-\frac{\text{dilog}(bx+a+1)}{2d}+\frac{\text{dilog}(-bx-a+1)}{2d}$	29
derivativedivides	$\frac{\frac{b\ln(bx+a)\,\text{arctanh}(bx+a)}{d}-\frac{b\left(\frac{\text{dilog}(bx+a+1)}{2}+\frac{\ln(bx+a)\ln(bx+a+1)}{2}+\frac{\text{dilog}(bx+a)}{2}\right)}{d}}{b}$	62
default	$\frac{\frac{b\ln(bx+a)\,\text{arctanh}(bx+a)}{d}-\frac{b\left(\frac{\text{dilog}(bx+a+1)}{2}+\frac{\ln(bx+a)\ln(bx+a+1)}{2}+\frac{\text{dilog}(bx+a)}{2}\right)}{d}}{b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
[Out] 1/b*(b/d*ln(b*x+a)*arctanh(b*x+a)-b/d*(1/2*dilog(b*x+a+1)+1/2*ln(b*x+a)*ln(
b*x+a+1)+1/2*dilog(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(26) = 52$.
time = 0.26, size = 132, normalized size = 4.12

$$-\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\text{Li}_2(-bx-a+1)}{bd}-\frac{\log(bx+a+1)\log(-bx-a)+\text{Li}_2(bx+a+1)}{bd}\right)-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{2\,d}+\frac{\text{artanh}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log(
b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x +
a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arctanh(b*x + a)*log(d*
x + a*d/b)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arctanh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{atanh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(atanh(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atanh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(atanh(a + b*x)/(d*x + (a*d)/b), x)
```

3.31 $\int (e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=168

$$\frac{bf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}+\frac{(e+fx)^4\left(a+b\tanh^{-1}(c+dx\right.}{4f}$$

```
[Out] 1/4*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d*
x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arctanh(d*x+c))/f+1/
8*b*(-c*f+d*e+f)^4*ln(-d*x-c+1)/d^4/f-1/8*b*(-c*f+d*e-f)^4*ln(d*x+c+1)/d^4/
f
```

Rubi [A]
time = 0.24, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6246, 6063, 716, 647, 31}

$$\frac{(e+fx)^4\left(a+b\tanh^{-1}(c+dx)\right)}{4f}+\frac{bfx((6c^2+1)f^2-12cdef+6d^2e^2)}{4d^3}+\frac{bf^2(c+dx)^2(de-cf)}{2d^4}-\frac{b(-cf+de-f)^4\log(c+dx+1)}{8d^4f}+\frac{b(-cf+de+f)^4\log(-c-dx+1)}{8d^4f}+\frac{bf^3(c+dx)^3}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcTanh[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*Log[1 - c - d*x])/(8*d^
4*f) - (b*(d*e - f - c*f)^4*Log[1 + c + d*x])/(8*d^4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)\,dx = \frac{\mathrm{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\tanh^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4 \left(a+b\tanh^{-1}(c+dx)\right)}{4f} - \frac{b\mathrm{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1-x^2}\,dx, x,\right.}{4f}$$

$$= \frac{(e+fx)^4 \left(a+b\tanh^{-1}(c+dx)\right)}{4f} - \frac{b\mathrm{Subst}\left(\int \left(-\frac{f^2(6d^2e^2-12cdef+}{d^4}\right.\right.}{}$$

$$= \frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} +$$

$$= \frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} +$$

$$= \frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} +$$

Mathematica [A]
time = 0.16, size = 270, normalized size = 1.61

$$\frac{6d\left(4ad^3e^3+bf\left(6d^2e^2-8cdef+\left(1+3c^2\right)f^2\right)\right)x+6d^2f\left(6ad^2e^2+bf(2de-cf)\right)x^2+2d^3f^2(12ade+bf)x^3+6ad^4f^3x^4+6bd^4x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\tanh^{-1}(c+dx)-3b(-1+c)\left(4d^3e^3-6(-1+c)d^2e^2f+4(-1+c)^2def^2-(-1+c)^3f^3\right)\log(1-c-dx)-3b(1+c)\left(-4d^3e^3+6(1+c)d^2e^2f-4(1+c)^2def^2+(1+c)^3f^3\right)\log(1+c+dx)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d^
2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^3
```

```
+ 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*A
rcTanh[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 +
c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*e^3 +
 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x
])/(24*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(156) = 312$.
time = 1.40, size = 982, normalized size = 5.85

method	result
risch	$\frac{b\,e^3\ln(dx+c+1)}{2d}+\frac{b\,e^3\ln(-dx-c+1)}{2d}-\frac{2f^2bcex}{d^2}+\frac{f^2\ln(dx+c+1)b\,c^3e}{2d^3}-\frac{3f\ln(dx+c+1)b\,c^2e^2}{4d^2}-\frac{f^2\ln(-dx-c+}{2d^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b*e^3*ln(d*x+c-1)+1/2*b*e^3*ln(d*x+c+1)-3*b/d^2*f^2*c*e*(d*x+c)+3/
2*b/d^3*f^3*c^2*(d*x+c)+1/8*b/d^3*f^3*ln(d*x+c-1)*c^4-1/2*b/d^3*f^3*ln(d*x+
c-1)*c^3+3/4*b/d^3*f^3*ln(d*x+c-1)*c^2-1/2*b/d^3*f^3*ln(d*x+c-1)*c+1/4*b/d^
3*f^3*arctanh(d*x+c)*c^4+1/4*b/d^3*f^3*arctanh(d*x+c)*(d*x+c)^4+1/4*b*d/f*a
rctanh(d*x+c)*e^4-3/4*b/d*f*ln(d*x+c+1)*e^2+1/2*b/d^2*f^2*ln(d*x+c+1)*e+3/4
*b/d*f*ln(d*x+c-1)*e^2+1/2*b/d^2*f^2*ln(d*x+c-1)*e-1/8*b*d/f*ln(d*x+c+1)*e^
4+1/8*b*d/f*ln(d*x+c-1)*e^4-1/8*b/d^3*f^3*ln(d*x+c+1)*c^4-1/2*b/d^3*f^3*ln(
d*x+c+1)*c^3-3/4*b/d^3*f^3*ln(d*x+c+1)*c^2-1/2*b/d^3*f^3*ln(d*x+c+1)*c+3/2*
b/d*f*e^2*(d*x+c)+1/2*b/d^2*f^2*e*(d*x+c)^2-1/2*b/d^3*f^3*c*(d*x+c)^2-b/d^3
*f^3*arctanh(d*x+c)*c^3*(d*x+c)+3/2*b/d^3*f^3*arctanh(d*x+c)*c^2*(d*x+c)^2-
b/d^3*f^3*arctanh(d*x+c)*c*(d*x+c)^3-1/2*b/d^2*f^2*ln(d*x+c-1)*c^3*e+3/4*b/
d*f*ln(d*x+c-1)*c^2*e^2+3/2*b/d^2*f^2*ln(d*x+c-1)*c^2*e-3/2*b/d*f*ln(d*x+c-
1)*c*e^2-3/2*b/d^2*f^2*ln(d*x+c-1)*c*e+1/2*b/d^2*f^2*ln(d*x+c+1)*c^3*e-3/4*
b/d*f*ln(d*x+c+1)*c^2*e^2+3/2*b/d^2*f^2*ln(d*x+c+1)*c^2*e-3/2*b/d*f*ln(d*x+
c+1)*c*e^2+3/2*b/d^2*f^2*ln(d*x+c+1)*c*e-b/d^2*f^2*arctanh(d*x+c)*c^3*e+3/2
*b/d*f*arctanh(d*x+c)*c^2*e^2+3/2*b/d*f*arctanh(d*x+c)*e^2*(d*x+c)^2+b/d^2*
f^2*arctanh(d*x+c)*e*(d*x+c)^3+1/4*(c*f-d*e-f*(d*x+c))^4*a/d^3/f+1/4*b/d^3*
f^3*(d*x+c)+1/12*b/d^3*f^3*(d*x+c)^3-1/2*b*ln(d*x+c-1)*c*e^3+1/2*b*ln(d*x+c
+1)*c*e^3-b*arctanh(d*x+c)*c*e^3+b*arctanh(d*x+c)*e^3*(d*x+c)+1/8*b/d^3*f^3
*ln(d*x+c-1)-1/8*b/d^3*f^3*ln(d*x+c+1)+3*b/d^2*f^2*arctanh(d*x+c)*c^2*e*(d*
x+c)-3*b/d*f*arctanh(d*x+c)*c*e^2*(d*x+c)-3*b/d^2*f^2*arctanh(d*x+c)*c*e*(d
*x+c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(161) = 322$.
time = 0.26, size = 331, normalized size = 1.97

$$\frac{1}{4}af^3x^4+af^2x^3e+\frac{1}{24}\left(6x^4\operatorname{artanh}(dx+c)+d\left(\frac{2(d^2x^3-3cdx^2+3(3c^2+1)x)}{d^4}-\frac{3(c^4+4c^3+6c^2+4c+1)\log(dx+c+1)}{d^5}+\frac{3(c^4-4c^3+6c^2-4c+1)\log(dx+c-1)}{d^5}\right)\right)bf^3+\frac{3}{2}afx^2e^2+\frac{1}{2}\left(2x^3\operatorname{artanh}(dx+c)+d\left(\frac{dx^2-4cx}{d^3}+\frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4}-\frac{(c^3-3c^2+3c-1)\log(dx+c-1)}{d^4}\right)\right)bf^2e+\frac{3}{4}\left(2x^2\operatorname{artanh}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bfe^2+axe^3+\frac{(2(dx+c)\operatorname{artanh}(dx+c)+\log(-(dx+c)^2+1))be^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")

[Out] 1/4*a*f^3*x^4 + a*f^2*x^3*e + 1/24*(6*x^4*arctanh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c - 1)/d^5))*b*f^3 + 3/2*a*f*x^2*e^2 + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*b*f^2*e + 3/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f*e^2 + a*x*e^3 + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*e^3/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(161) = 322$.
time = 0.41, size = 818, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(6*a*d^4*f^3*x^4 + 2*b*d^3*f^3*x^3 - 6*b*c*d^2*f^3*x^2 + 24*a*d^4*x*cosh(1)^3 + 24*a*d^4*x*sinh(1)^3 + 6*(3*b*c^2 + b)*d*f^3*x + 36*(a*d^4*f*x^2 + b*d^3*f*x)*cosh(1)^2 + 36*(a*d^4*f*x^2 + 2*a*d^4*x*cosh(1) + b*d^3*f*x)*sinh(1)^2 + 12*(2*a*d^4*f^2*x^3 + b*d^3*f^2*x^2 - 4*b*c*d^2*f^2*x)*cosh(1) + 3*(4*(b*c + b)*d^3*cosh(1)^3 + 4*(b*c + b)*d^3*sinh(1)^3 - 6*(b*c^2 + 2*b*c + b)*d^2*f*cosh(1)^2 + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*f^2*cosh(1) - (b*c^4 + 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3 + 6*(2*(b*c + b)*d^3*cosh(1) - (b*c^2 + 2*b*c + b)*d^2*f)*sinh(1)^2 + 4*(3*(b*c + b)*d^3*cosh(1)^2 - 3*(b*c^2 + 2*b*c + b)*d^2*f*cosh(1) + (b*c^3 + 3*b*c^2 + 3*b*c + b)*d*f^2)*sinh(1))*log(d*x + c + 1) - 3*(4*(b*c - b)*d^3*cosh(1)^3 + 4*(b*c - b)*d^3*sinh(1)^3 - 6*(b*c^2 - 2*b*c + b)*d^2*f*cosh(1)^2 + 4*(b*c^3 - 3*b*c^2 + 3*b*c - b)*d*f^2*cosh(1) - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3 + 6*(2*(b*c - b)*d^3*cosh(1) - (b*c^2 - 2*b*c + b)*d^2*f)*sinh(1)^2 + 4*(3*(b*c - b)*d^3*cosh(1)^2 - 3*(b*c^2 - 2*b*c + b)*d^2*f*cosh(1) + (b*c^3 - 3*b*c^2 + 3*b*c - b)*d*f^2)*sinh(1))*log(d*x + c - 1) + 3*(b*d^4*f^3*x^4 + 4*b*d^4*f^2*x^3*cosh(1) + 6*b*d^4*f*x^2*cosh(1)^2 + 4*b*d^4*x*cosh(1)^3 + 4*b*d^4*x*sinh(1)^3 + 6*(b*d^4*f*x^2 + 2*b*d^4*x*cosh(1))*sinh(1)^2 + 4*(b*d^4*f^2*x^3 + 3*b*d^4*f*x^2*cosh(1) + 3*b*d^4*x*cosh(1)^2)*sinh(1))*log(-(d*x + c + 1)/(d*x + c - 1)) + 12*(2*a*d^4*f^2*x^3 + b*d^3*f^2*x^2 + 6*a*d^4*x*cosh(1)^2 - 4*b*c*d^2*f^2*x + 6*(a*d^4*f*x^2 + b*d^3*f*x)*cosh(1))*sinh(1))/d^4

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(151) = 302$.

time = 2.07, size = 644, normalized size = 3.83

$$\begin{cases} ae^3x + \frac{3ae^2fx^2}{2} + aef^2x^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3\operatorname{atanh}(c+dx)}{4d^4} + \frac{bc^3ef^2\operatorname{atanh}(c+dx)}{d^3} - \frac{bc^3f^3\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^4} + \frac{bc^3f^3\operatorname{atanh}(c+dx)}{d^4} - \frac{3bc^2e^2f\operatorname{atanh}(c+dx)}{2d^2} + \frac{3bc^2ef^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{3bc^2ef^2\operatorname{atanh}(c+dx)}{d^3} + \frac{3bc^2f^3x}{4d^3} - \frac{3bc^2f^3\operatorname{atanh}(c+dx)}{2d^4} + \frac{bce^3\operatorname{atanh}(c+dx)}{d} - \frac{3bce^2f\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{3bce^2f\operatorname{atanh}(c+dx)}{d^2} - \frac{2bcef^2x}{d^2} - \frac{bcf^3x^2}{4d^2} + \frac{3bcef^2\operatorname{atanh}(c+dx)}{d^3} - \frac{bcf^3\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^4} + \frac{bcf^3\operatorname{atanh}(c+dx)}{d^4} + be^3x\operatorname{atanh}(c+dx) + \frac{3be^2fx^2\operatorname{atanh}(c+dx)}{2} + bef^2x^3\operatorname{atanh}(c+dx) + \frac{bf^3x^4\operatorname{atanh}(c+dx)}{4} + \frac{be^3\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{be^3\operatorname{atanh}(c+dx)}{d} + \frac{3be^2fx}{2d} + \frac{bef^2x^2}{2d} + \frac{bf^3x^3}{12d} - \frac{3be^2f\operatorname{atanh}(c+dx)}{2d^2} + \frac{bef^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bef^2\operatorname{atanh}(c+dx)}{d^3} + \frac{bf^3x}{4d^3} - \frac{bf^3\operatorname{atanh}(c+dx)}{4d^4} & \text{for } d \neq 0 \\ (a + b\operatorname{atanh}(c))\left(e^3x + \frac{3e^2fx^2}{2} + ef^2x^3 + \frac{f^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*atanh(c + d*x)/(4*d**4) + b*c**3*e*f**2*atanh(c + d*x)/d**3 - b*
c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*atanh(c + d*x)/d**4 - 3*b*c
**2*e**2*f*atanh(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)/d**
3 - 3*b*c**2*e*f**2*atanh(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3*b*c*
*2*f**3*atanh(c + d*x)/(2*d**4) + b*c*e**3*atanh(c + d*x)/d - 3*b*c*e**2*f*
log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*atanh(c + d*x)/d**2 - 2*b*c*e*f**2*x
/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*atanh(c + d*x)/d**3 - b*c*f**
3*log(c/d + x + 1/d)/d**4 + b*c*f**3*atanh(c + d*x)/d**4 + b*e**3*x*atanh(c
 + d*x) + 3*b*e**2*f*x**2*atanh(c + d*x)/2 + b*e*f**2*x**3*atanh(c + d*x) +
 b*f**3*x**4*atanh(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*e**3*atanh(
c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/(12*d)
- 3*b*e**2*f*atanh(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x + 1/d)/d**3 - b
*e*f**2*atanh(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*atanh(c + d*x)/(4*
d**4), Ne(d, 0)), ((a + b*atanh(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3
 + f**3*x**4/4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2336 vs. $2(156) = 312$.
time = 0.46, size = 2336, normalized size = 13.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*d^3*e^3/(d*x + c - 1)^3 -
 3*(d*x + c + 1)^2*b*d^3*e^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*d^3*e^3/(d
*x + c - 1) - b*d^3*e^3 - 3*(d*x + c + 1)^3*b*c*d^2*e^2*f/(d*x + c - 1)^3 +
 9*(d*x + c + 1)^2*b*c*d^2*e^2*f/(d*x + c - 1)^2 - 9*(d*x + c + 1)*b*c*d^2*
e^2*f/(d*x + c - 1) + 3*b*c*d^2*e^2*f + 3*(d*x + c + 1)^3*b*c^2*d*e*f^2/(d*
x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*d*e*f^2/(d*x + c - 1)^2 + 9*(d*x + c
 + 1)*b*c^2*d*e*f^2/(d*x + c - 1) - 3*b*c^2*d*e*f^2 - (d*x + c + 1)^3*b*c^3
*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c^3*f^3/(d*x + c - 1)^2 - 3*(d*x
 + c + 1)*b*c^3*f^3/(d*x + c - 1) + b*c^3*f^3 + 3*(d*x + c + 1)^3*b*d^2*e^2
*f/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*d^2*e^2*f/(d*x + c - 1)^2 + 3*(d*x
 + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 6*(d*x + c + 1)^3*b*c*d*e*f^2/(d*x +
c - 1)^3 + 12*(d*x + c + 1)^2*b*c*d*e*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)
```

*b*c*d*e*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*c^2*f^3/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*c^2*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*b*d*e*f^2/(d*x + c - 1)^2 + (d*x + c + 1)*b*d*e*f^2/(d*x + c - 1) - b*d*e*f^2 - 3*(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c*f^3/(d*x + c - 1)^2 - (d*x + c + 1)*b*c*f^3/(d*x + c - 1) + b*c*f^3 + (d*x + c + 1)^3*b*f^3/(d*x + c - 1)^3 + (d*x + c + 1)*b*f^3/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^5/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^5/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^5/(d*x + c - 1) + d^5) + (6*(d*x + c + 1)^3*a*d^3*e^3/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*a*d^3*e^3/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a*d^3*e^3/(d*x + c - 1) - 6*a*d^3*e^3 - 18*(d*x + c + 1)^3*a*c*d^2*e^2*f/(d*x + c - 1)^3 + 54*(d*x + c + 1)^2*a*c*d^2*e^2*f/(d*x + c - 1)^2 - 54*(d*x + c + 1)*a*c*d^2*e^2*f/(d*x + c - 1) + 18*a*c*d^2*e^2*f + 18*(d*x + c + 1)^3*a*c^2*d*e*f^2/(d*x + c - 1)^3 - 54*(d*x + c + 1)^2*a*c^2*d*e*f^2/(d*x + c - 1)^2 + 54*(d*x + c + 1)*a*c^2*d*e*f^2/(d*x + c - 1) - 18*a*c^2*d*e*f^2 - 6*(d*x + c + 1)^3*a*c^3*f^3/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*a*c^3*f^3/(d*x + c - 1)^2 - 18*(d*x + c + 1)*a*c^3*f^3/(d*x + c - 1) + 6*a*c^3*f^3 + 18*(d*x + c + 1)^3*a*d^2*e^2*f/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*d^2*e^2*f/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a*d^2*e^2*f/(d*x + c - 1) + 9*(d*x + c + 1)^3*b*d^2*e^2*f/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*d^2*e^2*f/(d*x + c - 1)^2 + 27*(d*x + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 9*b*d^2*e^2*f - 36*(d*x + c + 1)^3*a*c*d*e*f^2/(d*x + c - 1)^3 + 72*(d*x + c + 1)^2*a*c*d*e*f^2/(d*x + c - 1)^2 - 36*(d*x + c + 1)*a*c*d*e*f^2/(d*x + c - 1) - 18*(d*x + c + 1)^3*b*c*d*e*f^2/(d*x + c - 1)^3 + 54*(d*x + c + 1)^2*b*c*d*e*f^2/(d*x + c - 1)^2 - 54*(d*x + c + 1)*b*c*d*e*f^2/(d*x + c - 1) + 18*b*c*d*e*f^2 + 18*(d*x + c + 1)^3*a*c^2*f^3/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*c^2*f^3/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a*c^2*f^3/(d*x + c - 1) + 9*(d*x + c + 1)^3*b*c^2*f^3/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 27*(d*x + c + 1)*b*c^2*f^3/(d*x + c - 1) - 9*b*c^2*f^3 + 18*(d*x + c + 1)^3*a*d*e*f^2/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*a*d*e*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*a*d*e*f^2/(d*x + c - 1) - 6*a*d*e*f^2 + 6*(d*x + c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2*b*d*e*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*d*e*f^2/(d*x + c - 1) - 18*(d*x + c + 1)^3*a*c*f^3/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*a*c*f^3/(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*c*f^3/(d*x + c - 1) + 6*a*c*f^3 - 6*(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*b*c*f^3/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*c*f^3/(d*x + c - 1) + 6*(d*x + c + 1)^3*a*f^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*f^3/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*f^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*f^3/(d*x + c - 1) - 2*b*f^3)/((d*x + c + 1)^4*d^5/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^5/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^5/(d*x + c - 1) + d^5) - 3*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 + b*d*e*f^2 - b*c*f^3)*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^5 + 3*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 + b*d*e*f^2 - b

```
*c*f^3)*log(-(d*x + c + 1)/(d*x + c - 1))/d^5)
```

Mupad [B]
time = 1.72, size = 737, normalized size = 4.39

$$\ln(c+dx+1)\left(\frac{b e^3 x}{2}+\frac{3 b e^2 f x^2}{4}+\frac{b e f^2 x^3}{2}+\frac{b f^3 x^4}{8}\right)-\ln(1-dx-c)\left(\frac{b e^3 x}{2}+\frac{3 b e^2 f x^2}{4}+\frac{b e f^2 x^3}{2}+\frac{b f^3 x^4}{8}\right)+x\left(\frac{e\,(6 a c^2 f^2+12 a c d e f+2 a d^2 e^2+3 b d e f-6 a f^2)}{2 d^2}-\frac{(4 c^2-4)\left(\frac{f^2 (b f+8 a c f+12 a d e)}{4 d}-\frac{2 a c f^3}{d}\right)}{4 d^2}+\frac{2 c\left(\frac{2 c\left(\frac{f^2 (b f+8 a c f+12 a d e)}{4 d}-\frac{2 a c f^3}{d}\right)}{d}-\frac{4 a c^2 f^3+24 a c d e f^2+12 a d^2 e^2 f+4 b d e f^2-4 a f^3}{4 d^2}+\frac{a f^3 (4 c^2-4)}{4 d^2}\right)}{d}\right)-x^2\left(\frac{c\left(\frac{f^2 (b f+8 a c f+12 a d e)}{4 d}-\frac{2 a c f^3}{d}\right)}{d}-\frac{4 a c^2 f^3+24 a c d e f^2+12 a d^2 e^2 f+4 b d e f^2-4 a f^3}{8 d^2}+\frac{a f^3 (4 c^2-4)}{8 d^2}\right)+x^3\left(\frac{f^2 (b f+8 a c f+12 a d e)}{12 d}-\frac{2 a c f^3}{3 d}\right)+\frac{a f^3 x^4}{4}+\frac{\ln(c+dx-1)\,(b c^4 f^3-4 b c^3 d e f^2-4 b c^3 f^3+6 b c^2 d^2 e^2 f+12 b c^2 d e f^2+6 b c^2 f^3-4 b c d^3 e^3-12 b c d^2 e^2 f-12 b c d e f^2-4 b c f^3+4 b d^3 e^3+6 b d^2 e^2 f+4 b d e f^2+b f^3)}{8 d^4}-\frac{\ln(c+dx+1)\,(b c^4 f^3-4 b c^3 d e f^2+4 b c^3 f^3+6 b c^2 d^2 e^2 f-12 b c^2 d e f^2+6 b c^2 f^3-4 b c d^3 e^3+12 b c d^2 e^2 f-12 b c d e f^2+4 b c f^3-4 b d^3 e^3+6 b d^2 e^2 f-4 b d e f^2+b f^3)}{8 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*atanh(c + d*x)),x)
```

```
[Out] log(c + d*x + 1)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f^
2*x^3)/2) - log(1 - d*x - c)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)
/4 + (b*e*f^2*x^3)/2) + x*((e*(6*a*c^2*f^2 - 6*a*f^2 + 2*a*d^2*e^2 + 3*b*d*
e*f + 12*a*c*d*e*f))/(2*d^2) - ((4*c^2 - 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e
))/(4*d) - (2*a*c*f^3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a
*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 1
2*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 - 4))/(4*d^2)))/d)
- x^2*((c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*
a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2
) + (a*f^3*(4*c^2 - 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(1
2*d) - (2*a*c*f^3)/(3*d)) + (a*f^3*x^4)/4 + (log(c + d*x - 1)*(b*f^3 + 6*b*
c^2*f^3 - 4*b*c^3*f^3 + 4*b*d^3*e^3 + b*c^4*f^3 - 4*b*c*f^3 + 4*b*d*e*f^2 -
 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f - 12*b*c*d^2*e^2*f + 12*b*c^2*d*e*f^2 - 4*b*
c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4) - (log(c + d*x +
 1)*(b*f^3 + 6*b*c^2*f^3 + 4*b*c^3*f^3 - 4*b*d^3*e^3 + b*c^4*f^3 + 4*b*c*f^
3 - 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*c*d^2*e^2*f - 12*b*c
^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4)
```

3.32 $\int (e+fx)^2 \left(a+b\tanh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=120

$$\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}+\frac{b(de+f-cf)^3\log(1-c-dx)}{6d^3f}-\frac{b(de-$$

```
[Out] b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x
+c))/f+1/6*b*(-c*f+d*e+f)^3*ln(-d*x-c+1)/d^3/f-1/6*b*(d*e-(1+c)*f)^3*ln(d*x
+c+1)/d^3/f
```

Rubi [A]
time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6246, 6063, 716, 647, 31}

$$\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}+\frac{b(-cf+de+f)^3\log(-c-dx+1)}{6d^3f}-\frac{b(de-(c+1)f)^3\log(c+dx+1)}{6d^3f}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{bfx(de-cf)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcTanh[c + d*x]))/(3*f) + (b*(d*e + f - c*f)^3*Log[1 - c - d*x])/(6*d^3*f
) - (b*(d*e - (1 + c)*f)^3*Log[1 + c + d*x])/(6*d^3*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}-\frac{b\text{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1-x^2}\,dx,x,c\right.}{3f}$$

$$=\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}-\frac{b\text{Subst}\left(\int\left(-\frac{3f^2(de-cf)}{d^3}-\frac{f^3x}{d^3}\right.\right.}{}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)}{3f}$$

Mathematica [A]

time = 0.12, size = 174, normalized size = 1.45

$$\frac{2d(3ad^2e^2+bf(3de-2cf))\,x+d^2f(6ade+bf)x^2+2ad^3f^2x^3+2bd^3x(3e^2+3efx+f^2x^2)\tanh^{-1}(c+dx)-b(-1+c)\left(3d^2e^2-3(-1+c)def+(-1+c)^2f^2\right)\log(1-c-dx)+b(1+c)\left(3d^2e^2-3(1+c)def+(1+c)^2f^2\right)\log(1+c+dx)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2*
a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c + d*x] - b*
(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] +
 b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/
(6*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(112) = 224$.
time = 0.70, size = 590, normalized size = 4.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b*e^2*ln(d*x+c-1)+1/2*b*e^2*ln(d*x+c+1)-b/d^2*f^2*arctanh(d*x+c)*c
*(d*x+c)^2-b/d*f*ln(d*x+c-1)*c*e-1/2*b/d*f*ln(d*x+c+1)*c^2*e-b/d*f*ln(d*x+c
+1)*c*e+1/2*b/d*f*ln(d*x+c-1)*c^2*e+b/d*f*arctanh(d*x+c)*e*(d*x+c)^2+b/d*f*
arctanh(d*x+c)*c^2*e+b/d^2*f^2*arctanh(d*x+c)*c^2*(d*x+c)-b*arctanh(d*x+c)*
c*e^2+b*arctanh(d*x+c)*e^2*(d*x+c)+1/6*b/d^2*f^2*(d*x+c)^2+1/6*b/d^2*f^2*ln
(d*x+c-1)+1/6*b/d^2*f^2*ln(d*x+c+1)+1/2*b*ln(d*x+c+1)*c*e^2-1/2*b*ln(d*x+c-
1)*c*e^2-1/3*(c*f-d*e-f*(d*x+c))^3*a/d^2/f-2*b/d*f*arctanh(d*x+c)*c*e*(d*x+
c)-b/d^2*f^2*c*(d*x+c)+b/d*f*e*(d*x+c)-1/6*b/d^2*f^2*ln(d*x+c-1)*c^3+1/2*b/
d^2*f^2*ln(d*x+c-1)*c^2-1/2*b/d^2*f^2*ln(d*x+c-1)*c+1/6*b/d^2*f^2*ln(d*x+c+
1)*c^3+1/2*b/d^2*f^2*ln(d*x+c+1)*c^2+1/2*b/d^2*f^2*ln(d*x+c+1)*c-1/3*b/d^2*
f^2*arctanh(d*x+c)*c^3+1/3*b/d^2*f^2*arctanh(d*x+c)*(d*x+c)^3+1/6*b*d/f*ln(
d*x+c-1)*e^3+1/2*b/d*f*ln(d*x+c-1)*e-1/6*b*d/f*ln(d*x+c+1)*e^3-1/2*b/d*f*ln
(d*x+c+1)*e+1/3*b*d/f*arctanh(d*x+c)*e^3)
```

Maxima [A]
time = 0.26, size = 207, normalized size = 1.72

$$\frac{1}{3}\,af^2x^3 + afx^2e + \frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\left(dx+c\right) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{\left(c^3+3\,c^2+3\,c+1\right)\log\left(dx+c+1\right)}{d^4} - \frac{\left(c^3-3\,c^2+3\,c-1\right)\log\left(dx+c-1\right)}{d^4}\right)\right)bf^2 + \frac{1}{2}\left(2\,x^2\,\mathrm{artanh}\left(dx+c\right) + d\left(\frac{2\,x}{d^2} - \frac{\left(c^2+2\,c+1\right)\log\left(dx+c+1\right)}{d^3} + \frac{\left(c^2-2\,c+1\right)\log\left(dx+c-1\right)}{d^3}\right)\right)bfe + axe^2 + \frac{\left(2\left(dx+c\right)\mathrm{artanh}\left(dx+c\right)+\log\left(-\left(dx+c\right)^2+1\right)\right)be^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + 1/6*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x
)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c -
 1)*log(d*x + c - 1)/d^4))*b*f^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2
 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/
d^3))*b*f*e + a*x*e^2 + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^
2 + 1))*b*e^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(119) = 238$.
time = 0.41, size = 421, normalized size = 3.51

$$\frac{2\,ad^3f^2x^3 + bd^2f^2x^2 + 6\,ad^3x\cosh\left(1\right)^2 + 6\,ad^3x\sinh\left(1\right)^2 - 4\,bcdf^2x + 6\left(ad^3fx^2 + bd^2fx\right)\cosh\left(1\right) + \left(3\left(bc+b\right)d^2\cosh\left(1\right)^2 + 3\left(bc+b\right)d^2\sinh\left(1\right)^2 - 3\left(bc^2+2\,bc+b\right)df\cosh\left(1\right) + \left(bc^3+3\,bc^2+3\,bc+b\right)f^2 + 3\left(2\left(bc+b\right)d^2\cosh\left(1\right) - \left(bc^2+2\,bc+b\right)df\right)\sinh\left(1\right)\right)\log\left(dx+c+1\right) - \left(3\left(bc-b\right)d^2\cosh\left(1\right)^2 + 3\left(bc-b\right)d^2\sinh\left(1\right)^2 - 3\left(bc^2-2\,bc+b\right)df\cosh\left(1\right) + \left(bc^3-3\,bc^2+3\,bc-b\right)f^2 + 3\left(2\left(bc-b\right)d^2\cosh\left(1\right) - \left(bc^2-2\,bc+b\right)df\right)\sinh\left(1\right)\right)\log\left(dx+c-1\right) + \left(bd^3f^2x^3 + 3\,bd^3fx^2\cosh\left(1\right) + 3\,bd^3x\cosh\left(1\right)^2 + 3\,bd^3x\sinh\left(1\right)^2 + 3\left(bd^3fx^2 + 2\,bd^3x\cosh\left(1\right)\right)\sinh\left(1\right)\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right) + 6\left(ad^3fx^2 + 2\,ad^3x\cosh\left(1\right) + bd^2fx\right)\sinh\left(1\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + b*d^2*f^2*x^2 + 6*a*d^3*x*cosh(1)^2 + 6*a*d^3*x*sinh
(1)^2 - 4*b*c*d*f^2*x + 6*(a*d^3*f*x^2 + b*d^2*f*x)*cosh(1) + (3*(b*c + b)*
```

```
d^2*cosh(1)^2 + 3*(b*c + b)*d^2*sinh(1)^2 - 3*(b*c^2 + 2*b*c + b)*d*f*cosh(
1) + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2 + 3*(2*(b*c + b)*d^2*cosh(1) - (b*c^
2 + 2*b*c + b)*d*f)*sinh(1))*log(d*x + c + 1) - (3*(b*c - b)*d^2*cosh(1)^2
+ 3*(b*c - b)*d^2*sinh(1)^2 - 3*(b*c^2 - 2*b*c + b)*d*f*cosh(1) + (b*c^3 -
3*b*c^2 + 3*b*c - b)*f^2 + 3*(2*(b*c - b)*d^2*cosh(1) - (b*c^2 - 2*b*c + b)
*d*f)*sinh(1))*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*f*x^2*cosh(1) +
3*b*d^3*x*cosh(1)^2 + 3*b*d^3*x*sinh(1)^2 + 3*(b*d^3*f*x^2 + 2*b*d^3*x*cosh
(1))*sinh(1))*log(-(d*x + c + 1)/(d*x + c - 1)) + 6*(a*d^3*f*x^2 + 2*a*d^3*
x*cosh(1) + b*d^2*f*x)*sinh(1))/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(105) = 210$.
time = 2.26, size = 369, normalized size = 3.08

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{atanh}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{atanh}(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bc^2f^2\operatorname{atanh}(c+dx)}{d^3} + \frac{bce^2\operatorname{atanh}(c+dx)}{d} - \frac{2bcef\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{2bcef\operatorname{atanh}(c+dx)}{d^2} - \frac{2bcf^2x}{3d^2} + \frac{bcf^2\operatorname{atanh}(c+dx)}{d^3} + be^2x\operatorname{atanh}(c+dx) + befx^2\operatorname{atanh}(c+dx) + \frac{bf^2x^3\operatorname{atanh}(c+dx)}{3} + \frac{be^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{be^2\operatorname{atanh}(c+dx)}{d} + \frac{befx}{d} + \frac{bf^2x^2}{6d} - \frac{bef\operatorname{atanh}(c+dx)}{d^2} + \frac{bf^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{3d^3} - \frac{bf^2\operatorname{atanh}(c+dx)}{3d^3} & \text{for}\ d \neq 0 \\ (a+b\operatorname{atanh}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atanh(c + d*
x)/(3*d**3) - b*c**2*e*f*atanh(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x + 1/
d)/d**3 - b*c**2*f**2*atanh(c + d*x)/d**3 + b*c*e**2*atanh(c + d*x)/d - 2*b
*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*atanh(c + d*x)/d**2 - 2*b*c*f**2
*x/(3*d**2) + b*c*f**2*atanh(c + d*x)/d**3 + b*e**2*x*atanh(c + d*x) + b*e*
f*x**2*atanh(c + d*x) + b*f**2*x**3*atanh(c + d*x)/3 + b*e**2*log(c/d + x +
 1/d)/d - b*e**2*atanh(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) - b*e*f*a
tanh(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*atanh(c +
d*x)/(3*d**3), Ne(d, 0)), ((a + b*atanh(c))*(e**2*x + e*f*x**2 + f**2*x**3/
3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(112) = 224$.
time = 0.43, size = 976, normalized size = 8.13

$$\frac{1}{6}\left((c+1)d-(c-1)d\right)\left(\frac{\left(\frac{3(dx+c+1)^2bd^2e^2}{(dx+c-1)^2} - \frac{6(dx+c+1)bd^2e^2}{dx+c-1} + 3bd^2e^2 - \frac{6(dx+c+1)^2bcdef}{(dx+c-1)^2} + \frac{12(dx+c+1)bcdef}{dx+c-1} - 6bcdef + \frac{3(dx+c+1)^2bc^2f^2}{(dx+c-1)^2} - \frac{6(dx+c+1)bc^2f^2}{dx+c-1} + 3bc^2f^2 + \frac{6(dx+c+1)^2bdef}{(dx+c-1)^2} - \frac{6(dx+c+1)bdef}{dx+c-1} - \frac{6(dx+c+1)^2bcf^2}{(dx+c-1)^2} + \frac{6(dx+c+1)bcf^2}{dx+c-1} + \frac{3(dx+c+1)^2bf^2}{(dx+c-1)^2} + bf^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^3d^4}{(dx+c-1)^3} - \frac{3(dx+c+1)^2d^4}{(dx+c-1)^2} + \frac{3(dx+c+1)d^4}{dx+c-1} - d^4} + \frac{2\left(\frac{3(dx+c+1)^2ad^2e^2}{(dx+c-1)^2} - \frac{6(dx+c+1)ad^2e^2}{dx+c-1} + 3ad^2e^2 - \frac{6(dx+c+1)^2acdef}{(dx+c-1)^2} + \frac{12(dx+c+1)acdef}{dx+c-1} - 6acdef + \frac{3(dx+c+1)^2ac^2f^2}{(dx+c-1)^2} - \frac{6(dx+c+1)ac^2f^2}{dx+c-1} + 3ac^2f^2 + \frac{6(dx+c+1)^2adef}{(dx+c-1)^2} - \frac{6(dx+c+1)adef}{dx+c-1} + \frac{3(dx+c+1)^2bdef}{(dx+c-1)^2} - \frac{6(dx+c+1)bdef}{dx+c-1} + 3bdef - \frac{6(dx+c+1)^2acf^2}{(dx+c-1)^2} + \frac{6(dx+c+1)acf^2}{dx+c-1} - \frac{3(dx+c+1)^2bcf^2}{(dx+c-1)^2} + \frac{6(dx+c+1)bcf^2}{dx+c-1} - 3bcf^2 + \frac{3(dx+c+1)^2af^2}{(dx+c-1)^2} + af^2 + \frac{(dx+c+1)^2bf^2}{(dx+c-1)^2} - \frac{(dx+c+1)bf^2}{dx+c-1}\right)}{\frac{(dx+c+1)^3d^4}{(dx+c-1)^3} - \frac{3(dx+c+1)^2d^4}{(dx+c-1)^2} + \frac{3(dx+c+1)d^4}{dx+c-1} - d^4} - \frac{(3bd^2e^2 - 6bcdef + 3bc^2f^2 + bf^2)\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^4} + \frac{(3bd^2e^2 - 6bcdef + 3bc^2f^2 + bf^2)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*((c + 1)*d - (c - 1)*d)*((3*(d*x + c + 1)^2*b*d^2*e^2/(d*x + c - 1)^2 -
 6*(d*x + c + 1)*b*d^2*e^2/(d*x + c - 1) + 3*b*d^2*e^2 - 6*(d*x + c + 1)^2*
b*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*c*d*e*f/(d*x + c - 1) - 6*b*
c*d*e*f + 3*(d*x + c + 1)^2*b*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*c
^2*f^2/(d*x + c - 1) + 3*b*c^2*f^2 + 6*(d*x + c + 1)^2*b*d*e*f/(d*x + c - 1
)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*c*f^2/(d*
x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^2*b*
```

```
f^2/(d*x + c - 1)^2 + b*f^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c +
1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c - 1)^2 + 3*(d*x +
 c + 1)*d^4/(d*x + c - 1) - d^4) + 2*(3*(d*x + c + 1)^2*a*d^2*e^2/(d*x + c
- 1)^2 - 6*(d*x + c + 1)*a*d^2*e^2/(d*x + c - 1) + 3*a*d^2*e^2 - 6*(d*x + c
 + 1)^2*a*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*a*c*d*e*f/(d*x + c - 1
) - 6*a*c*d*e*f + 3*(d*x + c + 1)^2*a*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c
+ 1)*a*c^2*f^2/(d*x + c - 1) + 3*a*c^2*f^2 + 6*(d*x + c + 1)^2*a*d*e*f/(d*x
 + c - 1)^2 - 6*(d*x + c + 1)*a*d*e*f/(d*x + c - 1) + 3*(d*x + c + 1)^2*b*d
*e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) + 3*b*d*e*f -
6*(d*x + c + 1)^2*a*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*a*c*f^2/(d*x +
c - 1) - 3*(d*x + c + 1)^2*b*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^
2/(d*x + c - 1) - 3*b*c*f^2 + 3*(d*x + c + 1)^2*a*f^2/(d*x + c - 1)^2 + a*f
^2 + (d*x + c + 1)^2*b*f^2/(d*x + c - 1)^2 - (d*x + c + 1)*b*f^2/(d*x + c -
 1))/((d*x + c + 1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c
- 1)^2 + 3*(d*x + c + 1)*d^4/(d*x + c - 1) - d^4) - (3*b*d^2*e^2 - 6*b*c*d*
e*f + 3*b*c^2*f^2 + b*f^2)*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^4 + (3*b
*d^2*e^2 - 6*b*c*d*e*f + 3*b*c^2*f^2 + b*f^2)*log(-(d*x + c + 1)/(d*x + c -
 1))/d^4)
```

Mupad [B]
time = 1.39, size = 381, normalized size = 3.18

$$x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-\ln(1-d\,x-c)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)-x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f-3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2-3)}{3\,d^2}\right)+\ln(c+d\,x+1)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)+\frac{a\,f^2\,x^3}{3}+\frac{\ln(c+d\,x-1)\left(\frac{b\,f^2}{6}+d\left(\frac{b\,e\,f\,c^2}{2}-b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}-\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}-\frac{b\,c^3\,f^2}{6}-\frac{b\,c\,f^2}{2}\right)}{d^3}+\frac{\ln(c+d\,x+1)\left(\frac{b\,f^2}{6}-d\left(\frac{b\,e\,f\,c^2}{2}+b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}+\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}+\frac{b\,c^3\,f^2}{6}+\frac{b\,c\,f^2}{2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atanh(c + d*x)),x)
```

```
[Out] x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - log(1 - d*x - c)*
((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) - x*((2*c*((f*(b*f + 6*a*c*f
+ 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*c^2*f^2 - 3*a*f^2 + 3*a*d^2*e^
2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 - 3))/(3*d^2)) + log(
c + d*x + 1)*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) + (a*f^2*x^3)/3
+ (log(c + d*x - 1)*((b*f^2)/6 + d*((b*e*f)/2 + (b*c^2*e*f)/2 - b*c*e*f) +
d^2*((b*e^2)/2 - (b*c*e^2)/2) + (b*c^2*f^2)/2 - (b*c^3*f^2)/6 - (b*c*f^2)/2
))/d^3 + (log(c + d*x + 1)*((b*f^2)/6 - d*((b*e*f)/2 + (b*c^2*e*f)/2 + b*c*
e*f) + d^2*((b*e^2)/2 + (b*c*e^2)/2) + (b*c^2*f^2)/2 + (b*c^3*f^2)/6 + (b*c
*f^2)/2))/d^3
```

3.33 $\int (e+fx)\left(a+b\tanh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)^2\log(1-c-dx)}{4d^2f}-\frac{b(de-(1+c)f)^2\log(1+c+dx)}{4d^2f}$$

```
[Out] 1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arctanh(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*ln(-d
*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*ln(d*x+c+1)/d^2/f
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6246, 6063, 716, 647, 31}

$$\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}+\frac{b(-cf+de+f)^2\log(-c-dx+1)}{4d^2f}-\frac{b(de-(c+1)f)^2\log(c+dx+1)}{4d^2f}+\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcTanh[c + d*x]))/(2*f) + (b*(d*e + f
- c*f)^2*Log[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*Log[1 + c + d
*x])/(4*d^2*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tanh^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tanh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}-\frac{b\text{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1-x^2}\,dx,x,c\right.}{2f}$$

$$=\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}-\frac{b\text{Subst}\left(\int\left(-\frac{f^2}{d^2}+\frac{d^2e^2-2cdef+(\right.\right.}{}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}-\frac{b\text{Subst}\left(\int\frac{d^2e^2-2cdef+(1}{}\right.}{}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}-\frac{(b(de+f-cf)^2)\,\text{Subs}}{4d}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)^2\log(1-}{4d^2f}$$

Mathematica [A]
time = 0.07, size = 138, normalized size = 1.42

$$aex+\frac{bfx}{2d}+\frac{1}{2}afx^2+bex\tanh^{-1}(c+dx)+\frac{1}{2}bfx^2\tanh^{-1}(c+dx)+\frac{b(1-2c+c^2)f\log(1-c-dx)}{4d^2}+\frac{b(-1-2c-c^2)f\log(1+c+dx)}{4d^2}+\frac{be(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcTanh[c + d*x] + (b*f*x^2*Arc
Tanh[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-1
- 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c - d*x
]) + (1 + c)*Log[1 + c + d*x]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(89) = 178$.
time = 0.19, size = 185, normalized size = 1.91

method	result
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-e(dx+c)d-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arctanh}(dx+c)fc(dx+c)}{d}+e(dx+c)b\,\mathrm{arctanh}(dx+c)+\frac{b\,\mathrm{arctanh}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx\cdots}{2d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-e(dx+c)d-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arctanh}(dx+c)fc(dx+c)}{d}+e(dx+c)b\,\mathrm{arctanh}(dx+c)+\frac{b\,\mathrm{arctanh}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx\cdots}{2d}}{d}$
risch	$\frac{bx(fx+2e)\ln(dx+c+1)}{4}-\frac{bf\,x^2\ln(-dx-c+1)}{4}-\frac{bex\ln(-dx-c+1)}{2}+\frac{af\,x^2}{2}-\frac{\ln(dx+c+1)b\,c^2f}{4d^2}+\frac{\ln(dx+c+1)bc\cdots}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/d*(f*c*(d*x+c)-e*(d*x+c)*d-1/2*f*(d*x+c)^2)-b/d*arctanh(d*x+c)*f*c*
(d*x+c)+e*(d*x+c)*b*arctanh(d*x+c)+1/2*b/d*arctanh(d*x+c)*f*(d*x+c)^2+1/2*b
/d*f*(d*x+c)-1/2*b/d*ln(d*x+c-1)*f*c+1/2*b*e*ln(d*x+c-1)+1/4*b/d*ln(d*x+c-1
)*f-1/2*b/d*ln(d*x+c+1)*f*c+1/2*b*e*ln(d*x+c+1)-1/4*b/d*ln(d*x+c+1)*f)
```

Maxima [A]
time = 0.26, size = 111, normalized size = 1.14

$$\frac{1}{2}afx^2+\frac{1}{4}\left(2x^2\,\mathrm{artanh}\,(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bf+axe+\frac{(2(dx+c)\,\mathrm{artanh}\,(dx+c)+\log(-(dx+c)^2+1))be}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*lo
g(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f + a*x*e + 1
/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*e/d
```

Fricas [A]
time = 0.37, size = 177, normalized size = 1.82

$$\frac{2ad^2fx^2+4ad^2x\cosh(1)+4ad^2x\sinh(1)+2bdfx+(2(bc+b)d\cosh(1)+2(bc+b)d\sinh(1)-(bc^2+2bc+b)f)\log(dx+c+1)-(2(bc-b)d\cosh(1)+2(bc-b)d\sinh(1)-(bc^2-2bc+b)f)\log(dx+c-1)+(bd^2fx^2+2bd^2x\cosh(1)+2bd^2x\sinh(1))\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*f*x^2 + 4*a*d^2*x*cosh(1) + 4*a*d^2*x*sinh(1) + 2*b*d*f*x + (2
*(b*c + b)*d*cosh(1) + 2*(b*c + b)*d*sinh(1) - (b*c^2 + 2*b*c + b)*f)*log(d
*x + c + 1) - (2*(b*c - b)*d*cosh(1) + 2*(b*c - b)*d*sinh(1) - (b*c^2 - 2*b
*c + b)*f)*log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*x*cosh(1) + 2*b*d^2*x*
sinh(1))*log(-(d*x + c + 1)/(d*x + c - 1)))/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.
time = 1.04, size = 173, normalized size = 1.78

$$\begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2 f \operatorname{atanh}(c+dx)}{2d^2} + \frac{bce \operatorname{atanh}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{bcf \operatorname{atanh}(c+dx)}{d^2} + bex \operatorname{atanh}(c+dx) + \frac{bfx^2 \operatorname{atanh}(c+dx)}{2} + \frac{be \log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{be \operatorname{atanh}(c+dx)}{d} + \frac{bfx}{2d} - \frac{bf \operatorname{atanh}(c+dx)}{2d^2} & \text{for } d \neq 0 \\ (a + b \operatorname{atanh}(c)) \left(ex + \frac{fx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atanh(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atanh(c + d*x)/(2*d**2) + b*c*e*at
anh(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*atanh(c + d*x)/d**2
+ b*e*x*atanh(c + d*x) + b*f*x**2*atanh(c + d*x)/2 + b*e*log(c/d + x + 1/d)
/d - b*e*atanh(c + d*x)/d + b*f*x/(2*d) - b*f*atanh(c + d*x)/(2*d**2), Ne(d
, 0)), ((a + b*atanh(c))*(e*x + f*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(89) = 178.
time = 0.43, size = 341, normalized size = 3.52

$$\frac{1}{2}\left((c+1)d - (c-1)d\right)\left(\frac{\left(\frac{(dx+c+1)bde}{dx+c-1} - bde - \frac{(dx+c+1)bcf}{dx+c-1} + bcf + \frac{(dx+c+1)bf}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2 d^3}{(dx+c-1)^2} - \frac{2(dx+c+1)d^3}{dx+c-1} + d^3} + \frac{\frac{2(dx+c+1)ade}{dx+c-1} - 2ade - \frac{2(dx+c+1)acf}{dx+c-1} + 2acf + \frac{2(dx+c+1)af}{dx+c-1} + \frac{(dx+c+1)bf}{dx+c-1} - bf}{\frac{(dx+c+1)^2 d^3}{(dx+c-1)^2} - \frac{2(dx+c+1)d^3}{dx+c-1} + d^3} - \frac{(bde - bcf)\log\left(-\frac{dx+c+1}{dx+c-1} + 1\right)}{d^3} + \frac{(bde - bcf)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*(((d*x + c + 1)*b*d*e/(d*x + c - 1) - b*d*e - (
d*x + c + 1)*b*c*f/(d*x + c - 1) + b*c*f + (d*x + c + 1)*b*f/(d*x + c - 1))
*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^2*d^3/(d*x + c - 1)^2 - 2
*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) + (2*(d*x + c + 1)*a*d*e/(d*x + c -
 1) - 2*a*d*e - 2*(d*x + c + 1)*a*c*f/(d*x + c - 1) + 2*a*c*f + 2*(d*x + c
+ 1)*a*f/(d*x + c - 1) + (d*x + c + 1)*b*f/(d*x + c - 1) - b*f)/((d*x + c +
 1)^2*d^3/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) - (b*d
*e - b*c*f)*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^3 + (b*d*e - b*c*f)*log
(-(d*x + c + 1)/(d*x + c - 1))/d^3)
```

Mupad [B]
time = 1.34, size = 136, normalized size = 1.40

$$a\,e\,x + \frac{a\,f\,x^2}{2} + \frac{b\,e\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 - 1\right)}{2\,d} - \frac{b\,f\,\operatorname{atanh}(c + d\,x)}{2\,d^2} + \frac{b\,f\,x^2\,\operatorname{atanh}(c + d\,x)}{2} + \frac{b\,f\,x}{2\,d} + b\,e\,x\,\operatorname{atanh}(c + d\,x) - \frac{b\,c^2\,f\,\operatorname{atanh}(c + d\,x)}{2\,d^2} - \frac{b\,c\,f\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 - 1\right)}{2\,d^2} + \frac{b\,c\,e\,\operatorname{atanh}(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atanh(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/(2*d) - (b*f*a
tanh(c + d*x))/(2*d^2) + (b*f*x^2*atanh(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x
*atanh(c + d*x) - (b*c^2*f*atanh(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x
^2 + 2*c*d*x - 1))/(2*d^2) + (b*c*e*atanh(c + d*x))/d
```

3.34 $\int \left(a+b\tanh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$ax+\frac{b(c+dx)\tanh^{-1}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arctanh(d*x+c)/d+1/2*b*ln(1-(d*x+c)^2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {6238, 6021, 266}

$$ax+\frac{b\log\left(1-(c+dx)^2\right)}{2d}+\frac{b(c+dx)\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcTanh[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}(c+dx)\right)\,dx &= ax+b\int \tanh^{-1}(c+dx)\,dx\\ &= ax+\frac{b\text{Subst}\left(\int \tanh^{-1}(x)\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\tanh^{-1}(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\tanh^{-1}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.20

$$ax+bx\tanh^{-1}(c+dx)+\frac{b(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Log
[1 + c + d*x]))/(2*d)
```

Maple [A]
time = 0.58, size = 44, normalized size = 1.10

method	result
derivativedivides	$\frac{(dx+c)a+(dx+c)b\,\mathrm{arctanh}(dx+c)+\frac{b\ln\left(1-(dx+c)^2\right)}{2}}{d}$
default	$ax+b\,\mathrm{arctanh}\,(dx+c)\,x+\frac{b\,\mathrm{arctanh}(dx+c)c}{d}+\frac{b\ln\left(1-(dx+c)^2\right)}{2d}$
risch	$ax+\frac{b\ln(dx+c+1)x}{2}-\frac{bx\ln(-dx-c+1)}{2}+\frac{b\ln(-dx-c-1)c}{2d}-\frac{b\ln(dx+c-1)c}{2d}+\frac{b\ln(-dx-c-1)}{2d}+\frac{b\ln(dx+c}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*arctanh(d*x+c)*x+b/d*arctanh(d*x+c)*c+1/2*b*ln(1-(d*x+c)^2)/d
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.90

$$ax+\frac{\left(2\,(dx+c)\,\mathrm{artanh}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b/d
```

Fricas [A]
time = 0.38, size = 61, normalized size = 1.52

$$\frac{bdx\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,adx+(bc+b)\log\left(dx+c+1\right)-(bc-b)\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x
+ c + 1) - (b*c - b)*log(d*x + c - 1))/d
```

Sympy [A]
time = 0.23, size = 46, normalized size = 1.15

$$ax+b\left(\begin{cases}\frac{c\operatorname{atanh}\left(c+dx\right)}{d}+x\operatorname{atanh}\left(c+dx\right)+\frac{\log\left(c+dx+1\right)}{d}-\frac{\operatorname{atanh}\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\operatorname{atanh}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atanh(c + d*x)/d + x*atanh(c + d*x) + log(c + d*x + 1)
/d - atanh(c + d*x)/d, Ne(d, 0)), (x*atanh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(38) = 76.
time = 0.40, size = 200, normalized size = 5.00

$$\frac{1}{2}\left((c+1)d-(c-1)d\right)b\left(\frac{\log\left(\frac{|-dx-c-1|}{|dx+c-1|}\right)}{d^2}-\frac{\log\left(\left|-\frac{dx+c+1}{dx+c-1}+1\right|\right)}{d^2}+\frac{\log\left(-\frac{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}+1}{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}-1}\right)}{d^2\left(\frac{dx+c+1}{dx+c-1}-1\right)}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*b*(log(abs(-d*x - c - 1)/abs(d*x + c - 1))/d^2
- log(abs(-(d*x + c + 1)/(d*x + c - 1) + 1))/d^2 + log(-(c - ((d*x + c + 1)
```

```
*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d) + 1)/
(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x
+ c - 1) - d) - 1))/(d^2*((d*x + c + 1)/(d*x + c - 1) - 1))) + a*x
```

Mupad [B]
time = 1.44, size = 48, normalized size = 1.20

$$a\,x+\frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2}+b\,c\,\mathrm{atanh}\left(c+d\,x\right)}{d}+b\,x\,\mathrm{atanh}\left(c+d\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c + d*x),x)
```

```
[Out] a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*atanh(c + d*x))/d + b*x
*atanh(c + d*x)
```

3.35 $\int \frac{a+b\tanh^{-1}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=130

$$-\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+c}\right.}{2f}$$

```
[Out] -(a+b*arctanh(d*x+c))*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))*ln(2*d*(f*x+e)
/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*polylog(2,1-2/(d*x+c+1))/f-1/2*b*polylog(2
,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6246, 6057, 2449, 2352, 2497}

$$\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{f}-\frac{b\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTanh[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c +
d*x])*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*PolyLog[
2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e +
f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2d(e}{(de+f-cf}\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2d(e}{(de+f-cf}\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2d(e}{(de+f-cf}\right.}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 329, normalized size = 2.53

$$\frac{a\log(e+fx)+b\tanh^{-1}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+\log\left(i\sinh\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\right)\right)-\frac{1}{2}ib\left(-\frac{1}{4}i(\pi-2i\tanh^{-1}(c+dx))^2+i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)^2+(\pi-2i\tanh^{-1}(c+dx))\log\left(1+e^{2\tanh^{-1}(c+dx)}\right)+2i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)-(\pi-2i\tanh^{-1}(c+dx))\log\left(\frac{2}{\sqrt{1-(c+dx)^2}}\right)-2i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\log\left(2i\sinh\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\right)-i\text{PolyLog}\left(2,-e^{2\tanh^{-1}(c+dx)}\right)-i\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] + b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I
*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) - (I/2)*b*((-1/4*I)*(Pi
- (2*I)*ArcTanh[c + d*x])^2 + I*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])
^2 + (Pi - (2*I)*ArcTanh[c + d*x])*Log[1 + E^(2*ArcTanh[c + d*x])] + (2*I)*
```

```
(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])*Log[1 - E^(-2*(ArcTanh[(d*e - c
*f)/f] + ArcTanh[c + d*x]))] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2/Sqrt[1 -
 (c + d*x)^2]] - (2*I)*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])*Log[(2*I
)*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]] - I*PolyLog[2, -E^(2*Arc
Tanh[c + d*x])] - I*PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c +
d*x]))]))/f
```

Maple [A]
time = 13.64, size = 220, normalized size = 1.69

method	result
risch	$-\frac{b\,\mathrm{dilog}\left(\frac{(-dx-c+1)f+cf-de-f}{cf-de-f}\right)}{2f}-\frac{b\ln(-dx-c+1)\ln\left(\frac{(-dx-c+1)f+cf-de-f}{cf-de-f}\right)}{2f}+\frac{a\ln((-dx-c+1)f+cf-de-f)}{f}+$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\operatorname{arctanh}(dx+c)}{f}-\frac{bd\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)-f}{-cf+de-f}\right)}{2f}-\frac{bd\,\mathrm{dilog}\left(\frac{-f(dx+c)-f}{-cf+de-f}\right)}{2f}}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\operatorname{arctanh}(dx+c)}{f}-\frac{bd\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)-f}{-cf+de-f}\right)}{2f}-\frac{bd\,\mathrm{dilog}\left(\frac{-f(dx+c)-f}{-cf+de-f}\right)}{2f}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d*ln(c*f-d*e-f*(d*x+c))/f+b*d*ln(c*f-d*e-f*(d*x+c))/f*arctanh(d*x+c)
-1/2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))-1/2*b*d/f*
dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))+1/2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(
d*x+c)+f)/(-c*f+d*e+f))+1/2*b*d/f*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x) + a*lo
g(f*x + e)/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(d*x + c) + a)/(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atanh}{\left(c + dx \right)}}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(f*x+e),x)
[Out] Integral((a + b*atanh(c + d*x))/(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arctanh(d*x + c) + a)/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c+d\,x\right)}{e+f\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(e + f*x),x)
[Out] int((a + b*atanh(c + d*x))/(e + f*x), x)
```

3.36 $\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(1-c-dx)}{2f(de+f-cf)}+\frac{bd\log(1+c+dx)}{2f(de-f-cf)}-\frac{bd\log(e+fx)}{(de+f-cf)(de-(1+c)f)}$$

```
[Out] (-a-b*arctanh(d*x+c))/f/(f*x+e)-1/2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*d
*ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6244, 2007, 719, 31, 646}

$$-\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(-c-dx+1)}{2f(-cf+de+f)}+\frac{bd\log(c+dx+1)}{2f(-cf+de-f)}-\frac{bd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c + d*x])/(f*(e + f*x))) - (b*d*Log[1 - c - d*x])/(2*f*(d*
e + f - c*f)) + (b*d*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*Log[e +
 f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^2}\,dx &= -\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)(1-(c+dx)^2)}\,dx}{f}\\ &= -\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}\\ &= -\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2}\,dx}{f\left(-d^2e^2+2cdef+(1-c^2)f^2\right)}+\frac{(bdf)\int}{-d^2e^2+2cdef}\\ &= -\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(e+fx)}{(de-f-cf)(de+f-cf)}-\frac{(bd^3)\int \frac{1}{-d-cd-d^2x}\,dx}{2f(de-f-cf)}\\ &= -\frac{a+b\tanh^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(1-c-dx)}{2f(de+f-cf)}+\frac{bd\log(1+c+dx)}{2f(de-f-cf)}-\frac{b}{(de-f}\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 125, normalized size = 1.09

$$\frac{1}{2}\left(-\frac{2a}{f(e+fx)}-\frac{2b\tanh^{-1}(c+dx)}{f(e+fx)}+\frac{bd\log(1-c-dx)}{f(-de+(-1+c)f)}-\frac{bd\log(1+c+dx)}{f(-de+f+cf)}-\frac{2bd\log(e+fx)}{d^2e^2-2cdef+(-1+c^2)\,f^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^2,x]
```

```
[Out] ((-2*a)/(f*(e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (b*d*Log[1 -
 c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e) +
f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2
```

Maple [**A**]
time = 0.76, size = 170, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\,\mathrm{arctanh}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(dx+c+1)}{f(2cf-2de+2f)}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}+\frac{b\,d^2\ln(dx+c-1)}{f(2cf-2de-2f)}}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\,\mathrm{arctanh}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(dx+c+1)}{f(2cf-2de+2f)}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}+\frac{b\,d^2\ln(dx+c-1)}{f(2cf-2de-2f)}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{2f(fx+e)}-\frac{\ln(dx+c+1)bcd\,f^2x-\ln(dx+c+1)b\,d^2efx-\ln(-dx-c+1)bcd\,f^2x+\ln(-dx-c+1)b\,d^2efx+\ln(dx+c+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d^2/(c*f-d*e-f*(d*x+c))/f+b*d^2/(c*f-d*e-f*(d*x+c))/f*arctanh(d*x+c)
-b*d^2/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-b*d^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f
-d*e-f*(d*x+c))+b*d^2/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1))
```

Maxima [A]
time = 0.26, size = 128, normalized size = 1.11

$$-\frac{1}{2}\left(d\left(\frac{\log(dx+c+1)}{(c+1)f^2-dfe}-\frac{\log(dx+c-1)}{(c-1)f^2-dfe}-\frac{2\log(fx+e)}{2\,cdfe-(c^2-1)f^2-d^2e^2}\right)+\frac{2\,\mathrm{artanh}(dx+c)}{f^2x+fe}\right)b-\frac{a}{f^2x+fe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)
*f^2 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*a
rctanh(d*x + c)/(f^2*x + f*e))*b - a/(f^2*x + f*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(118) = 236$.
time = 0.50, size = 517, normalized size = 4.50

$$\frac{4\,acdf\cosh(1)-2\,ad^2\cosh(1)^2-2\,ad^2\sinh(1)^2-2\,(ac^2-a)f^2-((bc-b)df^2x-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bd^2fx-(bc-b)df)\cosh(1)-(bd^2fx+2\,bd^2\cosh(1)-(bc-b)df)\sinh(1))\log(dx+c+1)+((bc+b)df^2x-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bd^2fx-(bc+b)df)\cosh(1)-(bd^2fx+2\,bd^2\cosh(1)-(bc+b)df)\sinh(1))\log(dx+c-1)-2\,(bdf^2x+bdf\cosh(1)+bdf\sinh(1))\log(fx+\cosh(1)+\sinh(1))+(2\,bcdf\cosh(1)-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bc^2-b)f^2+2\,(bcdf-bd^2\cosh(1))\sinh(1))\log\left(-\frac{dx+c+1}{dx+c-1}\right)+4\,(acdf-ad^2\cosh(1))\sinh(1)}{2\,((c^2-1)f^4x+d^2f\cosh(1)^3+d^2f\sinh(1)^3+(d^2f^2x-2\,cdf^2)\cosh(1)^2+(d^2f^2x-2\,cdf^2+3\,d^2f\cosh(1))\sinh(1)^2-(2\,cdf^3x-(c^2-1)f^3)\cosh(1)-(2\,cdf^3x-3\,d^2f\cosh(1)^2-(c^2-1)f^3-2\,(d^2f^2x-2\,cdf^2)\cosh(1))\sinh(1))}$$

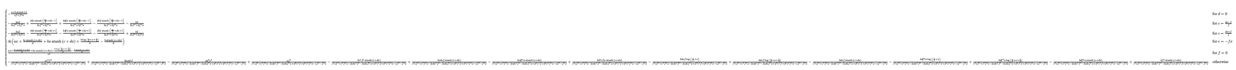
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*c*d*f*cosh(1) - 2*a*d^2*cosh(1)^2 - 2*a*d^2*sinh(1)^2 - 2*(a*c^2 -
 a)*f^2 - ((b*c - b)*d*f^2*x - b*d^2*cosh(1)^2 - b*d^2*sinh(1)^2 - (b*d^2*f
*x - (b*c - b)*d*f)*cosh(1) - (b*d^2*f*x + 2*b*d^2*cosh(1) - (b*c - b)*d*f)
*sinh(1))*log(d*x + c + 1) + ((b*c + b)*d*f^2*x - b*d^2*cosh(1)^2 - b*d^2*s
inh(1)^2 - (b*d^2*f*x - (b*c + b)*d*f)*cosh(1) - (b*d^2*f*x + 2*b*d^2*cosh(
1) - (b*c + b)*d*f)*sinh(1))*log(d*x + c - 1) - 2*(b*d*f^2*x + b*d*f*cosh(1
) + b*d*f*sinh(1))*log(f*x + cosh(1) + sinh(1)) + (2*b*c*d*f*cosh(1) - b*d^
2*cosh(1)^2 - b*d^2*sinh(1)^2 - (b*c^2 - b)*f^2 + 2*(b*c*d*f - b*d^2*cosh(1
```

```
))*sinh(1))*log(-(d*x + c + 1)/(d*x + c - 1)) + 4*(a*c*d*f - a*d^2*cosh(1))
*sinh(1))/((c^2 - 1)*f^4*x + d^2*f*cosh(1)^3 + d^2*f*sinh(1)^3 + (d^2*f^2*x
 - 2*c*d*f^2)*cosh(1)^2 + (d^2*f^2*x - 2*c*d*f^2 + 3*d^2*f*cosh(1))*sinh(1)
^2 - (2*c*d*f^3*x - (c^2 - 1)*f^3)*cosh(1) - (2*c*d*f^3*x - 3*d^2*f*cosh(1)
^2 - (c^2 - 1)*f^3 - 2*(d^2*f^2*x - 2*c*d*f^2)*cosh(1))*sinh(1))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(92) = 184$.
time = 4.62, size = 1658, normalized size = 14.42



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Piecewise((-(a + b*atanh(c))/(e*f + f**2*x), Eq(d, 0)), (-2*a*f/(2*e*f**2 +
 2*f**3*x) + b*d*e*atanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) + b*d*f*x*a
tanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*atanh(d*e/f + d*x - 1)/
(2*e*f**2 + 2*f**3*x) - b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e - f)/f)), (-2
*a*f/(2*e*f**2 + 2*f**3*x) - b*d*e*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**
3*x) - b*d*f*x*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*atanh(d
*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) + b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d
*e + f)/f)), (zoo*(a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c + d*x) + b*log(
c/d + x + 1/d)/d - b*atanh(c + d*x)/d), Eq(e, -f*x)), ((a*x + b*c*atanh(c +
 d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x + 1/d)/d - b*atanh(c + d*x)/d)
/e**2, Eq(f, 0)), (-a*c**2*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**
2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*
a*c*d*e*f/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d
**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3
 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) + a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2
*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x)
- b*c**2*f**2*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 -
 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d
*e*f*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*
f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*a
tanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x
 + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d**2*e*f*x*atanh(c
 + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**
2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*e*f*log(e/f + x)/(c**2
*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d*
*2*e**2*f**2*x - e*f**3 - f**4*x) + b*d*e*f*log(c/d + x + 1/d)/(c**2*e*f**3
 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) - b*d*e*f*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**
4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e
```

```
*f**3 - f**4*x) - b*d*f**2*x*log(e/f + x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*
d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f*
*4*x) + b*d*f**2*x*log(c/d + x + 1/d)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e*
*2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x
) - b*d*f**2*x*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2
- 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*f*
*2*atanh(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f*
*3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(112) = 224$.
time = 0.43, size = 474, normalized size = 4.12

$$-\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{b\log\left(-\frac{(dx+c+1)de}{dx+c-1}+de+\frac{(dx+c+1)cf}{dx+c-1}-cf-\frac{(dx+c+1)f}{dx+c-1}-f\right)}{d^2e^2-2cdef+c^2f^2-f^2}-\frac{b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d^2e^2-\frac{2(dx+c+1)cdef}{dx+c-1}+2cdef+\frac{(dx+c+1)c^2f^2}{dx+c-1}-c^2f^2+\frac{2(dx+c+1)def}{dx+c-1}-\frac{2(dx+c+1)cf^2}{dx+c-1}+\frac{(dx+c+1)f^2}{dx+c-1}+f^2}-\frac{b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2e^2-2cdef+c^2f^2-f^2}-\frac{2a}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d^2e^2-\frac{2(dx+c+1)cdef}{dx+c-1}+2cdef+\frac{(dx+c+1)c^2f^2}{dx+c-1}-c^2f^2+\frac{2(dx+c+1)def}{dx+c-1}-\frac{2(dx+c+1)cf^2}{dx+c-1}+\frac{(dx+c+1)f^2}{dx+c-1}+f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)*d*e/(d*x + c - 1) + d*e
+ (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*f/(d*x + c - 1) - f
)/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - b*log(-(d*x + c + 1)/(d*x + c - 1
))/((d*x + c + 1)*d^2*e^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f
/(d*x + c - 1) + 2*c*d*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2
+ 2*(d*x + c + 1)*d*e*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1)
 + (d*x + c + 1)*f^2/(d*x + c - 1) + f^2) - b*log(-(d*x + c + 1)/(d*x + c -
 1))/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - 2*a/((d*x + c + 1)*d^2*e^2/(d*
x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f/(d*x + c - 1) + 2*c*d*e*f +
(d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2 + 2*(d*x + c + 1)*d*e*f/(d*x
+ c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1) + (d*x + c + 1)*f^2/(d*x + c
 - 1) + f^2))
```

Mupad [**B**]
time = 1.64, size = 170, normalized size = 1.48

$$\ln\left(e+f\,x\right)\,\left(\frac{b\,\left(c-1\right)}{2\,e\,\left(d\,e-f\,\left(c-1\right)\right)}-\frac{b\,\left(c+1\right)}{2\,e\,\left(d\,e-f\,\left(c+1\right)\right)}\right)-\frac{a}{x\,f^2+e\,f}+\frac{b\,\ln\left(1-d\,x-c\right)}{f\,\left(2\,e+2\,f\,x\right)}-\frac{b\,\ln\left(c+d\,x+1\right)}{2\,f\,\left(e+f\,x\right)}-\frac{b\,d\,\ln\left(c+d\,x-1\right)}{2\,f^2-2\,c\,f^2+2\,d\,e\,f}-\frac{b\,d\,\ln\left(c+d\,x+1\right)}{2\,c\,f^2+2\,f^2-2\,d\,e\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(e + f*x)^2,x)
```

```
[Out] log(e + f*x)*((b*(c - 1))/(2*e*(d*e - f*(c - 1))) - (b*(c + 1))/(2*e*(d*e -
 f*(c + 1)))) - a/(e*f + f^2*x) + (b*log(1 - d*x - c))/(f*(2*e + 2*f*x)) -
(b*log(c + d*x + 1))/(2*f*(e + f*x)) - (b*d*log(c + d*x - 1))/(2*f^2 - 2*c*
f^2 + 2*d*e*f) - (b*d*log(c + d*x + 1))/(2*c*f^2 + 2*f^2 - 2*d*e*f)
```

3.37 $\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=167

$$\frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(1-c-dx)}{4f(de+f-cf)^2}+\frac{bd^2\log(1+c+dx)}{4f(de-f-cf)^2}-$$

```
[Out] 1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*arctanh(d*x+c))/f/(f*x+
e)^2-1/4*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*ln(d*x+c+1)/f/(-c*f+
d*e-f)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6244, 2007, 723, 814}

$$-\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(-c-dx+1)}{4f(-cf+de+f)^2}+\frac{bd^2\log(c+dx+1)}{4f(-cf+de-f)^2}-\frac{bd^2(de-cf)\log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2}+\frac{bd}{2(e+fx)(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcTanh[c +
d*x])/(2*f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/(4*f*(d*e + f - c*f)^2)
+ (b*d^2*Log[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*Log
[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
```

```
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
 + 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2(1-(c+dx)^2)}\,dx}{2f}$$

$$= -\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2(1-c^2-2cdx-d^2x^2)}\,dx}{2f}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{}{(e+ \ldots}}{2f\left(-d^2e^2+\ldots\right.}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \left(\frac{d^2}{2(de \ldots}\right.}{}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\tanh^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(1-\ldots}{4f(de+f\ldots}$$

Mathematica [A]
time = 0.26, size = 174, normalized size = 1.04

$$\frac{1}{4}\left(-\frac{2a}{f(e+fx)^2}+\frac{2bd}{(d^2e^2-2cdef+(-1+c^2)f^2)(e+fx)}-\frac{2b\tanh^{-1}(c+dx)}{f(e+fx)^2}-\frac{bd^2\log(1-c-dx)}{f(de+f-cf)^2}+\frac{bd^2\log(1+c+dx)}{f(-de+f+cf)^2}-\frac{4bd^2(de-cf)\log(e+fx)}{(d^2e^2-2cdef+(-1+c^2)f^2)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^3,x]
```

```
[Out] ((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(
e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x
])/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)^2
) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^
2)^2)/4
```

Maple [A]
time = 0.84, size = 266, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3\,\operatorname{arctanh}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{b\,d^3\ln(dx+c+1)}{4f(cf-de+f)^2}-\frac{b\,d^3}{2(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}+\frac{b\,d^3 f\ln(cf-de-}{(cf-de-f)^2(cf}}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3\,\operatorname{arctanh}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{b\,d^3\ln(dx+c+1)}{4f(cf-de+f)^2}-\frac{b\,d^3}{2(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}+\frac{b\,d^3 f\ln(cf-de-}{(cf-de-f)^2(cf}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{4f(fx+e)^2}-\frac{b\,d^4e^2f^2x^2\ln(-dx-c+1)+2b\,d^4e^3fx\ln(-dx-c+1)+2\ln(-dx-c+1)bc\,d^2e^2f^2-4\ln(-dx-c+1)bc}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a*d^3/(c*f-d*e-f*(d*x+c))^2/f-1/2*b*d^3/(c*f-d*e-f*(d*x+c))^2/f*a
rctanh(d*x+c)+1/4*b*d^3/f/(c*f-d*e+f)^2*ln(d*x+c+1)-1/2*b*d^3/(c*f-d*e-f)/(
c*f-d*e+f)/(c*f-d*e-f*(d*x+c))+b*d^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d
*e-f*(d*x+c))*c-b*d^4/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*e-1
/4*b*d^3/f/(c*f-d*e-f)^2*ln(d*x+c-1))
```

Maxima [A]
time = 0.28, size = 316, normalized size = 1.89

$$-\frac{1}{4}\left(d\left(\frac{d\log\left(dx+c+1\right)}{2\,(ce+e)df^2-(c^2+2\,c+1)f^3-d^2fe^2}-\frac{d\log\left(dx+c-1\right)}{2\,(ce-e)df^2-(c^2-2\,c+1)f^3-d^2fe^2}+\frac{4\,(cdf-d^2e)\log\left(fx+e\right)}{4\,cd^3fe^3-2\,(3\,c^2e^2-e^2)d^2f^2+4\,(c^3e-ce)df^3-(c^4-2\,c^2+1)f^4-d^4e^4}+\frac{2}{2\,cdfe^2-(c^2e-e)f^2-d^2e^3+(2\,cdf^2e-(c^2-1)f^3-d^2fe^2)x}\right)+\frac{2\,\operatorname{artanh}\left(dx+c\right)}{f^3x^2+2\,f^2xe+fe^2}\right)b-\frac{a}{2\,(f^3x^2+2\,f^2xe+fe^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(d*(d*log(d*x + c + 1)/(2*(c*e + e)*d*f^2 - (c^2 + 2*c + 1)*f^3 - d^2*
f*e^2) - d*log(d*x + c - 1)/(2*(c*e - e)*d*f^2 - (c^2 - 2*c + 1)*f^3 - d^2*
f*e^2) + 4*(c*d*f - d^2*e)*log(f*x + e)/(4*c*d^3*f*e^3 - 2*(3*c^2*e^2 - e^2
)*d^2*f^2 + 4*(c^3*e - c*e)*d*f^3 - (c^4 - 2*c^2 + 1)*f^4 - d^4*e^4) + 2/(2
*c*d*f*e^2 - (c^2*e - e)*f^2 - d^2*e^3 + (2*c*d*f^2*e - (c^2 - 1)*f^3 - d^2
*f*e^2)*x)) + 2*arctanh(d*x + c)/(f^3*x^2 + 2*f^2*x*e + f*e^2))*b - 1/2*a/(
f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. $2(171) = 342$.
time = 1.09, size = 2443, normalized size = 14.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^4*cosh(1)^4 + 2*a*d^4*sinh(1)^4 - 2*(4*a*c + b)*d^3*f*cosh(1)^3
 - 2*(b*c^2 - b)*d*f^4*x + 2*(a*c^4 - 2*a*c^2 + a)*f^4 + 2*(4*a*d^4*cosh(1)
```

```
 - (4*a*c + b)*d^3*f)*sinh(1)^3 - 2*(b*d^3*f^2*x - 2*(3*a*c^2 + b*c - a)*d^
2*f^2)*cosh(1)^2 - 2*(b*d^3*f^2*x - 6*a*d^4*cosh(1)^2 + 3*(4*a*c + b)*d^3*f
*cosh(1) - 2*(3*a*c^2 + b*c - a)*d^2*f^2)*sinh(1)^2 + 2*(2*b*c*d^2*f^3*x -
(4*a*c^3 + b*c^2 - 4*a*c - b)*d*f^3)*cosh(1) - ((b*c^2 - 2*b*c + b)*d^2*f^4
*x^2 + b*d^4*cosh(1)^4 + b*d^4*sinh(1)^4 + 2*(b*d^4*f*x - (b*c - b)*d^3*f)*
cosh(1)^3 + 2*(b*d^4*f*x + 2*b*d^4*cosh(1) - (b*c - b)*d^3*f)*sinh(1)^3 + (
b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f^2*x + (b*c^2 - 2*b*c + b)*d^2*f^2)*cosh(1
)^2 + (b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f^2*x + 6*b*d^4*cosh(1)^2 + (b*c^2 -
 2*b*c + b)*d^2*f^2 + 6*(b*d^4*f*x - (b*c - b)*d^3*f)*cosh(1))*sinh(1)^2 -
2*((b*c - b)*d^3*f^3*x^2 - (b*c^2 - 2*b*c + b)*d^2*f^3*x)*cosh(1) - 2*((b*c
 - b)*d^3*f^3*x^2 - 2*b*d^4*cosh(1)^3 - (b*c^2 - 2*b*c + b)*d^2*f^3*x - 3*(
b*d^4*f*x - (b*c - b)*d^3*f)*cosh(1)^2 - (b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f
^2*x + (b*c^2 - 2*b*c + b)*d^2*f^2)*cosh(1))*sinh(1))*log(d*x + c + 1) + ((
b*c^2 + 2*b*c + b)*d^2*f^4*x^2 + b*d^4*cosh(1)^4 + b*d^4*sinh(1)^4 + 2*(b*d
^4*f*x - (b*c + b)*d^3*f)*cosh(1)^3 + 2*(b*d^4*f*x + 2*b*d^4*cosh(1) - (b*c
 + b)*d^3*f)*sinh(1)^3 + (b*d^4*f^2*x^2 - 4*(b*c + b)*d^3*f^2*x + (b*c^2 +
2*b*c + b)*d^2*f^2)*cosh(1)^2 + (b*d^4*f^2*x^2 - 4*(b*c + b)*d^3*f^2*x + 6*
b*d^4*cosh(1)^2 + (b*c^2 + 2*b*c + b)*d^2*f^2 + 6*(b*d^4*f*x - (b*c + b)*d^
3*f)*cosh(1))*sinh(1)^2 - 2*((b*c + b)*d^3*f^3*x^2 - (b*c^2 + 2*b*c + b)*d^
2*f^3*x)*cosh(1) - 2*((b*c + b)*d^3*f^3*x^2 - 2*b*d^4*cosh(1)^3 - (b*c^2 +
2*b*c + b)*d^2*f^3*x - 3*(b*d^4*f*x - (b*c + b)*d^3*f)*cosh(1)^2 - (b*d^4*f
^2*x^2 - 4*(b*c + b)*d^3*f^2*x + (b*c^2 + 2*b*c + b)*d^2*f^2)*cosh(1))*sinh
(1))*log(d*x + c - 1) - 4*(b*c*d^2*f^4*x^2 - b*d^3*f*cosh(1)^3 - b*d^3*f*si
nh(1)^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*cosh(1)^2 - (2*b*d^3*f^2*x - b*c*d^
2*f^2 + 3*b*d^3*f*cosh(1))*sinh(1)^2 - (b*d^3*f^3*x^2 - 2*b*c*d^2*f^3*x)*co
sh(1) - (b*d^3*f^3*x^2 - 2*b*c*d^2*f^3*x + 3*b*d^3*f*cosh(1)^2 + 2*(2*b*d^3
*f^2*x - b*c*d^2*f^2)*cosh(1))*sinh(1))*log(f*x + cosh(1) + sinh(1)) - (4*b
*c*d^3*f*cosh(1)^3 - b*d^4*cosh(1)^4 - b*d^4*sinh(1)^4 - 2*(3*b*c^2 - b)*d^
2*f^2*cosh(1)^2 + 4*(b*c^3 - b*c)*d*f^3*cosh(1) - (b*c^4 - 2*b*c^2 + b)*f^4
 + 4*(b*c*d^3*f - b*d^4*cosh(1))*sinh(1)^3 + 2*(6*b*c*d^3*f*cosh(1) - 3*b*d
^4*cosh(1)^2 - (3*b*c^2 - b)*d^2*f^2)*sinh(1)^2 + 4*(3*b*c*d^3*f*cosh(1)^2
 - b*d^4*cosh(1)^3 - (3*b*c^2 - b)*d^2*f^2*cosh(1) + (b*c^3 - b*c)*d*f^3)*si
nh(1))*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*(2*b*c*d^2*f^3*x + 4*a*d^4*cos
h(1)^3 - 3*(4*a*c + b)*d^3*f*cosh(1)^2 - (4*a*c^3 + b*c^2 - 4*a*c - b)*d*f^
3 - 2*(b*d^3*f^2*x - 2*(3*a*c^2 + b*c - a)*d^2*f^2)*cosh(1))*sinh(1))/(d^4*
f*cosh(1)^6 + d^4*f*sinh(1)^6 + (c^4 - 2*c^2 + 1)*f^7*x^2 + 2*(d^4*f^2*x -
2*c*d^3*f^2)*cosh(1)^5 + 2*(d^4*f^2*x - 2*c*d^3*f^2 + 3*d^4*f*cosh(1))*sinh
(1)^5 + (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cosh(1)^4 + (
d^4*f^3*x^2 - 8*c*d^3*f^3*x + 15*d^4*f*cosh(1)^2 + 2*(3*c^2 - 1)*d^2*f^3 +
10*(d^4*f^2*x - 2*c*d^3*f^2)*cosh(1))*sinh(1)^4 - 4*(c*d^3*f^4*x^2 - (3*c^2
 - 1)*d^2*f^4*x + (c^3 - c)*d*f^4)*cosh(1)^3 - 4*(c*d^3*f^4*x^2 - (3*c^2 -
1)*d^2*f^4*x - 5*d^4*f*cosh(1)^3 + (c^3 - c)*d*f^4 - 5*(d^4*f^2*x - 2*c*d^3
*f^2)*cosh(1)^2 - (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cos
h(1))*sinh(1)^3 + (2*(3*c^2 - 1)*d^2*f^5*x^2 - 8*(c^3 - c)*d*f^5*x + (c^4 -
 2*c^2 + 1)*f^5)*cosh(1)^2 + (2*(3*c^2 - 1)*d^2*f^5*x^2 + 15*d^4*f*cosh(1)^
```

```
4 - 8*(c^3 - c)*d*f^5*x + (c^4 - 2*c^2 + 1)*f^5 + 20*(d^4*f^2*x - 2*c*d^3*f
^2)*cosh(1)^3 + 6*(d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cos
h(1)^2 - 12*(c*d^3*f^4*x^2 - (3*c^2 - 1)*d^2*f^4*x + (c^3 - c)*d*f^4)*cosh(
1))*sinh(1)^2 - 2*(2*(c^3 - c)*d*f^6*x^2 - (c^4 - 2*c^2 + 1)*f^6*x)*cosh(1)
 - 2*(2*(c^3 - c)*d*f^6*x^2 - 3*d^4*f*cosh(1)^5 - (c^4 - 2*c^2 + 1)*f^6*x -
 5*(d^4*f^2*x - 2*c*d^3*f^2)*cosh(1)^4 - 2*(d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2
*(3*c^2 - 1)*d^2*f^3)*cosh(1)^3 + 6*(c*d^3*f^4*x^2 - (3*c^2 - 1)*d^2*f^4*x
+ (c^3 - c)*d*f^4)*cosh(1)^2 - (2*(3*c^2 - 1)*d^2*f^5*x^2 - 8*(c^3 - c)*d*f
^5*x + (c^4 - 2*c^2 + 1)*f^5)*cosh(1))*sinh(1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19912 vs. $2(143) = 286$.
time = 12.57, size = 19912, normalized size = 119.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Piecewise(((a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x
 + 1/d)/d - b*atanh(c + d*x)/d)/e**3, Eq(f, 0)), (-(a + b*atanh(c))/(2*e**2
*f + 4*e*f**2*x + 2*f**3*x**2), Eq(d, 0)), (-4*a*f**2/(8*e**2*f**3 + 16*e*f
**4*x + 8*f**5*x**2) + b*d**2*e**2*atanh(d*e/f + d*x - 1)/(8*e**2*f**3 + 16
*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*atanh(d*e/f + d*x - 1)/(8*e**2*f*
*3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*x**2*atanh(d*e/f + d*x - 1)/(
8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*e*f/(8*e**2*f**3 + 16*e*f**4
*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) -
4*b*f**2*atanh(d*e/f + d*x - 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) -
 b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2), Eq(c, (d*e - f)/f)), (-4
*a*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*atanh(d*e/f
 + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*atan
h(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*
x**2*atanh(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d
*e*f/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 +
16*e*f**4*x + 8*f**5*x**2) - 4*b*f**2*atanh(d*e/f + d*x + 1)/(8*e**2*f**3 +
 16*e*f**4*x + 8*f**5*x**2) + b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x*
*2), Eq(c, (d*e + f)/f)), (zoo*(a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c +
d*x) + b*log(c/d + x + 1/d)/d - b*atanh(c + d*x)/d), Eq(e, -f*x)), (-a*c**4
*f**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**
3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f
**3 + 24*c**2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*
f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*
e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*
x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f*
*3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2
```

```
*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2) + 4*a*c**3*d*e*f**3/(2*c**4*e**2*f**
5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**
2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**
3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x
 - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3
*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2
+ 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*
f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*
x + 2*f**7*x**2) - 6*a*c**2*d**2*e**2*f**2/(2*c**4*e**2*f**5 + 4*c**4*e*f**
6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**
3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*c*
*2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x
**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2
+ 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f
+ 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e*
*3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2)
 + 2*a*c**2*f**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8
*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*
d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4
*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 -
 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d
*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d
**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f*
*5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2) + 4*a*c*d**3*e**3*f/(2*c*
*4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16
*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c*
*2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c*
*2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x
 - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e
*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4
*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5
+ 4*e*f**6*x + 2*f**7*x**2) - 4*a*c*d*e*f**3/(2*c**4*e**2*f**5 + 4*c**4*e*f
**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c
**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*
c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7
*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**
2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*
f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*
e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**
2) - a*d**4*e**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8
*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(160) = 320$.

time = 0.50, size = 2567, normalized size = 15.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*((c + 1)*d - (c - 1)*d)*((b*d^2*e - b*c*d*f)*log(-(d*x + c + 1)*d*e/(d
*x + c - 1) + d*e + (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*f
/(d*x + c - 1) - f)/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3*d*
e*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - ((d*x +
c + 1)*b*d^2*e/(d*x + c - 1) - b*d^2*e - (d*x + c + 1)*b*c*d*f/(d*x + c - 1
) + b*c*d*f + (d*x + c + 1)*b*d*f/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x +
c - 1))/((d*x + c + 1)^2*d^4*e^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^4*e^4/
(d*x + c - 1) + d^4*e^4 - 4*(d*x + c + 1)^2*c*d^3*e^3*f/(d*x + c - 1)^2 + 8
*(d*x + c + 1)*c*d^3*e^3*f/(d*x + c - 1) - 4*c*d^3*e^3*f + 6*(d*x + c + 1)^
2*c^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*f^2/(d*x +
 c - 1) + 6*c^2*d^2*e^2*f^2 - 4*(d*x + c + 1)^2*c^3*d*e*f^3/(d*x + c - 1)^2
 + 8*(d*x + c + 1)*c^3*d*e*f^3/(d*x + c - 1) - 4*c^3*d*e*f^3 + (d*x + c + 1
)^2*c^4*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^4/(d*x + c - 1) + c^4*f
^4 + 4*(d*x + c + 1)^2*d^3*e^3*f/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^3*e^3*
f/(d*x + c - 1) - 12*(d*x + c + 1)^2*c*d^2*e^2*f^2/(d*x + c - 1)^2 + 12*(d*
x + c + 1)*c*d^2*e^2*f^2/(d*x + c - 1) + 12*(d*x + c + 1)^2*c^2*d*e*f^3/(d*
x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d*e*f^3/(d*x + c - 1) - 4*(d*x + c + 1)
^2*c^3*f^4/(d*x + c - 1)^2 + 4*(d*x + c + 1)*c^3*f^4/(d*x + c - 1) + 6*(d*x
 + c + 1)^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 2*d^2*e^2*f^2 - 12*(d*x + c + 1)^
2*c*d*e*f^3/(d*x + c - 1)^2 + 4*c*d*e*f^3 + 6*(d*x + c + 1)^2*c^2*f^4/(d*x
+ c - 1)^2 - 2*c^2*f^4 + 4*(d*x + c + 1)^2*d*e*f^3/(d*x + c - 1)^2 + 4*(d*x
 + c + 1)*d*e*f^3/(d*x + c - 1) - 4*(d*x + c + 1)^2*c*f^4/(d*x + c - 1)^2 -
 4*(d*x + c + 1)*c*f^4/(d*x + c - 1) + (d*x + c + 1)^2*f^4/(d*x + c - 1)^2
+ 2*(d*x + c + 1)*f^4/(d*x + c - 1) + f^4) - (b*d^2*e - b*c*d*f)*log(-(d*x
+ c + 1)/(d*x + c - 1))/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^
3*d*e*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - (2*(
d*x + c + 1)*a*d^3*e^2/(d*x + c - 1) - 2*a*d^3*e^2 - 4*(d*x + c + 1)*a*c*d^
2*e*f/(d*x + c - 1) + 4*a*c*d^2*e*f + 2*(d*x + c + 1)*a*c^2*d*f^2/(d*x + c
- 1) - 2*a*c^2*d*f^2 + 2*a*d^2*e*f - (d*x + c + 1)*b*d^2*e*f/(d*x + c - 1)
+ b*d^2*e*f - 2*a*c*d*f^2 + (d*x + c + 1)*b*c*d*f^2/(d*x + c - 1) - b*c*d*f
^2 - 2*(d*x + c + 1)*a*d*f^2/(d*x + c - 1) - (d*x + c + 1)*b*d*f^2/(d*x + c
 - 1) - b*d*f^2)/((d*x + c + 1)^2*d^5*e^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)
*d^5*e^5/(d*x + c - 1) + d^5*e^5 - 5*(d*x + c + 1)^2*c*d^4*e^4*f/(d*x + c -
 1)^2 + 10*(d*x + c + 1)*c*d^4*e^4*f/(d*x + c - 1) - 5*c*d^4*e^4*f + 10*(d*
x + c + 1)^2*c^2*d^3*e^3*f^2/(d*x + c - 1)^2 - 20*(d*x + c + 1)*c^2*d^3*e^3
*f^2/(d*x + c - 1) + 10*c^2*d^3*e^3*f^2 - 10*(d*x + c + 1)^2*c^3*d^2*e^2*f^
3/(d*x + c - 1)^2 + 20*(d*x + c + 1)*c^3*d^2*e^2*f^3/(d*x + c - 1) - 10*c^3
*d^2*e^2*f^3 + 5*(d*x + c + 1)^2*c^4*d*e*f^4/(d*x + c - 1)^2 - 10*(d*x + c
```

```
+ 1)*c^4*d*e*f^4/(d*x + c - 1) + 5*c^4*d*e*f^4 - (d*x + c + 1)^2*c^5*f^5/(d
*x + c - 1)^2 + 2*(d*x + c + 1)*c^5*f^5/(d*x + c - 1) - c^5*f^5 + 3*(d*x +
c + 1)^2*d^4*e^4*f/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^4*e^4*f/(d*x + c - 1
) - d^4*e^4*f - 12*(d*x + c + 1)^2*c*d^3*e^3*f^2/(d*x + c - 1)^2 + 8*(d*x +
 c + 1)*c*d^3*e^3*f^2/(d*x + c - 1) + 4*c*d^3*e^3*f^2 + 18*(d*x + c + 1)^2*
c^2*d^2*e^2*f^3/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*f^3/(d*x + c
 - 1) - 6*c^2*d^2*e^2*f^3 - 12*(d*x + c + 1)^2*c^3*d*e*f^4/(d*x + c - 1)^2
+ 8*(d*x + c + 1)*c^3*d*e*f^4/(d*x + c - 1) + 4*c^3*d*e*f^4 + 3*(d*x + c +
1)^2*c^4*f^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^5/(d*x + c - 1) - c^4*
f^5 + 2*(d*x + c + 1)^2*d^3*e^3*f^2/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^3*e
^3*f^2/(d*x + c - 1) - 2*d^3*e^3*f^2 - 6*(d*x + c + 1)^2*c*d^2*e^2*f^3/(d*x
 + c - 1)^2 - 12*(d*x + c + 1)*c*d^2*e^2*f^3/(d*x + c - 1) + 6*c*d^2*e^2*f^
3 + 6*(d*x + c + 1)^2*c^2*d*e*f^4/(d*x + c - 1)^2 + 12*(d*x + c + 1)*c^2*d*
e*f^4/(d*x + c - 1) - 6*c^2*d*e*f^4 - 2*(d*x + c + 1)^2*c^3*f^5/(d*x + c -
1)^2 - 4*(d*x + c + 1)*c^3*f^5/(d*x + c - 1) + 2*c^3*f^5 - 2*(d*x + c + 1)^
2*d^2*e^2*f^3/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^2*f^3/(d*x + c - 1) +
 2*d^2*e^2*f^3 + 4*(d*x + c + 1)^2*c*d*e*f^4/(d*x + c - 1)^2 - 8*(d*x + c +
 1)*c*d*e*f^4/(d*x + c - 1) - 4*c*d*e*f^4 - 2*(d*x + c + 1)^2*c^2*f^5/(d*x
+ c - 1)^2 + 4*(d*x + c + 1)*c^2*f^5/(d*x + c - 1) + 2*c^2*f^5 - 3*(d*x + c
 + 1)^2*d*e*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d*e*f^4/(d*x + c - 1) + d
*e*f^4 + 3*(d*x + c + 1)^2*c*f^5/(d*x + c - 1)^2 + 2*(d*x + c + 1)*c*f^5/(d
*x + c - 1) - c*f^5 - (d*x + c + 1)^2*f^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)
*f^5/(d*x + c - 1) - f^5))
```

Mupad [B]
time = 3.03, size = 417, normalized size = 2.50

$$\frac{b\,d^2\,\ln\left(c+d\,x+1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2+8\,c\,f^3+4\,d^2\,e^2\,f-8\,d\,e\,f^2+4\,f^3}-\frac{\ln\left(e+f\,x\right)\,\left(b\,d^3\,e-b\,c\,d^2\,f\right)}{c^4\,f^4-4\,c^3\,d\,e\,f^3+6\,c^2\,d^2\,e^2\,f^2-2\,c^2\,f^4-4\,c\,d^3\,e^3\,f+4\,c\,d\,e\,f^3+d^4\,e^4-2\,d^2\,e^2\,f^2+f^4}-\frac{b\,\ln\left(c+d\,x+1\right)}{4\,f\,\left(e^2+2\,e\,f\,x+f^2\,x^2\right)}-\frac{b\,d^2\,\ln\left(c+d\,x-1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2-8\,c\,f^3+4\,d^2\,e^2\,f+8\,d\,e\,f^2+4\,f^3}-\frac{\frac{-a\,c^2\,f^2+2\,a\,c\,d\,e\,f-a\,d^2\,e^2+b\,d\,e\,f+a\,f^2}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}+\frac{b\,d\,f^2\,x}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}}{2\,e^2\,f+4\,e\,f^2\,x+2\,f^3\,x^2}+\frac{b\,\ln\left(1-d\,x-c\right)}{2\,f\,\left(2\,e^2+4\,e\,f\,x+2\,f^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d^2*log(c + d*x + 1))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*e
*f^2 - 8*c*d*e*f^2) - (log(e + f*x)*(b*d^3*e - b*c*d^2*f))/(f^4 - 2*c^2*f^4
 + c^4*f^4 + d^4*e^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 + 6*c^2*d^2*e^2*f^2 - 4*
c*d^3*e^3*f - 4*c^3*d*e*f^3) - (b*log(c + d*x + 1))/(4*f*(e^2 + f^2*x^2 + 2
*e*f*x)) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*f^3 + 4*c^2*f^3 + 4*d^2*e^
2*f + 8*d*e*f^2 - 8*c*d*e*f^2) - ((a*f^2 - a*c^2*f^2 - a*d^2*e^2 + b*d*e*f
+ 2*a*c*d*e*f)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(f^2 - c
^2*f^2 - d^2*e^2 + 2*c*d*e*f))/(2*e^2*f + 2*f^3*x^2 + 4*e*f^2*x) + (b*log(1
 - d*x - c))/(2*f*(2*e^2 + 2*f^2*x^2 + 4*e*f*x))
```

3.38 $\int (e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=562

$$\frac{b^2f^2(de-cf)x}{d^3}+\frac{abf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{2d^3}+\frac{b^2f^3(c+dx)^2}{12d^4}-\frac{b^2f^2(de-cf)\tanh^{-1}(c+dx)}{d^4}+\frac{b^2}{}$$

```
[Out] b^2*f^2*(-c*f+d*e)*x/d^3+1/2*a*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/d
^3+1/12*b^2*f^3*(d*x+c)^2/d^4-b^2*f^2*(-c*f+d*e)*arctanh(d*x+c)/d^4+1/2*b^2
*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*(d*x+c)*arctanh(d*x+c)/d^4+b*f^2*(-
c*f+d*e)*(d*x+c)^2*(a+b*arctanh(d*x+c))/d^4+1/6*b*f^3*(d*x+c)^3*(a+b*arctan
h(d*x+c))/d^4+(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(d*x+c
))^2/d^4-1/4*(d^4*e^4-4*c*d^3*e^3*f+6*(c^2+1)*d^2*e^2*f^2-4*c*(c^2+3)*d*e*f
^3+(c^4+6*c^2+1)*f^4)*(a+b*arctanh(d*x+c))^2/d^4/f+1/4*(f*x+e)^4*(a+b*arcta
nh(d*x+c))^2/f-2*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(
d*x+c))*ln(2/(-d*x-c+1))/d^4+1/12*b^2*f^3*ln(1-(d*x+c)^2)/d^4+1/4*b^2*f*(6*
d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*ln(1-(d*x+c)^2)/d^4-b^2*(-c*f+d*e)*(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^4
```

Rubi [A]
time = 0.71, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6246, 6065, 6021, 266, 6037, 327, 212, 272, 45, 6195, 6095, 6131, 6055, 2449, 2352}

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*(d*e - c*f)*x)/d^3 + (a*b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*
f^2)*x)/(2*d^3) + (b^2*f^3*(c + d*x)^2)/(12*d^4) - (b^2*f^2*(d*e - c*f)*Arc
Tanh[c + d*x])/d^4 + (b^2*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*(c +
 d*x)*ArcTanh[c + d*x])/(2*d^4) + (b*f^2*(d*e - c*f)*(c + d*x)^2*(a + b*Arc
Tanh[c + d*x]))/d^4 + (b*f^3*(c + d*x)^3*(a + b*ArcTanh[c + d*x]))/(6*d^4)
+ ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*ArcTanh[c + d*x
])^2)/d^4 - ((d^4*e^4 - 4*c*d^3*e^3*f + 6*(1 + c^2)*d^2*e^2*f^2 - 4*c*(3 +
c^2)*d*e*f^3 + (1 + 6*c^2 + c^4)*f^4)*(a + b*ArcTanh[c + d*x])^2)/(4*d^4*f)
 + ((e + f*x)^4*(a + b*ArcTanh[c + d*x])^2)/(4*f) - (2*b*(d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x
)])/d^4 + (b^2*f^3*Log[1 - (c + d*x)^2])/(12*d^4) + (b^2*f*(6*d^2*e^2 - 12*
c*d*e*f + (1 + 6*c^2)*f^2)*Log[1 - (c + d*x)^2])/(4*d^4) - (b^2*(d*e - c*f)
*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\tanh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4 \left(a+b\tanh^{-1}(c+dx)\right)^2}{4f} - \frac{b\text{Subst}\left(\int \left(-\frac{f^2(6d^2e^2-12cdef+}{}\right.\right.}{}$$

$$= \frac{(e+fx)^4 \left(a+b\tanh^{-1}(c+dx)\right)^2}{4f} - \frac{b\text{Subst}\left(\int \frac{(d^4e^4-4cd^3e^3f+6(1+c^2}{}\right.}{}$$

$$= \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} + \frac{bf^2(de-cf)(c+dx)^2(a}{d^4}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} + \frac{b^2f(6c}{}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} - \frac{b^2f^2(d}{}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c}{12}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c}{12}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{abf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{2d^3} + \frac{b^2f^3(c}{12}$$

Mathematica [A]
time = 5.99, size = 1082, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (12*a^2*e^3*x + 18*a^2*e^2*f*x^2 + 12*a^2*e*f^2*x^3 + 3*a^2*f^3*x^4 + a*b*(
6*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[c + d*x] - (-2*d*f*
x*(3*(1 + 3*c^2)*f^2 - 3*c*d*f*(8*e + f*x) + d^2*(18*e^2 + 6*e*f*x + f^2*x^
2)) + 3*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 + c)^2*d*e*f^2 -
 (-1 + c)^3*f^3)*Log[1 - c - d*x] + 3*(1 + c)*(-4*d^3*e^3 + 6*(1 + c)*d^2*e
^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x])/d^4) + (12*b^
2*e^3*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*
ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d - (18*b^2*e^
2*f*((1 - 2*c + c^2 - d^2*x^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*(c +
 d*x + 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 2*Log[1/Sqrt[1 - (c + d*x)^2
]] + 2*c*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^2 + (b^2*f^3*(-1 - 11*c^2
 - 10*c*d*x + d^2*x^2 - 3*(1 - 4*c + 6*c^2 - 4*c^3 + c^4 - d^4*x^4)*ArcTanh
[c + d*x]^2 + 2*ArcTanh[c + d*x]*(9*c + 13*c^3 + 3*d*x + 9*c^2*d*x - 3*c*d^
2*x^2 + d^3*x^3 + 12*(c + c^3)*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 8*Log[1/
Sqrt[1 - (c + d*x)^2]] - 36*c^2*Log[1/Sqrt[1 - (c + d*x)^2]] - 12*(c + c^3)
*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^4 - (3*b^2*e*f^2*(1 - (c + d*x)^2
)^(3/2)*(-((c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*c*(c + d*x)*ArcTanh[c + d*
x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c +
d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + ArcT
anh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh[c + d*x]^2*Cosh[3*A
rcTanh[c + d*x]] + 2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-
2*ArcTanh[c + d*x])] + 6*c^2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[
1 + E^(-2*ArcTanh[c + d*x])] - 6*c*Cosh[3*ArcTanh[c + d*x]]*Log[1/Sqrt[1 -
(c + d*x)^2]] + (3*(1 - 4*c + 3*c^2)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x
]*(2 + (3 + 9*c^2)*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 18*c*Log[1/Sqrt[1 -
(c + d*x)^2]])/Sqrt[1 - (c + d*x)^2] - (4*(1 + 3*c^2)*PolyLog[2, -E^(-2*Arc
Tanh[c + d*x])])/(1 - (c + d*x)^2)^(3/2) - Sinh[3*ArcTanh[c + d*x]] + 6*c*A
rcTanh[c + d*x]*Sinh[3*ArcTanh[c + d*x]] - ArcTanh[c + d*x]^2*Sinh[3*ArcTan
h[c + d*x]] - 3*c^2*ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]]))/d^3)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4768 vs. $2(548) = 1096$.
time = 1.10, size = 4769, normalized size = 8.49

method	result	size
risch	Expression too large to display	4356
derivativedivides	Expression too large to display	4769
default	Expression too large to display	4769

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*b^2*ln(d*x+c-1)^2-1/4*e^3*b^2*ln(d*x+c+1)^2+e^3*b^2*arctanh(d*
x+c)*ln(d*x+c-1)+e^3*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/2*e^3*b^2*ln(d*x+c-1)
```

```
*ln(1/2*d*x+1/2*c+1/2)+1/2*e^3*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/2*e
^3*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+e^3*a*b*ln(d*x+c-1)+e^3
*a*b*ln(d*x+c+1)+1/4*a*b/d^3*f^3*ln(d*x+c-1)*c^4+3/8*b^2/d*f*ln(d*x+c+1)^2*
e^2-1/4*b^2/d^2*f^2*ln(d*x+c+1)^2*e-b^2/d^2*f^2*dilog(1/2*d*x+1/2*c+1/2)*e-
b^2*dilog(1/2*d*x+1/2*c+1/2)*e^3+1/12*b^2/d^3*f^3*(d*x+c)^2+1/3*b^2/d^3*f^3
*ln(d*x+c-1)+1/3*b^2/d^3*f^3*ln(d*x+c+1)+1/16*b^2/d^3*f^3*ln(d*x+c+1)^2+1/4
*(c*f-d*e-f*(d*x+c))^4*a^2/d^3/f+1/16*b^2/d^3*f^3*ln(d*x+c-1)^2-1/4*b^2*ln(
d*x+c+1)^2*c*e^3-1/4*b^2*ln(d*x+c-1)^2*c*e^3-b^2*arctanh(d*x+c)^2*c*e^3+b^2
*arctanh(d*x+c)^2*e^3*(d*x+c)+1/6*a*b/d^3*f^3*(d*x+c)^3+1/4*a*b/d^3*f^3*ln(
d*x+c-1)-1/4*a*b/d^3*f^3*ln(d*x+c+1)-a*b*ln(d*x+c-1)*c*e^3+a*b*ln(d*x+c+1)*
c*e^3-2*a*b*arctanh(d*x+c)*c*e^3+2*a*b*arctanh(d*x+c)*e^3*(d*x+c)+3/8*b^2/d
*f*ln(d*x+c-1)^2*e^2+1/4*b^2/d^2*f^2*ln(d*x+c-1)^2*e+3/8*b^2/d^3*f^3*ln(d*x
+c-1)^2*c^2-1/4*b^2/d^3*f^3*ln(d*x+c-1)^2*c+1/4*b^2/d^3*f^3*arctanh(d*x+c)*
ln(d*x+c-1)-1/4*b^2/d^3*f^3*arctanh(d*x+c)*ln(d*x+c+1)+1/2*b^2/d^3*f^3*arct
anh(d*x+c)*(d*x+c)+1/6*b^2/d^3*f^3*arctanh(d*x+c)*(d*x+c)^3-1/8*b^2/d^3*f^3
*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)+1/8*b^2/d^3*f^3*ln(-1/2*d*x-1/2*c+1/2)*
ln(1/2*d*x+1/2*c+1/2)+1/16*b^2/d^3*f^3*ln(d*x+c+1)^2*c^4+1/4*b^2/d^3*f^3*ln
(d*x+c+1)^2*c^3+3/8*b^2/d^3*f^3*ln(d*x+c+1)^2*c^2+1/4*b^2/d^3*f^3*ln(d*x+c+
1)^2*c+b^2/d^3*f^3*dilog(1/2*d*x+1/2*c+1/2)*c^3+b^2/d^3*f^3*dilog(1/2*d*x+1
/2*c+1/2)*c-1/8*b^2/d^3*f^3*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/16*b^2/d^3*
f^3*ln(d*x+c-1)^2*c^4-1/4*b^2/d^3*f^3*ln(d*x+c-1)^2*c^3+1/4*b^2/d^3*f^3*arc
tanh(d*x+c)^2*c^4+1/4*b^2/d^3*f^3*arctanh(d*x+c)^2*(d*x+c)^4+3/2*b^2/d*f*ln
(d*x+c+1)*e^2-1/2*b^2/d^2*f^2*ln(d*x+c+1)*e+3/2*b^2/d*f*ln(d*x+c-1)*e^2+1/2
*b^2/d^2*f^2*ln(d*x+c-1)*e+3/2*b^2/d^3*f^3*ln(d*x+c+1)*c^2+1/2*b^2/d^3*f^3*
ln(d*x+c+1)*c+3/2*b^2/d^3*f^3*ln(d*x+c-1)*c^2-b^2*arctanh(d*x+c)*ln(d*x+c-1
)*c*e^3+1/2*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*e^3-1/2*b^2*ln(-1/2*d*
x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c*e^3+1/2*b^2*ln(d*x+c-1)*ln(1/2*d*x+1/2
*c+1/2)*c*e^3+b^2*arctanh(d*x+c)*ln(d*x+c+1)*c*e^3-b^2/d^3*f^3*c*(d*x+c)+b^
2/d^2*f^2*e*(d*x+c)-1/2*b^2/d^3*f^3*ln(d*x+c-1)*c+1/4*b^2*d/f*arctanh(d*x+c
)^2*e^4+1/16*b^2*d/f*ln(d*x+c+1)^2*e^4+1/16*b^2*d/f*ln(d*x+c-1)^2*e^4+1/2*a
*b/d^3*f^3*(d*x+c)-3/2*b^2/d*f*arctanh(d*x+c)*ln(d*x+c+1)*e^2+b^2/d^2*f^2*a
rctanh(d*x+c)*ln(d*x+c+1)*e+3/2*b^2/d*f*arctanh(d*x+c)*ln(d*x+c-1)*e^2+b^2/
d^2*f^2*arctanh(d*x+c)*ln(d*x+c-1)*e+3*b^2/d*f*arctanh(d*x+c)*e^2*(d*x+c)-3
/4*b^2/d*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^2+1/2*b^2/d^2*f^2*ln(d*x+c+
1)*ln(-1/2*d*x-1/2*c+1/2)*e+3/4*b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1
/2*c+1/2)*e^2-1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*
e-1/4*b^2/d^2*f^2*ln(d*x+c+1)^2*c^3*e+3/8*b^2/d*f*ln(d*x+c+1)^2*c^2*e^2-3/4
*b^2/d^2*f^2*ln(d*x+c+1)^2*c^2*e+3/4*b^2/d*f*ln(d*x+c+1)^2*c*e^2-3/4*b^2/d^
2*f^2*ln(d*x+c+1)^2*c*e-1/8*b^2*d/f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^4+
1/8*b^2*d/f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*e^4-1/8*b^2*d/f*ln
(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*e^4-1/4*b^2*d/f*arctanh(d*x+c)*ln(d*x+c+1)*
e^4+1/4*b^2*d/f*arctanh(d*x+c)*ln(d*x+c-1)*e^4-3*b^2/d^2*f^2*dilog(1/2*d*x+
1/2*c+1/2)*c^2*e+3*b^2/d*f*dilog(1/2*d*x+1/2*c+1/2)*c*e^2-3/4*b^2/d*f*ln(d*
x+c-1)*ln(1/2*d*x+1/2*c+1/2)*e^2-1/2*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2
*c+1/2)*e-1/4*b^2/d^2*f^2*ln(d*x+c-1)^2*c^3*e+3/8*b^2/d*f*ln(d*x+c-1)^2*c^2
```

```
*e^2-b^2/d^2*f^2*arctanh(d*x+c)^2*c^3*e+3/2*b^2/d*f*arctanh(d*x+c)^2*c^2*e^
2+3/2*b^2/d*f*arctanh(d*x+c)^2*e^2*(d*x+c)^2+b^2/d^2*f^2*arctanh(d*x+c)^2*e
*(d*x+c)^3+3/2*a*b/d*f*ln(d*x+c-1)*e^2+a*b/d^2*f^2*ln(d*x+c-1)*e-1/4*a*b*d/
f*ln(d*x+c+1)*e^4+1/4*a*b*d/f*ln(d*x+c-1)*e^4-1/4*a*b/d^3*f^3*ln(d*x+c+1)*c
^4-a*b/d^3*f^3*ln(d*x+c+1)*c^3-3/2*a*b/d^3*f^3*ln(d*x+c+1)*c^2-a*b/d^3*f^3*
ln(d*x+c+1)*c-3*b^2/d^2*f^2*ln(d*x+c-1)*c*e-3*b^2/d^2*f^2*ln(d*x+c+1)*c*e-b
^2/d^3*f^3*arctanh(d*x+c)^2*c^3*(d*x+c)+3/2*b^2/d^3*f^3*arctanh(d*x+c)^2*c^
2*(d*x+c)^2-b^2/d^3*f^3*arctanh(d*x+c)^2*c*(d*x+c)^3-3/2*b^2/d^3*f^3*arctan
h(d*x+c)*ln(d*x+c+1)*c^2-b^2/d^3*f^3*arctanh(d*x+c)*ln(d*x+c+1)*c+1/4*b^2/d
^3*f^3*arctanh(d*x+c)*ln(d*x+c-1)*c^4-b^2/d^3*f^3*arctanh(d*x+c)*ln(d*x+c-1
)*c^3+3/2*b^2/d^3*f^3*arctanh(d*x+c)*ln(d*x+c-1)*c^2-b^2/d^3*f^3*arctanh(d*
x+c)*ln(d*x+c-1)*c-b^2/d^3*f^3*arctanh(d*x+c)*c*(d*x+c)^2+3*b^2/d^3*f^3*arc
tanh(d*x+c)*c^2*(d*x+c)-1/8*b^2/d^3*f^3*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*
c^4-1/2*b^2/d^3*f^3*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^3-3/4*b^2/d^3*f^3*
ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^2-1/2*b^2/d^3*f^3*ln(d*x+c+1)*ln(-1/2*
d*x-1/2*c+1/2)*c+1/8*b^2/d^3*f^3*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/
2)*c^4+1/2*b^2/d^3*f^3*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c^3+3/4
*b^2/d^3*f^3*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c^2+1/2*b^2/d^3*f
^3*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c-1/8*b^2/d^3*f^3*ln(d*x+c-
1)*ln(1/2*d*x+1/2*c+1/2)*c^4+1/2*b^2/d^3*f^3*ln...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1438 vs. $2(555) = 1110$.
time = 0.46, size = 1438, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*f^3*x^4 + a^2*f^2*x^3*e + 1/12*(6*x^4*arctanh(d*x + c) + d*(2*(d^2*
x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*
log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c - 1)/d
^5))*a*b*f^3 + 3/2*a^2*f*x^2*e^2 + (2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*
c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*
c - 1)*log(d*x + c - 1)/d^4))*a*b*f^2*e + 3/2*(2*x^2*arctanh(d*x + c) + d*(
2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x +
c - 1)/d^3))*a*b*f*e^2 + a^2*x*e^3 + (2*(d*x + c)*arctanh(d*x + c) + log(-(
d*x + c)^2 + 1))*a*b*e^3/d - (3*b^2*c*d^2*f*e^2 - b^2*d^3*e^3 - (3*c^2*d*f^
2 + d*f^2)*b^2*e + (c^3*f^3 + c*f^3)*b^2)*(log(d*x + c + 1)*log(-1/2*d*x -
1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2))/d^4 + 1/12*(18*(c*d^2*f + d^2*
f)*b^2*e^2 - 6*(5*c^2*d*f^2 + 6*c*d*f^2 + d*f^2)*b^2*e + (13*c^3*f^3 + 18*c
^2*f^3 + 9*c*f^3 + 4*f^3)*b^2)*log(d*x + c + 1)/d^4 - 1/12*(18*(c*d^2*f - d
^2*f)*b^2*e^2 - 6*(5*c^2*d*f^2 - 6*c*d*f^2 + d*f^2)*b^2*e + (13*c^3*f^3 - 1
8*c^2*f^3 + 9*c*f^3 - 4*f^3)*b^2)*log(d*x + c - 1)/d^4 + 1/48*(4*b^2*d^2*f^
```

```
3*x^2 + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*f^2*x^3*e + 6*b^2*d^4*f*x^2*e^2 + 4*
b^2*d^4*x*e^3 + 4*(c*d^3 + d^3)*b^2*e^3 - 6*(c^2*d^2*f + 2*c*d^2*f + d^2*f)
*b^2*e^2 + 4*(c^3*d*f^2 + 3*c^2*d*f^2 + 3*c*d*f^2 + d*f^2)*b^2*e - (c^4*f^3
 + 4*c^3*f^3 + 6*c^2*f^3 + 4*c*f^3 + f^3)*b^2)*log(d*x + c + 1)^2 + 3*(b^2*
d^4*f^3*x^4 + 4*b^2*d^4*f^2*x^3*e + 6*b^2*d^4*f*x^2*e^2 + 4*b^2*d^4*x*e^3 +
 4*(c*d^3 - d^3)*b^2*e^3 - 6*(c^2*d^2*f - 2*c*d^2*f + d^2*f)*b^2*e^2 + 4*(c
^3*d*f^2 - 3*c^2*d*f^2 + 3*c*d*f^2 - d*f^2)*b^2*e - (c^4*f^3 - 4*c^3*f^3 +
6*c^2*f^3 - 4*c*f^3 + f^3)*b^2)*log(-d*x - c + 1)^2 - 8*(5*b^2*c*d*f^3 - 6*
b^2*d^2*f^2*e)*x + 4*(b^2*d^3*f^3*x^3 - 3*(b^2*c*d^2*f^3 - 2*b^2*d^3*f^2*e)
*x^2 - 3*(8*b^2*c*d^2*f^2*e - 6*b^2*d^3*f*e^2 - (3*c^2*d*f^3 + d*f^3)*b^2)*
x)*log(d*x + c + 1) - 2*(2*b^2*d^3*f^3*x^3 - 6*(b^2*c*d^2*f^3 - 2*b^2*d^3*f
^2*e)*x^2 - 6*(8*b^2*c*d^2*f^2*e - 6*b^2*d^3*f*e^2 - (3*c^2*d*f^3 + d*f^3)*
b^2)*x + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*f^2*x^3*e + 6*b^2*d^4*f*x^2*e^2 + 4
*b^2*d^4*x*e^3 + 4*(c*d^3 + d^3)*b^2*e^3 - 6*(c^2*d^2*f + 2*c*d^2*f + d^2*f
)*b^2*e^2 + 4*(c^3*d*f^2 + 3*c^2*d*f^2 + 3*c*d*f^2 + d*f^2)*b^2*e - (c^4*f^
3 + 4*c^3*f^3 + 6*c^2*f^3 + 4*c*f^3 + f^3)*b^2)*log(d*x + c + 1))*log(-d*x
- c + 1))/d^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^3*x^3 + 3*a^2*f^2*x^2*e + 3*a^2*f*x*e^2 + (b^2*f^3*x^3 + 3*b
^2*f^2*x^2*e + 3*b^2*f*x*e^2 + b^2*e^3)*arctanh(d*x + c)^2 + a^2*e^3 + 2*(a
*b*f^3*x^3 + 3*a*b*f^2*x^2*e + 3*a*b*f*x*e^2 + a*b*e^3)*arctanh(d*x + c), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}{\left(c + dx \right)})^{2} (e + fx)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*(b*arctanh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^3\,(a+b\,\mathrm{atanh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^3*(a + b*atanh(c + d*x))^2, x)
```

3.39 $\int (e+fx)^2 \left(a+b\tanh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=374

$$\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3}+\frac{2b^2f(de-cf)(c+dx)\tanh^{-1}(c+dx)}{d^3}+\frac{bf^2(c+dx)^2\left(a+b\right.}{3d^3}$$

```
[Out] 1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctanh(d*x+c)/d^3+2
*b^2*f*(-c*f+d*e)*(d*x+c)*arctanh(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arcta
nh(d*x+c))/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arctanh(
d*x+c))^2/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c)
)^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x+c))^2/f-2/3*b*(3*d^2*e^2-6*c*d*e*f+(
3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^3+b^2*f*(-c*f+d*e)*ln
(1-(d*x+c)^2)/d^3-1/3*b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(2,(-d
*x-c-1)/(-d*x-c+1))/d^3
```

Rubi [A]
time = 0.44, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6246, 6065, 6021, 266, 6037, 327, 212, 6195, 6095, 6131, 6055, 2449, 2352}

$$-\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{3d^3f}+\frac{\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{3d^3}-\frac{2b\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{3d^3}+\frac{bf^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)}{3d^3}+\frac{2abfx(de-cf)}{d^2}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)^2}{3f}-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{3d^3}+\frac{b^2f(de-cf)\log\left(1-(c+dx)^2\right)}{d^3}+\frac{2b^2f(c+dx)(de-cf)\tanh^{-1}(c+dx)}{d^3}-\frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3}+\frac{b^2f^2x}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTanh[c + d*
x])/(3*d^3) + (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTanh[c + d*x])/d^3 + (b*f^2
*(c + d*x)^2*(a + b*ArcTanh[c + d*x]))/(3*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*
c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(3*d^3*f) + ((3*d^2*e^
2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(3*d^3) + ((e
+ f*x)^3*(a + b*ArcTanh[c + d*x])^2)/(3*f) - (2*b*(3*d^2*e^2 - 6*c*d*e*f +
(1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/(3*d^3) + (
b^2*f*(d*e - c*f)*Log[1 - (c + d*x)^2])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(3*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
```

```
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tanh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{(e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{2abf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2 \left(a+b\tanh^{-1}(c+dx)\right)}{3d^3} + \frac{(e+\ldots}{\ldots}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\tanh^{-1}(c+d\ldots}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3} + \frac{2b^2f(de-c\ldots}{\ldots}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3} + \frac{2b^2f(de-c\ldots}{\ldots}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3} + \frac{2b^2f(de-c\ldots}{\ldots}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3} + \frac{2b^2f(de-c\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 795 vs. 2(374) = 748.
time = 2.90, size = 795, normalized size = 2.13

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f^
2*x^2)*ArcTanh[c + d*x] + (d*f*x*(6*d*e - 4*c*f + d*f*x) - (-1 + c)*(3*d^2*
e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + (1 + c)*(3*d^2*
e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/d^3))/3 + (b^2*e^2
*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTa
nh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d + (b^2*e*f*((-1 +
```

```
 2*c - c^2 + d^2*x^2)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(c + d*x + 2*
c*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 2*Log[1/Sqrt[1 - (c + d*x)^2]] - 2*c*
PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^2 - (b^2*f^2*(1 - (c + d*x)^2)^(3/
2)*(-((c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*c*(c + d*x)*ArcTanh[c + d*x])/S
qrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^
2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + ArcTanh[c
 + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh[c + d*x]^2*Cosh[3*ArcTan
h[c + d*x]] + 2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*Arc
Tanh[c + d*x])] + 6*c^2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E
^(-2*ArcTanh[c + d*x])] - 6*c*Cosh[3*ArcTanh[c + d*x]]*Log[1/Sqrt[1 - (c +
d*x)^2]] + (3*(1 - 4*c + 3*c^2)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(2
+ (3 + 9*c^2)*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 18*c*Log[1/Sqrt[1 - (c +
d*x)^2]])/Sqrt[1 - (c + d*x)^2] - (4*(1 + 3*c^2)*PolyLog[2, -E^(-2*ArcTanh[
c + d*x])])/(1 - (c + d*x)^2)^(3/2) - Sinh[3*ArcTanh[c + d*x]] + 6*c*ArcTan
h[c + d*x]*Sinh[3*ArcTanh[c + d*x]] - ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c +
 d*x]] - 3*c^2*ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]]))/(12*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2889 vs. $2(360) = 720$.
time = 0.72, size = 2890, normalized size = 7.73

method	result	size
risch	Expression too large to display	2486
derivativedivides	Expression too large to display	2890
default	Expression too large to display	2890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*e^2*dilog(1/2*d*x+1/2*c+1/2)+1/4*b^2*e^2*ln(d*x+c-1)^2-1/4*b^2*e^
2*ln(d*x+c+1)^2+b^2*e^2*arctanh(d*x+c)*ln(d*x+c+1)-1/2*b^2*e^2*ln(d*x+c-1)*
ln(1/2*d*x+1/2*c+1/2)+1/2*b^2*e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/2*b^
2*e^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+a*b*e^2*ln(d*x+c-1)+a*b*
e^2*ln(d*x+c+1)+b^2*e^2*arctanh(d*x+c)*ln(d*x+c-1)+2/3*a*b*d/f*arctanh(d*x+
c)*e^3-4*a*b/d*f*arctanh(d*x+c)*c*e*(d*x+c)-b^2*arctanh(d*x+c)^2*c*e^2+b^2*
arctanh(d*x+c)^2*e^2*(d*x+c)-1/3*(c*f-d*e-f*(d*x+c))^3*a^2/d^2/f+1/3*b^2/d^
2*f^2*(d*x+c)+1/6*b^2/d^2*f^2*ln(d*x+c-1)-1/6*b^2/d^2*f^2*ln(d*x+c+1)-1/12*
b^2/d^2*f^2*ln(d*x+c+1)^2+1/12*b^2/d^2*f^2*ln(d*x+c-1)^2-1/3*b^2/d^2*f^2*di
log(1/2*d*x+1/2*c+1/2)-1/4*b^2*ln(d*x+c+1)^2*c*e^2-1/4*b^2*ln(d*x+c-1)^2*c*
e^2-a*b*ln(d*x+c-1)*c*e^2-2*a*b*arctanh(d*x+c)*c*e^2+2*a*b*arctanh(d*x+c)*e
^2*(d*x+c)-b^2/d^2*f^2*ln(d*x+c-1)*c-b^2/d^2*f^2*ln(d*x+c+1)*c+b^2/d*f*ln(d
*x+c-1)*e+b^2/d*f*ln(d*x+c+1)*e+1/4*b^2/d^2*f^2*ln(d*x+c-1)^2*c^2-1/4*b^2/d
^2*f^2*ln(d*x+c-1)^2*c+1/3*b^2/d^2*f^2*arctanh(d*x+c)*ln(d*x+c-1)+1/3*b^2/d
^2*f^2*arctanh(d*x+c)*ln(d*x+c+1)+1/6*b^2/d^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1
/2*c+1/2)-1/6*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)-1/12
```

```
*b^2/d^2*f^2*ln(d*x+c+1)^2*c^3-1/4*b^2/d^2*f^2*ln(d*x+c+1)^2*c^2-1/4*b^2/d^
2*f^2*ln(d*x+c+1)^2*c-1/6*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/1
2*b^2/d^2*f^2*ln(d*x+c-1)^2*c^3-b^2/d^2*f^2*dilog(1/2*d*x+1/2*c+1/2)*c^2+1/
3*b^2/d^2*f^2*arctanh(d*x+c)*(d*x+c)^2-1/3*b^2/d^2*f^2*arctanh(d*x+c)^2*c^3
+1/3*b^2/d^2*f^2*arctanh(d*x+c)^2*(d*x+c)^3+1/12*b^2*d/f*ln(d*x+c+1)^2*e^3+
1/4*b^2/d*f*ln(d*x+c+1)^2*e+1/12*b^2*d/f*ln(d*x+c-1)^2*e^3+1/4*b^2/d*f*ln(d
*x+c-1)^2*e+1/3*b^2*d/f*arctanh(d*x+c)^2*e^3+b^2*arctanh(d*x+c)*ln(d*x+c+1)
*c*e^2-b^2*arctanh(d*x+c)*ln(d*x+c-1)*c*e^2+1/2*b^2*ln(d*x+c+1)*ln(-1/2*d*x
-1/2*c+1/2)*c*e^2-1/2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c*e^
2+1/2*b^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*c*e^2+1/3*a*b/d^2*f^2*(d*x+c)^2
+1/3*a*b/d^2*f^2*ln(d*x+c-1)+1/3*a*b/d^2*f^2*ln(d*x+c+1)+a*b*ln(d*x+c+1)*c*
e^2+1/2*b^2/d^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^2+1/2*b^2/d^2*f^2*
ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c-1/6*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)
*ln(1/2*d*x+1/2*c+1/2)*c^3-1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*
x+1/2*c+1/2)*c^2-1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/
2)*c-2/3*a*b/d^2*f^2*arctanh(d*x+c)*c^3+2/3*a*b/d^2*f^2*arctanh(d*x+c)*(d*x
+c)^3+1/3*a*b*d/f*ln(d*x+c-1)*e^3+a*b/d*f*ln(d*x+c-1)*e-1/3*a*b*d/f*ln(d*x+
c+1)*e^3-a*b/d*f*ln(d*x+c+1)*e-2*a*b/d^2*f^2*c*(d*x+c)+2*a*b/d*f*e*(d*x+c)+
1/6*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*c^3-1/2*b^2/d^2*f^2*ln(d*
x+c-1)*ln(1/2*d*x+1/2*c+1/2)*c^2+1/2*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2
*c+1/2)*c+1/3*b^2/d^2*f^2*arctanh(d*x+c)*ln(d*x+c+1)*c^3-2*b^2/d^2*f^2*arct
anh(d*x+c)*c*(d*x+c)+b^2/d^2*f^2*arctanh(d*x+c)^2*c^2*(d*x+c)-b^2/d^2*f^2*a
rctanh(d*x+c)^2*c*(d*x+c)^2+1/4*b^2/d*f*ln(d*x+c-1)^2*c^2*e-1/2*b^2/d*f*ln(
d*x+c-1)^2*c*e-1/3*b^2*d/f*arctanh(d*x+c)*ln(d*x+c+1)*e^3-b^2/d*f*arctanh(d
*x+c)*ln(d*x+c+1)*e+b^2/d*f*arctanh(d*x+c)^2*c^2*e+b^2/d*f*arctanh(d*x+c)^2
*e*(d*x+c)^2+1/3*b^2*d/f*arctanh(d*x+c)*ln(d*x+c-1)*e^3+b^2/d*f*arctanh(d*x
+c)*ln(d*x+c-1)*e-1/6*b^2*d/f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^3-1/2*b^
2/d*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e+1/6*b^2*d/f*ln(-1/2*d*x-1/2*c+1/
2)*ln(1/2*d*x+1/2*c+1/2)*e^3+1/2*b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+
1/2*c+1/2)*e+1/4*b^2/d*f*ln(d*x+c+1)^2*c^2*e+1/2*b^2/d*f*ln(d*x+c+1)^2*c*e-
1/6*b^2*d/f*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*e^3-1/2*b^2/d*f*ln(d*x+c-1)*l
n(1/2*d*x+1/2*c+1/2)*e+2*b^2/d*f*dilog(1/2*d*x+1/2*c+1/2)*c*e+2*b^2/d*f*arc
tanh(d*x+c)*e*(d*x+c)+b^2/d^2*f^2*arctanh(d*x+c)*ln(d*x+c+1)*c^2+b^2/d^2*f^
2*arctanh(d*x+c)*ln(d*x+c+1)*c-1/3*b^2/d^2*f^2*arctanh(d*x+c)*ln(d*x+c-1)*c
^3+b^2/d^2*f^2*arctanh(d*x+c)*ln(d*x+c-1)*c^2-b^2/d^2*f^2*arctanh(d*x+c)*ln
(d*x+c-1)*c+1/6*b^2/d^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^3-1/3*a*b/
d^2*f^2*ln(d*x+c-1)*c^3+a*b/d^2*f^2*ln(d*x+c-1)*c^2-a*b/d^2*f^2*ln(d*x+c-1)
*c+1/3*a*b/d^2*f^2*ln(d*x+c+1)*c^3+a*b/d^2*f^2*ln(d*x+c+1)*c^2+a*b/d^2*f^2*
ln(d*x+c+1)*c+2*a*b/d^2*f^2*arctanh(d*x+c)*c^2*(d*x+c)-2*a*b/d^2*f^2*arctan
h(d*x+c)*c*(d*x+c)^2-2*a*b/d*f*ln(d*x+c-1)*c*e-a*b/d*f*ln(d*x+c+1)*c^2*e-2*
a*b/d*f*ln(d*x+c+1)*c*e+a*b/d*f*ln(d*x+c-1)*c^2*e+2*a*b/d*f*arctanh(d*x+c)*
e*(d*x+c)^2+2*a*b/d*f*arctanh(d*x+c)*c^2*e-1/2*b^2/d*f*ln(d*x+c-1)*ln(1/2*d
*x+1/2*c+1/2)*c^2*e-b^2/d*f*arctanh(d*x+c)*ln(d*x+c+1)*c^2*e-2*b^2/d*f*arct
anh(d*x+c)*ln(d*x+c+1)*c*e+b^2/d*f*arctanh(d*x+c)*ln(d*x+c-1)*c^2*e-2*b^2/d
*f*arctanh(d*x+c)*ln(d*x+c-1)*c*e+b^2/d*f*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)
```

```
*c*e-1/2*b^2/d*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c^2*e-b^2/d*f*ln(d*x+c+
1)*ln(-1/2*d*x-1/2*c+1/2)*c*e+1/2*b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x
+1/2*c+1/2)*c^2*e+b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c*e-
2*b^2/d*f*arctanh(d*x+c)^2*c*e*(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(361) = 722$.
time = 0.45, size = 857, normalized size = 2.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*f^2*x^3 + a^2*f*x^2*e + 1/3*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3
*c - 1)*log(d*x + c - 1)/d^4))*a*b*f^2 + (2*x^2*arctanh(d*x + c) + d*(2*x/d
^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1
)/d^3))*a*b*f*e + a^2*x*e^2 + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c
)^2 + 1))*a*b*e^2/d - 1/3*(6*b^2*c*d*f*e - 3*b^2*d^2*e^2 - (3*c^2*f^2 + f^2
)*b^2)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*
c + 1/2))/d^3 + 1/6*(6*(c*d*f + d*f)*b^2*e - (5*c^2*f^2 + 6*c*f^2 + f^2)*b^
2)*log(d*x + c + 1)/d^3 - 1/6*(6*(c*d*f - d*f)*b^2*e - (5*c^2*f^2 - 6*c*f^2
 + f^2)*b^2)*log(d*x + c - 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3
+ 3*b^2*d^3*f*x^2*e + 3*b^2*d^3*x*e^2 + 3*(c*d^2 + d^2)*b^2*e^2 - 3*(c^2*d*
f + 2*c*d*f + d*f)*b^2*e + (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^2)*log(d
*x + c + 1)^2 + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*f*x^2*e + 3*b^2*d^3*x*e^2 + 3*
(c*d^2 - d^2)*b^2*e^2 - 3*(c^2*d*f - 2*c*d*f + d*f)*b^2*e + (c^3*f^2 - 3*c^
2*f^2 + 3*c*f^2 - f^2)*b^2)*log(-d*x - c + 1)^2 + 2*(b^2*d^2*f^2*x^2 - 2*(2
*b^2*c*d*f^2 - 3*b^2*d^2*f*e)*x)*log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 - 2*
(2*b^2*c*d*f^2 - 3*b^2*d^2*f*e)*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*f*x^2*e +
3*b^2*d^3*x*e^2 + 3*(c*d^2 + d^2)*b^2*e^2 - 3*(c^2*d*f + 2*c*d*f + d*f)*b^2
*e + (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^2)*log(d*x + c + 1))*log(-d*x
- c + 1))/d^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*f*x*e + (b^2*f^2*x^2 + 2*b^2*f*x*e + b^2*e^2)*
arctanh(d*x + c)^2 + a^2*e^2 + 2*(a*b*f^2*x^2 + 2*a*b*f*x*e + a*b*e^2)*arct
anh(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(c + dx))^2 (e + fx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c))**2,x)
[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2*(b*arctanh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2 \, (a + b\operatorname{atanh}(c + d\,x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atanh(c + d*x))^2,x)
[Out] int((e + f*x)^2*(a + b*atanh(c + d*x))^2, x)
```

3.40 $\int (e+fx) \left(a+b\tanh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=221

$$\frac{abfx}{d}+\frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^2}-\frac{\left(d^2e^2-2cdef+(1+c^2)f^2\right)\left(a+b\right.}{2d^2f}$$

```
[Out] a*b*f*x/d+b^2*f*(d*x+c)*arctanh(d*x+c)/d^2+(-c*f+d*e)*(a+b*arctanh(d*x+c))^
2/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(d*x+c))^2/d^2/f+1/2*
(f*x+e)^2*(a+b*arctanh(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arctanh(d*x+c))*ln(2
/(-d*x-c+1))/d^2+1/2*b^2*f*ln(1-(d*x+c)^2)/d^2-b^2*(-c*f+d*e)*polylog(2,(-d
*x-c-1)/(-d*x-c+1))/d^2
```

Rubi [A]
time = 0.32, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$,
Rules used = {6246, 6065, 6021, 266, 6195, 6095, 6131, 6055, 2449, 2352}

$$\frac{\left(-\frac{(c^2+1)f}{d}+2ce-\frac{de^2}{f}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d}+\frac{(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^2}-\frac{2b(de-cf)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d^2}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f}+\frac{abfx}{d}-\frac{b^2(de-cf)\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{d^2}+\frac{b^2f\log\left(1-(c+dx)^2\right)}{2d^2}+\frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcTanh[c + d*x])/d^2 + ((d*e - c*f)*(a + b*
ArcTanh[c + d*x])^2)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*Ar
cTanh[c + d*x])^2)/(2*d) + ((e + f*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*f) -
 (2*b*(d*e - c*f)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/d^2 + (b^2
*f*Log[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*PolyLog[2, -((1 + c + d
*x)/(1 - c - d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tanh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tanh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f} - \frac{b\text{Subst}\left(\int \left(-\frac{f^2\left(a+b\tanh^{-1}(x)\right)}{d^2}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f} - \frac{b\text{Subst}\left(\int \frac{\left(d^2e^2-2cdef+(1+c^2)f^2\right.}{}\right.}{}$$

$$= \frac{abfx}{d} + \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f} - \frac{b\text{Subst}\left(\int\left(\frac{d^2e^2\left(1+\right.}{}\right.\right.}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2} + \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+\right.}{2f}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\tanh^{-1}(c+\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\tanh^{-1}(c+\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\tanh^{-1}(c+\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\tanh^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\tanh^{-1}(c+\right.}{d^2}$$

Mathematica [A]
time = 0.32, size = 271, normalized size = 1.23

$$\frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(-1+c+dx)(2de+f-cf+dfx)\tanh^{-1}(c+dx)^2+2b\tanh^{-1}(c+dx)\left(-((c+dx)(-bf+acf-ad(2e+fx)))-2b(de-cf)\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+abf\log(1-c-dx)-abf\log(1+c+dx)-4abde\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)-2b^2f\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+4abcf\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+2b^2(de-cf)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcTanh[c + d*x]^2 +
```

```
 2*b*ArcTanh[c + d*x]*(-((c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*
b*(d*e - c*f)*Log[1 + E^(-2*ArcTanh[c + d*x])]) + a*b*f*Log[1 - c - d*x] -
a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/Sqrt[1 - (c + d*x)^2]] - 2*b^2*f*L
og[1/Sqrt[1 - (c + d*x)^2]] + 4*a*b*c*f*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*b^
2*(d*e - c*f)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(217) = 434$.
time = 0.32, size = 812, normalized size = 3.67 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(e*b^2*arctanh(d*x+c)*ln(d*x+c-1)+e*b^2*arctanh(d*x+c)*ln(d*x+c+1)-1/2*
e*b^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/2*e*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln
(d*x+c+1)-1/2*e*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+b*a*e*ln(d
*x+c-1)+b*a*e*ln(d*x+c+1)+1/4*e*b^2*ln(d*x+c-1)^2-1/4*e*b^2*ln(d*x+c+1)^2-b
^2*dilog(1/2*d*x+1/2*c+1/2)*e-a^2/d*(f*c*(d*x+c)-e*(d*x+c)*d-1/2*f*(d*x+c)^
2)+e*(d*x+c)*b^2*arctanh(d*x+c)^2+1/2*b^2/d*ln(d*x+c-1)*f+1/2*b^2/d*ln(d*x+
c+1)*f+1/8*b^2/d*ln(d*x+c+1)^2*f+1/8*b^2/d*ln(d*x+c-1)^2*f+2*e*(d*x+c)*a*b*
arctanh(d*x+c)+1/2*a*b/d*ln(d*x+c-1)*f-1/2*a*b/d*ln(d*x+c+1)*f-1/4*b^2/d*ln
(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*f-1/4*b^2/d*ln(d*x+c-1)^2*c*f-1/4*b^2/d*ln(
d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*f+1/4*b^2/d*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d
*x+1/2*c+1/2)*f+1/4*b^2/d*ln(d*x+c+1)^2*c*f+b^2/d*dilog(1/2*d*x+1/2*c+1/2)*
c*f+b^2/d*arctanh(d*x+c)*f*(d*x+c)+1/2*b^2/d*arctanh(d*x+c)*ln(d*x+c-1)*f-1
/2*b^2/d*arctanh(d*x+c)*ln(d*x+c+1)*f+1/2*b^2/d*arctanh(d*x+c)^2*f*(d*x+c)^
2+a*b/d*f*(d*x+c)-2*a*b/d*arctanh(d*x+c)*f*c*(d*x+c)-a*b/d*ln(d*x+c-1)*f*c-
a*b/d*ln(d*x+c+1)*f*c-1/2*b^2/d*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*f+1/2*
b^2/d*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c*f+1/2*b^2/d*ln(d*x+c-1
)*ln(1/2*d*x+1/2*c+1/2)*c*f-b^2/d*arctanh(d*x+c)*ln(d*x+c-1)*f*c-b^2/d*arct
anh(d*x+c)*ln(d*x+c+1)*f*c-b^2/d*arctanh(d*x+c)^2*f*c*(d*x+c)+a*b/d*arctanh
(d*x+c)*f*(d*x+c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(213) = 426$.
time = 0.45, size = 435, normalized size = 1.97

$$\frac{1}{2}a^2fx^2 + \frac{1}{2}\left(2x^2\operatorname{artanh}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)abf + a^2xe + \frac{(2(dx+c)\operatorname{artanh}(dx+c) + \log(-(dx+c)^2+1))abe}{d} + \frac{(cf+f)b^2\log(dx+c+1)}{2d^2} - \frac{(cf-f)b^2\log(dx+c-1)}{2d^2} - \frac{(b^2cf-b^2de)\left(\log(dx+c+1)\log\left(-\frac{1}{2}dx-\frac{1}{2}c+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}dx+\frac{1}{2}c+\frac{1}{2}\right)\right)}{d^2} + \frac{4b^2dfx\log(dx+c+1) + (b^2d^2fx^2+2b^2d^2xe+2(cd+d)b^2e-(c^2f+2cf+f)b^2)\log(dx+c+1)^2 + (b^2d^2fx^2+2b^2d^2xe+2(cd-d)b^2e-(c^2f-2cf+f)b^2)\log(-dx-c+1)^2 - 2(2b^2dfx+(b^2d^2fx^2+2b^2d^2xe+2(cd+d)b^2e-(c^2f+2cf+f)b^2)\log(dx+c+1))\log(-dx-c+1)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2*x
*e + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d + 1/2*(
c*f + f)*b^2*log(d*x + c + 1)/d^2 - 1/2*(c*f - f)*b^2*log(d*x + c - 1)/d^2
```

```
- (b^2*c*f - b^2*d*e)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog
(1/2*d*x + 1/2*c + 1/2))/d^2 + 1/8*(4*b^2*d*f*x*log(d*x + c + 1) + (b^2*d^2
*f*x^2 + 2*b^2*d^2*x*e + 2*(c*d + d)*b^2*e - (c^2*f + 2*c*f + f)*b^2)*log(d
*x + c + 1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*x*e + 2*(c*d - d)*b^2*e - (c^2*f
 - 2*c*f + f)*b^2)*log(-d*x - c + 1)^2 - 2*(2*b^2*d*f*x + (b^2*d^2*f*x^2 +
2*b^2*d^2*x*e + 2*(c*d + d)*b^2*e - (c^2*f + 2*c*f + f)*b^2)*log(d*x + c +
1))*log(-d*x - c + 1))/d^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + (b^2*f*x + b^2*e)*arctanh(d*x + c)^2 + a^2*e + 2*(a*b*f*
x + a*b*e)*arctanh(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atanh}(c + dx))^2 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atanh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctanh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\,\left(a + b\,\mathrm{atanh}(c + d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] int((e + f*x)*(a + b*atanh(c + d*x))^2, x)
```

3.41 $\int \left(a+b\tanh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}-\frac{2b\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{b^2\text{Pol}}{}$$

[Out] (a+b*arctanh(d*x+c))^2/d+(d*x+c)*(a+b*arctanh(d*x+c))^2/d-2*b*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-b^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6238, 6021, 6131, 6055, 2449, 2352}

$$\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}-\frac{2b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d}-\frac{b^2\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c + d*x])^2,x]

[Out] (a + b*ArcTanh[c + d*x])^2/d + ((c + d*x)*(a + b*ArcTanh[c + d*x])^2)/d - (2*b*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/d - (b^2*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/d

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\tanh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x\left(a+b\tanh^{-1}(x)\right)}{1-x^2}\,dx,x,c+d\right.}{d}$$

$$= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\tan\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\tan\right.}{}$$

Mathematica [A]

time = 0.15, size = 107, normalized size = 1.10

$$\frac{b^2(-1+c+dx)\tanh^{-1}(c+dx)^2+2b\tanh^{-1}(c+dx)\left(adx-b\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+a(adx+(b-bc)\log(1-c-dx)+b(1+c)\log(1+c+dx))+b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] (b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(a*d*x - b*Lo
g[1 + E^(-2*ArcTanh[c + d*x])]) + a*(a*d*x + (b - b*c)*Log[1 - c - d*x] + b
*(1 + c)*Log[1 + c + d*x]) + b^2*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d
```

Maple [A]
time = 1.75, size = 140, normalized size = 1.44

method	result
derivativedivides	$\frac{(dx+c)a^2+(dx+c)b^2\operatorname{arctanh}(dx+c)^2+b^2\operatorname{arctanh}(dx+c)^2-2\operatorname{arctanh}(dx+c)\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)b^2-\operatorname{polylog}\left(2,-\frac{(dx+c}{1-(dx}\right.}{d}$
default	$\frac{(dx+c)a^2+(dx+c)b^2\operatorname{arctanh}(dx+c)^2+b^2\operatorname{arctanh}(dx+c)^2-2\operatorname{arctanh}(dx+c)\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)b^2-\operatorname{polylog}\left(2,-\frac{(dx+c}{1-(dx}\right.}{d}$
risch	$\frac{b^2(dx+c+1)\ln(dx+c+1)^2}{4d}+\left(-\frac{x\,b^2\ln(-dx-c+1)}{2}+\frac{b(2adx-\ln(-dx-c+1)bc+b\ln(-dx-c+1))}{2d}\right)\ln(dx+c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2+(d*x+c)*b^2*arctanh(d*x+c)^2+b^2*arctanh(d*x+c)^2-2*arctan
h(d*x+c)*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))*b^2-polylog(2,-(d*x+c+1)^2/(1-(d*x
+c)^2))*b^2+2*(d*x+c)*a*b*arctanh(d*x+c)+a*b*ln(1-(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c*d*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2) + d
^2*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*
x + c - 1)/d^3) - 2*c*d*integrate(x*log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c
^2 - 1), x) - 2*c^2*integrate(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1
), x) + d*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2) - 6
*d*integrate(x*log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x) - 4*c*int
egrate(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x) - (d*x + c - 1)*(
log(-d*x - c + 1)^2 - 2*log(-d*x - c + 1) + 2)/d - (d*x*log(d*x + c + 1)^2
+ 2*(d*x - (d*x + c + 1)*log(d*x + c + 1))*log(-d*x - c + 1))/d - 2*integra
te(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x))*b^2 + a^2*x + (2*(d*
x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2,x)
[Out] Integral((a + b*atanh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c + d\,x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2,x)
[Out] int((a + b*atanh(c + d*x))^2, x)
```

3.42 $\int \frac{(a+b\tanh^{-1}(c+dx))^2}{e+fx}\,dx$

Optimal. Leaf size=214

$$-\frac{(a+b\tanh^{-1}(c+dx))^2\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\tanh^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{b(a+b\tanh^{-1}(c$$

```
[Out] -(a+b*arctanh(d*x+c))^2*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))^2*ln(2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+b*(a+b*arctanh(d*x+c))*polylog(2,1-2/(d*x+c+
1))/f-b*(a+b*arctanh(d*x+c))*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1)
)/f+1/2*b^2*polylog(3,1-2/(d*x+c+1))/f-1/2*b^2*polylog(3,1-2*d*(f*x+e)/(-c*
f+d*e+f)/(d*x+c+1))/f
```

Rubi [A]

time = 0.11, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6246, 6059}

$$-\frac{b(a+b\tanh^{-1}(c+dx))\,\mathrm{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f}+\frac{(a+b\tanh^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))^2}{f}-\frac{b^2\mathrm{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b^2\mathrm{Li}_3\left(1-\frac{2}{c+dx+1}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTanh[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c
 + d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a +
 b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcTanh
[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d
*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{e+fx}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2 \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2 \log\left(\frac{}{(de+}\right.}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.62, size = 2404, normalized size = 11.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x),x]
```

```
[Out] (a^2*Log[e + f*x])/f - ((2*I)*a*b*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c +
 d*x)^2]] + Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-I)
*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*ArcT
anh[c + d*x])^2 + 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 -
 E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2*I)*A
rcTanh[c + d*x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi - (2*I)
*ArcTanh[c + d*x])*Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(I*ArcTa
nh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f)/
f] + ArcTanh[c + d*x]]] - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] +
 I*ArcTanh[c + d*x]))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))])
/2))/f + (b^2*(d*e - c*f + f*(c + d*x))*((2*ArcTanh[c + d*x]^2*(d*e*ArcTanh
[c + d*x] - (1 + c)*f*ArcTanh[c + d*x] + 3*(d*e - c*f)*Log[1 + E^(-2*ArcTan
h[c + d*x])]) + (-6*d*e*ArcTanh[c + d*x] + 6*c*f*ArcTanh[c + d*x])*PolyLog[
2, -E^(-2*ArcTanh[c + d*x])] + (-3*d*e + 3*c*f)*PolyLog[3, -E^(-2*ArcTanh[c
 + d*x])])/(6*f*(-(d*e) + c*f)) + ((-(d*e) - f + c*f)*(-(d*e) + f + c*f)*((
ArcTanh[c + d*x]^2*(f*ArcTanh[c + d*x] + (-(d*e) + c*f)*Log[(d*e)/Sqrt[1 -
(c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*
x)^2]]))/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (ArcTanh[c + d*x]*(I*d*e*Pi*
ArcTanh[c + d*x] - I*c*f*Pi*ArcTanh[c + d*x] + 2*f*ArcTanh[c + d*x]^2 - (Sq
rt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]^2)/E^ArcTanh[(
d*e - c*f)/f] - I*d*e*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] + I*c*f*Pi*Log[1 +
 E^(2*ArcTanh[c + d*x])] + 2*d*e*ArcTanh[c + d*x]*Log[1 - E^(-2*(ArcTanh[(d
```

```
*e - c*f)/f] + ArcTanh[c + d*x]))] - 2*c*f*ArcTanh[c + d*x]*Log[1 - E^(-2*(
ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + I*d*e*Pi*Log[1/Sqrt[1 - (c +
 d*x)^2]] - I*c*f*Pi*Log[1/Sqrt[1 - (c + d*x)^2]] - 2*d*e*ArcTanh[c + d*x]*
Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x
))/Sqrt[1 - (c + d*x)^2]] + 2*c*f*ArcTanh[c + d*x]*Log[(d*e)/Sqrt[1 - (c +
d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]
] + 2*(d*e - c*f)*ArcTanh[(d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*
(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - Log[I*Sinh[ArcTanh[(d*e - c
*f)/f] + ArcTanh[c + d*x]]]) + (-(d*e) + c*f)*PolyLog[2, E^(-2*(ArcTanh[(d*
e - c*f)/f] + ArcTanh[c + d*x]))]))/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (
((-2*d*e + (2 + 2*c - Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]/E^ArcTanh
[(d*e - c*f)/f])*f)*ArcTanh[c + d*x]^3)/3 + (d*e - c*f)*ArcTanh[c + d*x]^2*
Log[-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + (d*e - c*f)*A
rcTanh[c + d*x]*(I*Pi*(ArcTanh[c + d*x] - Log[1 + E^(2*ArcTanh[c + d*x])] +
 Log[(1 + E^(2*ArcTanh[c + d*x]))/(2*E^ArcTanh[c + d*x])]) + 2*ArcTanh[(d*e
 - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcT
anh[c + d*x]))] - Log[(I/2)*E^(-ArcTanh[(d*e - c*f)/f] - ArcTanh[c + d*x])*
(-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])))])) - (d*e - c*f)*A
rcTanh[c + d*x]^2*Log[d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E^(2*ArcT
anh[c + d*x]) + c*E^(2*ArcTanh[c + d*x]))*f] + (d*e - c*f)*ArcTanh[c + d*x]
^2*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c +
d*x]))] - Log[-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + Log
[d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E^(2*ArcTanh[c + d*x]) + c*E^(
2*ArcTanh[c + d*x]))*f] - Log[(d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c -
E^(2*ArcTanh[c + d*x]) + c*E^(2*ArcTanh[c + d*x]))*f)/(2*E^ArcTanh[c + d*x]
)]) + ((d*e - c*f)*(2*ArcTanh[c + d*x]^3 + 3*ArcTanh[c + d*x]^2*Log[1 - E^(
-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - 3*ArcTanh[c + d*x]^2*Log
[1 - E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*ArcTanh[c + d*x]^2*
Log[1 + E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*ArcTanh[c + d*x]
*PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - 6*ArcTanh
[c + d*x]*PolyLog[2, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 6*Ar
cTanh[c + d*x]*PolyLog[2, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] +
6*PolyLog[3, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6*PolyLog[3,
 E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])]))/3 + ((d*e - c*f)*(4*ArcTa
nh[c + d*x]^3 - 6*ArcTanh[c + d*x]^2*Log[1 - E^(2*(ArcTanh[(d*e - c*f)/f] +
 ArcTanh[c + d*x]))] - 6*ArcTanh[c + d*x]*PolyLog[2, E^(2*(ArcTanh[(d*e - c
*f)/f] + ArcTanh[c + d*x]))] + 3*PolyLog[3, E^(2*(ArcTanh[(d*e - c*f)/f] +
ArcTanh[c + d*x]))]))/6 - ((d*e - c*f)*(4*ArcTanh[c + d*x]^3 - 6*ArcTanh[c
+ d*x]^2*Log[1 + (E^(2*ArcTanh[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c)*f)
] - 6*ArcTanh[c + d*x]*PolyLog[2, -((E^(2*ArcTanh[c + d*x])*(d*e + f - c*f)
)/(d*e - (1 + c)*f))] + 3*PolyLog[3, -((E^(2*ArcTanh[c + d*x])*(d*e + f - c
*f))/(d*e - (1 + c)*f))]))/6)/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(f*(
-(d*e) + c*f))))/(d*(e + f*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 30.40, size = 1872, normalized size = 8.75

method	result	size
derivativedivides	Expression too large to display	1872
default	Expression too large to display	1872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f+b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arctanh(d*
x+c)^2-b^2*d/f*arctanh(d*x+c)^2*ln(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x
+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)+1/2*I*b^2*d/
f*Pi*arctanh(d*x+c)^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)
^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1
-(d*x+c)^2)))^3-1/2*I*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-
(d*x+c)^2)))*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*
x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^
2)))^2-1/2*I*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x
+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f
)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2)
)+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f))+1/2
*I*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I
*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d
*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(f*c*(1
+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^
2/(1-(d*x+c)^2)+1)*f))-b^2*d/f*arctanh(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*
x+c)^2))+1/2*b^2*d/f*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+b^2*d*c/(c*f-d*e
-f)*arctanh(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f
))+b^2*d*c/(c*f-d*e-f)*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-
(d*x+c)^2)/(-c*f+d*e-f))-1/2*b^2*d*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x
+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-b^2*d/(c*f-d*e-f)*arctanh(d*x+c)^2*ln(1
-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-b^2*d/(c*f-d*e-f)*arct
anh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/
2*b^2*d/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d
*e-f))-b^2*d^2/f*e/(c*f-d*e-f)*arctanh(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)^
2/(1-(d*x+c)^2)/(-c*f+d*e-f))-b^2*d^2/f*e/(c*f-d*e-f)*arctanh(d*x+c)*polylo
g(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/2*b^2*d^2/f*e/(c*
f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+2*a*
b*d*ln(c*f-d*e-f*(d*x+c))/f*arctanh(d*x+c)-a*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln
((-f*(d*x+c)-f)/(-c*f+d*e-f))-a*b*d/f*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))+a*
b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))+a*b*d/f*dilog((
-f*(d*x+c)+f)/(-c*f+d*e+f)))
```

<u>**Maxima**</u> **[F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(d*x + c + 1) - log(-d*x - c + 1
))^2/(f*x + e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f*x + e),
 x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c + dx\right)\right)^{2}}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2/(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2}{e+f\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(e + f*x),x)
```

```
[Out] int((a + b*atanh(c + d*x))^2/(e + f*x), x)
```

3.43 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=480

$$-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{b^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}-\frac{abd\log(1-c-dx)}{f(de+f-cf)}-\frac{b^2d\tanh^{-1}(c+dx)\log\left(\frac{\cdot}{1}\right.}{f(de-f-cf)}$$

```
[Out] -(a+b*arctanh(d*x+c))^2/f/(f*x+e)+b^2*d*arctanh(d*x+c)*ln(2/(-d*x-c+1))/f/(
-c*f+d*e+f)-a*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*arctanh(d*x+c)*ln(2/(d*
x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*arctanh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-f)/
(-c*f+d*e+f)+a*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*ln(f*x+e)/(f^2-(-c*f+
d*e)^2)-2*b^2*d*arctanh(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f
+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e
+f)+1/2*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*polylog(2,1-2/(
d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e
+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 1.24, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {6244, 2007, 719, 31, 646, 6873, 6256, 720, 647, 6820, 12, 6857, 84, 6874, 6055, 2449, 2352, 6057, 2497}

$$-\frac{abd\log(-c-dx+1)}{f(-cf+de+f)}+\frac{abd\log(c+dx+1)}{f(-cf+de-f)}-\frac{2abd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{b^2d\,\mathrm{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)}+\frac{b^2d\,\mathrm{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{2f(-cf+de-f)}-\frac{b^2d\,\mathrm{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\,\mathrm{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\log\left(\frac{2}{-c-dx+1}\right)\tanh^{-1}(c+dx)}{f(-cf+de+f)}-\frac{b^2d\log\left(\frac{2}{c+dx+1}\right)\tanh^{-1}(c+dx)}{f(-cf+de-f)}+\frac{2b^2d\log\left(\frac{2}{c+dx+1}\right)\tanh^{-1}(c+dx)}{(-cf+de+f)(de-(c+1)f)}-\frac{2b^2d\tanh^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcTanh[c + d*x]*Log[2
/(1 - c - d*x)])/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
 - c*f)) - (b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e -
 (1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) - (2*a*b*d*Log[
e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*b^2*d*ArcTanh[c + d*x]*L
og[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e
- (1 + c)*f)) + (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*
e + f - c*f)) + (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f
)) - (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) + (b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x
))])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
```

```
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)(1-(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{d(a+b\tanh^{-1}(x))}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \left(-\frac{a}{(-1+x)(1+x)(de-cf+fx)} - \frac{}{(-1+x}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx, x, c+\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\text{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+c)}\right.\right.}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} + \frac{abd\log(1+c+dx)}{f(de-f-cf)} - $$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.92, size = 425, normalized size = 0.89

$$\frac{-\frac{a^2}{f} + \frac{2ab\left(\left(f-c^2f+d^2ex+cd(e-fx)\right)\tanh^{-1}(c+dx)-d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{1-(c+dx)^2}}\right)\right)}{(de+f-cf)(de-(1+c)f)} + \frac{b^2d(e+fx)\left(-\frac{e^{-\tanh^{-1}\left(\frac{de-cf}{f}\right)}\tanh^{-1}(c+dx)^2}{f\sqrt{1-\frac{(de-cf)^2}{f^2}}} + \frac{(c+dx)\tanh^{-1}(c+dx)^2}{d(e+fx)} + \frac{(de-cf)\left(i\pi\log\left(1+e^{2\tanh^{-1}(c+dx)}\right)-2\tanh^{-1}(c+dx)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)-i\pi\left(\tanh^{-1}(c+dx)+\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)\right)-2\tanh^{-1}\left(\frac{de-cf}{f}\right)\left(\tanh^{-1}(c+dx)+\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)-\log\left(i\sinh\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\right)\right)+\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)\right)}{d^2e^2-2cdef+\left(-1+c^2\right)f^2}\right)}{de-cf}}{e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*ArcTanh[c + d*x]
- d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 - (c + d*x)^2]]))/((d*e + f - c*f)*(
d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(-(ArcTanh[c + d*x]^2/(E^ArcTanh[(d*e
- c*f)/f]*f*Sqrt[1 - (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTanh[c + d*x]^2)/
(d*(e + f*x)) + ((d*e - c*f)*(I*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - 2*ArcT
anh[c + d*x]*Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] -
I*Pi*(ArcTanh[c + d*x] + Log[1/Sqrt[1 - (c + d*x)^2]]) - 2*ArcTanh[(d*e - c
*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[
c + d*x]))] - Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + Pol
yLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]))/(d^2*e^2 - 2*
c*d*e*f + (-1 + c^2)*f^2)))/(d*e - c*f))/(e + f*x)
```

Maple [A]
time = 4.88, size = 857, normalized size = 1.79

method	result
derivativedivides	$\frac{a^2 d^2}{(cf-de-f(dx+c))f}+\frac{b^2 d^2 \operatorname{arctanh}(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2b^2 d^2 \operatorname{arctanh}(dx+c)\ln(dx+c+1)}{f(2cf-2de+2f)}-\frac{2b^2 d^2 \operatorname{arctanh}(dx+c)\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}+\frac{2b^2 d^2}{}$
default	$\frac{a^2 d^2}{(cf-de-f(dx+c))f}+\frac{b^2 d^2 \operatorname{arctanh}(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2b^2 d^2 \operatorname{arctanh}(dx+c)\ln(dx+c+1)}{f(2cf-2de+2f)}-\frac{2b^2 d^2 \operatorname{arctanh}(dx+c)\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}+\frac{2b^2 d^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d^2/(c*f-d*e-f*(d*x+c))/f+b^2*d^2/(c*f-d*e-f*(d*x+c))/f*arctanh(d*
x+c)^2-2*b^2*d^2/f*arctanh(d*x+c)/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-2*b^2*d^2*a
rctanh(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))+2*b^2*d^2/f*arc
tanh(d*x+c)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-1/2*b^2*d^2/f/(c*f-d*e-f)*dilog(1
/2*d*x+1/2*c+1/2)-1/2*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/
2)+1/4*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2-1/2*b^2*d^2/f/(c*f-d*e+f)*ln(-1/
2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/2*b^2*d^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/
2)*ln(1/2*d*x+1/2*c+1/2)+1/2*b^2*d^2/f/(c*f-d*e+f)*dilog(1/2*d*x+1/2*c+1/2)
+1/4*b^2*d^2/f/(c*f-d*e+f)*ln(d*x+c+1)^2+b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)*ln
(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))+b^2*d^2/(c*f-d*e-f)/(c*
f-d*e+f)*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))-b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)
*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))-b^2*d^2/(c*f-d*e-f)/
(c*f-d*e+f)*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))+2*a*b*d^2/(c*f-d*e-f*(d*x+c)
)/f*arctanh(d*x+c)-2*a*b*d^2/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-2*a*b*d^2/(c*f
-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))+2*a*b*d^2/f/(2*c*f-2*d*e-2*f)*ln(
d*x+c-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)*f^2
 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*arcta
nh(d*x + c)/(f^2*x + f*e))*a*b - 1/4*b^2*(log(-d*x - c + 1)^2/(f^2*x + f*e)
 + integrate(-((d*f*x + c*f - f)*log(d*x + c + 1)^2 + 2*(d*f*x + d*e - (d*f
*x + c*f - f)*log(d*x + c + 1))*log(-d*x - c + 1))/(d*f^3*x^3 + (c*f^3 + 2*
d*f^2*e - f^3)*x^2 + (d*f*e^2 + 2*(c*f^2 - f^2)*e)*x + (c*f - f)*e^2), x))
- a^2/(f^2*x + f*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^2*x^2 +
 2*f*x*e + e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^2}{(e + fx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2/(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c + d\,x))^2}{(e + f\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(e + f*x)^2,x)
[Out] int((a + b*atanh(c + d*x))^2/(e + f*x)^2, x)
```

3.44 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx$

Optimal. Leaf size=750

$$-\frac{abd}{(f^2-(de-cf)^2)\,(e+fx)}+\frac{b^2d\tanh^{-1}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2}+\frac{b^2d^2\tanh^{-1}}{2f(}$$

```
[Out] -a*b*d/(f^2-(-c*f+d*e)^2)/(f*x+e)+b^2*d*arctanh(d*x+c)/(-c*f+d*e-f)/(-c*f+d
*e+f)/(f*x+e)-1/2*(a+b*arctanh(d*x+c))^2/f/(f*x+e)^2+1/2*b^2*d^2*arctanh(d*
x+c)*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)^2-1/2*a*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e
+f)^2+1/2*b^2*d^2*ln(-d*x-c+1)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)-1/2*b^2*d^2*arc
tanh(d*x+c)*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)^2+2*b^2*d^2*(-c*f+d*e)*arctanh(d
*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+1/2*a*b*d^2*ln(d*x+c+1
)/f/(-c*f+d*e-f)^2-1/2*b^2*d^2*ln(d*x+c+1)/(-c*f+d*e+f)/(d*e-(1+c)*f)^2+b^2
*d^2*f*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*a*b*d^2*(-c*f+d*e)*ln(f*x
+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*b^2*d^2*(-c*f+d*e)*arctanh(d*x+c)*ln(2
*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+1/4*b^2*d
^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)^2+1/4*b^2*d^2*polylog(2,
1-2/(d*x+c+1))/f/(-c*f+d*e-f)^2-b^2*d^2*(-c*f+d*e)*polylog(2,1-2/(d*x+c+1))
/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+b^2*d^2*(-c*f+d*e)*polylog(2,1-2*d*(f*x+e)/
(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

Rubi [A]
time = 1.56, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6244, 2007, 723, 814, 6873, 6256, 724, 815, 6857, 6055, 2449, 2352, 6063, 720, 31, 647, 6057, 2497}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3,x]
```

```
[Out] -((a*b*d)/((f^2 - (d*e - c*f)^2)*(e + f*x))) + (b^2*d*ArcTanh[c + d*x])/((d
*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcTanh[c + d*x])^2/(2*
f*(e + f*x)^2) + (b^2*d^2*ArcTanh[c + d*x]*Log[2/(1 - c - d*x)])/(2*f*(d*e
+ f - c*f)^2) - (a*b*d^2*Log[1 - c - d*x])/(2*f*(d*e + f - c*f)^2) + (b^2*d
^2*Log[1 - c - d*x])/(2*(d*e + f - c*f)^2*(d*e - (1 + c)*f)) - (b^2*d^2*Arc
Tanh[c + d*x]*Log[2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)^2) + (2*b^2*d^2*(d
*e - c*f)*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)^2*(d*e -
(1 + c)*f)^2) + (a*b*d^2*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)^2) - (b^2*d
^2*Log[1 + c + d*x])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)^2) + (b^2*d^2*f*L
og[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) - (2*a*b*d^2*(d*e - c*
```

```
f)*Log[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) - (2*b^2*d^2*(d*e
- c*f)*ArcTanh[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))
])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) + (b^2*d^2*PolyLog[2, -((1 + c +
 d*x)/(1 - c - d*x))])/(4*f*(d*e + f - c*f)^2) + (b^2*d^2*PolyLog[2, 1 - 2/
(1 + c + d*x)])/(4*f*(d*e - f - c*f)^2) - (b^2*d^2*(d*e - c*f)*PolyLog[2, 1
 - 2/(1 + c + d*x)])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) + (b^2*d^2*(d*
e - c*f)*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(
(d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
```

```
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
```

```
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{(bd)\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^2(1-(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{(bd)\int \frac{a+b\tanh^{-1}(c+dx)}{(e+fx)^2(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{b\text{Subst}\left(\int \left(-\frac{ad^2}{(de-cf+fx)^2(-1+x^2)} - \frac{bd^2\tanh^{-1}(}{(de-cf+fx)^2(-}\right.\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} - \frac{(abd^2)\,\text{Subst}\left(\int \frac{1}{(de-cf+fx)^2(-1+x^2)}\,dx, x, c+dx\right.}{f}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} - \frac{(b^2d^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} - \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{2f(e+fx)^2} - \frac{(b^2d^2)\,\text{Subst}\left(\int\right.}{2f(de}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{(a}{}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{(a}{}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{(a}{}$$

$$= -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} + \frac{b^2d\tanh^{-1}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{(a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.70, size = 1968, normalized size = 2.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3,x]
```

```
[Out] -1/2*a^2/(f*(e + f*x)^2) + (a*b*(d*e - c*f + f*(c + d*x))^3*((f*(2 + ((d*e
+ f - c*f)*(d*e - (1 + c)*f))/((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c +
d*x))/Sqrt[1 - (c + d*x)^2])^2)*ArcTanh[c + d*x])/((d*e + f - c*f)^2*(-(d*e
) + f + c*f)^2) - ((c + d*x)*(f - 2*d*e*ArcTanh[c + d*x] + 2*c*f*ArcTanh[c
+ d*x]))/((d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Sqrt[1 - (c + d*x)^
2]*((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]
)) - (2*(d*e - c*f)*Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d
*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]])/(d^2*e^2 - 2*c*d*e*f + (-1 +
 c^2)*f^2)^2))/(d*(e + f*x)^3) + (b^2*(d*e - c*f + f*(c + d*x))^3*((f*(1 -
(c + d*x)^2)^(3/2)*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^
2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^3*ArcTanh[c + d*x]^2)/(2*(d*e - f
 - c*f)*(d*e + f - c*f)*(d*e - c*f + f*(c + d*x))^3*(-((d*e)/Sqrt[1 - (c +
d*x)^2]) + (c*f)/Sqrt[1 - (c + d*x)^2] - (f*(c + d*x))/Sqrt[1 - (c + d*x)^2
])^2) + ((1 - (c + d*x)^2)^(3/2)*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[
1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^3*((f*(c + d*x)*Arc
Tanh[c + d*x])/Sqrt[1 - (c + d*x)^2] - (d*e*(c + d*x)*ArcTanh[c + d*x]^2)/S
qrt[1 - (c + d*x)^2] + (c*f*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x
)^2]))/((d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*(d*e - c*f + f*(c + d*x
))^3*(-((d*e)/Sqrt[1 - (c + d*x)^2]) + (c*f)/Sqrt[1 - (c + d*x)^2] - (f*(c
+ d*x))/Sqrt[1 - (c + d*x)^2])) + (f*(1 - (c + d*x)^2)^(3/2)*((d*e)/Sqrt[1
- (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c +
d*x)^2])^3*(-(f*ArcTanh[c + d*x]) + (d*e - c*f)*Log[(d*e - c*f)/Sqrt[1 - (c
 + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]]))/((d*e - c*f)*(d*e - f -
 c*f)*(d*e + f - c*f)*(-f^2 + (d*e - c*f)^2)*(d*e - c*f + f*(c + d*x))^3) +
 (c*(1 - (c + d*x)^2)^(3/2)*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (
c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^3*(ArcTanh[c + d*x]^2/E^
ArcTanh[(d*e - c*f)/f] - (I*(d*e - c*f)*(-((-Pi + (2*I)*ArcTanh[(d*e - c*f)
/f])*ArcTanh[c + d*x]) - 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*
Log[1 - E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] - Pi*Log
[1 + E^(2*ArcTanh[c + d*x])] + Pi*Log[1/Sqrt[1 - (c + d*x)^2]] + (2*I)*ArcT
anh[(d*e - c*f)/f]*Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]] +
 I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))]))/
(f*Sqrt[1 - (d*e - c*f)^2/f^2])))/((d*e - c*f)*(d*e - f - c*f)*(d*e + f - c
*f)*Sqrt[(f^2 - (d*e - c*f)^2)/f^2]*(d*e - c*f + f*(c + d*x))^3) - (d*e*(1
- (c + d*x)^2)^(3/2)*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x
)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^3*(ArcTanh[c + d*x]^2/E^ArcTanh
[(d*e - c*f)/f] - (I*(d*e - c*f)*(-((-Pi + (2*I)*ArcTanh[(d*e - c*f)/f])*Ar
cTanh[c + d*x]) - 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 -
 E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] - Pi*Log[1 + E^
(2*ArcTanh[c + d*x])] + Pi*Log[1/Sqrt[1 - (c + d*x)^2]] + (2*I)*ArcTanh[(d*
e - c*f)/f]*Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]] + I*Poly
Log[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))]))/(f*Sqrt
[1 - (d*e - c*f)^2/f^2])))/(f*(d*e - c*f)*(d*e - f - c*f)*(d*e + f - c*f)*S
qrt[(f^2 - (d*e - c*f)^2)/f^2]*(d*e - c*f + f*(c + d*x))^3)))/(d*(e + f*x)^
3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1490 vs. $2(735) = 1470$.
time = 2.16, size = 1491, normalized size = 1.99

method	result	size
derivativedivides	Expression too large to display	1491
default	Expression too large to display	1491

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*d^3/(c*f-d*e-f)/(c*f-d*e+f)/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)+b^2*d^3*
f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))-b^2*d^3/(c*f-d*e-f)/(c*
f-d*e+f)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)+b^2*d^4/(c*f-d*e-f)^2/(c*f-d*e+f)^2*
dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))*e-b^2*d^4/(c*f-d*e-f)^2/(c*f-d*e+f)^2*di
log((-f*(d*x+c)+f)/(-c*f+d*e+f))*e-a*b*d^3/(c*f-d*e-f)/(c*f-d*e+f)/(c*f-d*e
-f*(d*x+c))-a*b*d^3/(c*f-d*e-f*(d*x+c))^2/f*arctanh(d*x+c)+1/2*a*b*d^3/f/(c
*f-d*e+f)^2*ln(d*x+c+1)-1/2*a*b*d^3/f/(c*f-d*e-f)^2*ln(d*x+c-1)+1/2*b^2*d^3
/f*arctanh(d*x+c)/(c*f-d*e+f)^2*ln(d*x+c+1)-b^2*d^3*arctanh(d*x+c)/(c*f-d*e
-f)/(c*f-d*e+f)/(c*f-d*e-f*(d*x+c))-1/2*b^2*d^3/f*arctanh(d*x+c)/(c*f-d*e-f
)^2*ln(d*x+c-1)+1/4*b^2*d^3/f/(c*f-d*e-f)^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/
2)+1/4*b^2*d^3/f/(c*f-d*e+f)^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/4*b^2*d
^3/f/(c*f-d*e+f)^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+b^2*d^3*f/(
c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*
e+f))*c+2*a*b*d^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*c+b^2
*d^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))*c-2*a
*b*d^4/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*e-2*b^2*d^4*arctan
h(d*x+c)/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*e+b^2*d^4/(c*f-d
*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))
*e-b^2*d^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))
*c-b^2*d^4/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)
+f)/(-c*f+d*e+f))*e-1/2*a^2*d^3/(c*f-d*e-f*(d*x+c))^2/f-1/2*b^2*d^3/(c*f-d*
e-f*(d*x+c))^2/f*arctanh(d*x+c)^2+1/4*b^2*d^3/f/(c*f-d*e-f)^2*dilog(1/2*d*x
+1/2*c+1/2)-1/8*b^2*d^3/f/(c*f-d*e-f)^2*ln(d*x+c-1)^2-1/4*b^2*d^3/f/(c*f-d*
e+f)^2*dilog(1/2*d*x+1/2*c+1/2)-1/8*b^2*d^3/f/(c*f-d*e+f)^2*ln(d*x+c+1)^2+2
*b^2*d^3*f*arctanh(d*x+c)/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))
*c-b^2*d^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+
c)-f)/(-c*f+d*e-f))*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(d*log(d*x + c + 1)/(2*(c*e + e)*d*f^2 - (c^2 + 2*c + 1)*f^3 - d^2*
f*e^2) - d*log(d*x + c - 1)/(2*(c*e - e)*d*f^2 - (c^2 - 2*c + 1)*f^3 - d^2*
f*e^2) + 4*(c*d*f - d^2*e)*log(f*x + e)/(4*c*d^3*f*e^3 - 2*(3*c^2*e^2 - e^2
)*d^2*f^2 + 4*(c^3*e - c*e)*d*f^3 - (c^4 - 2*c^2 + 1)*f^4 - d^4*e^4) + 2/(2
*c*d*f*e^2 - (c^2*e - e)*f^2 - d^2*e^3 + (2*c*d*f^2*e - (c^2 - 1)*f^3 - d^2
*f*e^2)*x)) + 2*arctanh(d*x + c)/(f^3*x^2 + 2*f^2*x*e + f*e^2))*a*b - 1/8*b
^2*(log(-d*x - c + 1)^2/(f^3*x^2 + 2*f^2*x*e + f*e^2) + 2*integrate(-((d*f*
x + c*f - f)*log(d*x + c + 1)^2 + (d*f*x + d*e - 2*(d*f*x + c*f - f)*log(d*
x + c + 1))*log(-d*x - c + 1))/(d*f^4*x^4 + (c*f^4 + 3*d*f^3*e - f^4)*x^3 +
 3*(d*f^2*e^2 + (c*f^3 - f^3)*e)*x^2 + (d*f*e^3 + 3*(c*f^2 - f^2)*e^2)*x +
(c*f - f)*e^3), x)) - 1/2*a^2/(f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^3*x^3 +
 3*f^2*x^2*e + 3*f*x*e^2 + e^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c + dx\right)\right)^{2}}{\left(e + fx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**2/(f*x+e)**3,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**2/(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{atanh}(c+d\,x))^2}{(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^2/(e + f*x)^3,x)
[Out] int((a + b*atanh(c + d*x))^2/(e + f*x)^3, x)
```

3.45 $\int (e+fx)^2 \left(a+b\tanh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=546

$$\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\tanh^{-1}(c+dx)}{d^3}-\frac{bf^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^3}+\frac{3bf(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3}$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctanh(d*x+c)/d^3-1/2*b*f^2*(a+b*arctanh(d
*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(a+b*arctanh(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(
d*x+c)*(a+b*arctanh(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arctanh(d*x+c))^
2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arctanh(d*x+c))^3
/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))^3/d^3+1
/3*(f*x+e)^3*(a+b*arctanh(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctanh(d*x+c
))*ln(2/(-d*x-c+1))/d^3-b*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(
d*x+c))^2*ln(2/(-d*x-c+1))/d^3+1/2*b^3*f^2*ln(1-(d*x+c)^2)/d^3-3*b^3*f*(-c*
f+d*e)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^3-b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2
+1)*f^2)*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^3+1/2*b^3*(3*d^2*
e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(3,1-2/(-d*x-c+1))/d^3
```

Rubi [A]
time = 0.75, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6246, 6065, 6021, 6131, 6055, 2449, 2352, 6037, 6127, 266, 6095, 6195, 6205, 6745}

$$-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d^3}-\frac{6b^2f(de-cf)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d^3}+\frac{ab^2f^2x}{d^2}-\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d^3f}+\frac{\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\left(a+b\tanh^{-1}(c+dx)\right)^3}{3d^3}-\frac{b\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3}+\frac{3bf(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3}+\frac{3bf(c+dx)(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3}-\frac{bf^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^3}+\frac{bf^2(c+dx)^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^3}+\frac{(e+fx)^3\left(a+b\tanh^{-1}(c+dx)\right)^3}{3f}+\frac{b^3\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d^3}-\frac{3b^3f(de-cf)\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{d^3}+\frac{b^3f^2\log\left(1-(c+dx)^2\right)}{2d^3}+\frac{b^3f^2(c+dx)\tanh^{-1}(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTanh[c + d*x])/d^3 - (b*f^2*(a +
b*ArcTanh[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcTanh[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTanh[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcTanh[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])^3)/(3*d^3)
 + ((e + f*x)^3*(a + b*ArcTanh[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a
 + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*
f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 +
 (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -
((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*
c^2)*f^2)*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + (
b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x
)])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6037

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6127

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tanh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tanh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+b\tan}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\tanh^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef+}{}\right.}{}$$

$$= \frac{3bf(de-cf)(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2 (a}{}$$

$$= \frac{3bf(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^3} + \frac{3bf(de-cf)(c+dx)(a}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(de-cf)(a+b\tanh}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tanh^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tanh^{-1}(c+d\right.}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tanh^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tanh^{-1}(c+d\right.}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tanh^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tanh^{-1}(c+d\right.}{2d^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1646 vs. $2(546) = 1092$.
time = 8.07, size = 1646, normalized size = 3.01

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] a^2*(a*e^2 + (b*f*(3*d*e - 2*c*f))/d^2)*x + (a^2*f*(2*a*d*e + b*f)*x^2)/(2*
d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c + d*x]
 - (a^2*b*(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 -
c - d*x])/(2*d^3) + (a^2*b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2
```

```
*f^2)*Log[1 + c + d*x])/(2*d^3) + (3*a*b^2*e^2*(ArcTanh[c + d*x]*((-1 + c +
 d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -
E^(-2*ArcTanh[c + d*x])]))/d - (3*a*b^2*e*f*((1 - 2*c + c^2 - d^2*x^2)*ArcT
anh[c + d*x]^2 - 2*ArcTanh[c + d*x]*(c + d*x + 2*c*Log[1 + E^(-2*ArcTanh[c
+ d*x])]) + 2*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*c*PolyLog[2, -E^(-2*ArcTanh[
c + d*x])]))/d^2 + (b^3*e^2*(2*ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c
 + d*x] - 3*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 6*ArcTanh[c + d*x]*PolyLog[
2, -E^(-2*ArcTanh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])]))/(2*
d) - (b^3*e*f*(ArcTanh[c + d*x]*((1 - 2*c + c^2 - d^2*x^2)*ArcTanh[c + d*x]
^2 + 6*Log[1 + E^(-2*ArcTanh[c + d*x])] - 3*ArcTanh[c + d*x]*(-1 + c + d*x
+ 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])])) + (-3 + 6*c*ArcTanh[c + d*x])*Poly
Log[2, -E^(-2*ArcTanh[c + d*x])] + 3*c*PolyLog[3, -E^(-2*ArcTanh[c + d*x])]
))/d^2 - (a*b^2*f^2*(1 - (c + d*x)^2)^(3/2)*(-((c + d*x)/Sqrt[1 - (c + d*x)
^2]) + (6*c*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x
)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c +
d*x]^2)/Sqrt[1 - (c + d*x)^2] + ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]]
 + 3*c^2*ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 2*ArcTanh[c + d*x]*C
osh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])] + 6*c^2*ArcTanh[c
+ d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*c*Cosh
[3*ArcTanh[c + d*x]]*Log[1/Sqrt[1 - (c + d*x)^2]] + (3*(1 - 4*c + 3*c^2)*Ar
cTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(2 + (3 + 9*c^2)*Log[1 + E^(-2*ArcTan
h[c + d*x])]) - 18*c*Log[1/Sqrt[1 - (c + d*x)^2]])/Sqrt[1 - (c + d*x)^2] -
(4*(1 + 3*c^2)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(1 - (c + d*x)^2)^(3/2
) - Sinh[3*ArcTanh[c + d*x]] + 6*c*ArcTanh[c + d*x]*Sinh[3*ArcTanh[c + d*x]
] - ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]] - 3*c^2*ArcTanh[c + d*x]^2*
Sinh[3*ArcTanh[c + d*x]]))/(4*d^3) + (b^3*f^2*((-3*c + (1 + 3*c^2)*ArcTanh[
c + d*x])*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] - ((1 - (c + d*x)^2)^(3/2)*(
(-3*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x)^2] + (9*c*(c + d*x)*ArcT
anh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^3)/Sq
rt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^3)/Sqrt[1 - (c + d*
x)^2] - 9*c*ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + ArcTanh[c + d*x]^
3*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh[c + d*x]^3*Cosh[3*ArcTanh[c + d*
x]] - 18*c*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*ArcTanh[
c + d*x])] + 3*ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]]*Log[1 + E^(-2*Ar
cTanh[c + d*x])] + 9*c^2*ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]]*Log[1
+ E^(-2*ArcTanh[c + d*x])] + 3*Cosh[3*ArcTanh[c + d*x]]*Log[1/Sqrt[1 - (c +
 d*x)^2]] + (3*((1 - 4*c + 3*c^2)*ArcTanh[c + d*x]^3 - 18*c*ArcTanh[c + d*x
]*Log[1 + E^(-2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]^2*(2 - 9*c + (3 + 9*c
^2)*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*Log[1/Sqrt[1 - (c + d*x)^2]]))/Sq
rt[1 - (c + d*x)^2] - (6*(1 + 3*c^2)*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/
(1 - (c + d*x)^2)^(3/2) - 3*ArcTanh[c + d*x]*Sinh[3*ArcTanh[c + d*x]] + 9*c
*ArcTanh[c + d*x]^2*Sinh[3*ArcTanh[c + d*x]] - ArcTanh[c + d*x]^3*Sinh[3*Ar
cTanh[c + d*x]] - 3*c^2*ArcTanh[c + d*x]^3*Sinh[3*ArcTanh[c + d*x]]))/12))/
d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 50.18, size = 12235, normalized size = 22.41

method	result	size
derivativedivides	Expression too large to display	12235
default	Expression too large to display	12235

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*f^2*x^3 + a^3*f*x^2*e + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3
*c - 1)*log(d*x + c - 1)/d^4))*a^2*b*f^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*
(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x +
 c - 1)/d^3))*a^2*b*f*e + a^3*x*e^2 + 3/2*(2*(d*x + c)*arctanh(d*x + c) + l
og(-(d*x + c)^2 + 1))*a^2*b*e^2/d - 1/24*((b^3*d^3*f^2*x^3 + 3*b^3*d^3*f*x^
2*e + 3*b^3*d^3*x*e^2 + 3*(c*d^2 - d^2)*b^3*e^2 - 3*(c^2*d*f - 2*c*d*f + d*
f)*b^3*e + (c^3*f^2 - 3*c^2*f^2 + 3*c*f^2 - f^2)*b^3)*log(-d*x - c + 1)^3 -
 3*(2*a*b^2*d^3*f^2*x^3 + (6*a*b^2*d^3*f*e + b^3*d^2*f^2)*x^2 - 2*(2*b^3*c*
d*f^2 - 3*a*b^2*d^3*e^2 - 3*b^3*d^2*f*e)*x + (b^3*d^3*f^2*x^3 + 3*b^3*d^3*f
*x^2*e + 3*b^3*d^3*x*e^2 + 3*(c*d^2 + d^2)*b^3*e^2 - 3*(c^2*d*f + 2*c*d*f +
 d*f)*b^3*e + (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^3)*log(d*x + c + 1))*
log(-d*x - c + 1)^2)/d^3 - integrate(-1/8*((b^3*d^3*f^2*x^3 + (c*d^2 - d^2)
*b^3*e^2 + (2*b^3*d^3*f*e + (c*d^2*f^2 - d^2*f^2)*b^3)*x^2 + (b^3*d^3*e^2 +
 2*(c*d^2*f - d^2*f)*b^3*e)*x)*log(d*x + c + 1)^3 + 6*(a*b^2*d^3*f^2*x^3 +
(c*d^2 - d^2)*a*b^2*e^2 + (2*a*b^2*d^3*f*e + (c*d^2*f^2 - d^2*f^2)*a*b^2)*x
^2 + (a*b^2*d^3*e^2 + 2*(c*d^2*f - d^2*f)*a*b^2*e)*x)*log(d*x + c + 1)^2 -
(4*a*b^2*d^3*f^2*x^3 + 2*(6*a*b^2*d^3*f*e + b^3*d^2*f^2)*x^2 + 3*(b^3*d^3*f
^2*x^3 + (c*d^2 - d^2)*b^3*e^2 + (2*b^3*d^3*f*e + (c*d^2*f^2 - d^2*f^2)*b^3
)*x^2 + (b^3*d^3*e^2 + 2*(c*d^2*f - d^2*f)*b^3*e)*x)*log(d*x + c + 1)^2 - 4
*(2*b^3*c*d*f^2 - 3*a*b^2*d^3*e^2 - 3*b^3*d^2*f*e)*x - 2*(3*(c^2*d*f + 2*c*
d*f + d*f)*b^3*e - (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^3 - (6*a*b^2*d^3
*f^2 + b^3*d^3*f^2)*x^3 - 3*(2*(c*d^2*f^2 - d^2*f^2)*a*b^2 + (4*a*b^2*d^3*f
 + b^3*d^3*f)*e)*x^2 - 3*(4*(c*d^2*f - d^2*f)*a*b^2*e + (2*a*b^2*d^3 + b^3*
```

```
d^3)*e^2)*x - 3*(2*(c*d^2 - d^2)*a*b^2 + (c*d^2 + d^2)*b^3)*e^2)*log(d*x +
c + 1))*log(-d*x - c + 1))/(d^3*x + c*d^2 - d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*f*x*e + (b^3*f^2*x^2 + 2*b^3*f*x*e + b^3*e^2)*
arctanh(d*x + c)^3 + a^3*e^2 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*f*x*e + a*b^2*e^2
)*arctanh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e + a^2*b*e^2)*arctan
h(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^3 (e + fx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arctanh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2 \,(a + b\,\operatorname{atanh}(c + d\,x))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^2*(a + b*atanh(c + d*x))^3, x)
```

3.46 $\int (e+fx)\left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=326

$$\frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2}+\frac{(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d^2}-\frac{(d^2e$$

```
[Out] 3/2*b*f*(a+b*arctanh(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arctanh(d*x+c))^2/d
^2+(-c*f+d*e)*(a+b*arctanh(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)*(a+b*arctanh(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctanh(d*x+c))^3/f-3*b^2
*f*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*arctanh(d*
x+c))^2*ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^2
-3*b^2*(-c*f+d*e)*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^2+3/2*b^
3*(-c*f+d*e)*polylog(3,1-2/(-d*x-c+1))/d^2
```

Rubi [A]
time = 0.58, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6246, 6065, 6021, 6131, 6055, 2449, 2352, 6195, 6095, 6205, 6745}

$$-\frac{3b^2(de-cf)\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d^2}-\frac{3b^2f\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d^2}+\frac{\left(-\frac{(c^2+1)f}{d}+2ce-\frac{de^2}{f}\right)\left(a+b\tanh^{-1}(c+dx)\right)^3}{2d}+\frac{(de-cf)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d^2}-\frac{3b(de-cf)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d^2}+\frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2}+\frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2f}+\frac{3b^3(de-cf)\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d^2}-\frac{3b^3f\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (3*b*f*(a + b*ArcTanh[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcTan
h[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcTanh[c + d*x])^3)/d^2 + ((2
*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*ArcTanh[c + d*x])^3)/(2*d) + ((e
 + f*x)^2*(a + b*ArcTanh[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTanh[c + d
*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcTanh[c + d*x])^
2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1
- 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)
])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6065

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6195

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\tanh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tanh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \left(-\frac{f^2(a+b\tanh^{-1}}{d^2}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\tanh^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \frac{(d^2e^2-2cdef+(1+}{}\right.}{}$$

$$= \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\tanh^{-1}(c\right.}{2f}$$

$$= \frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\tanh^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)}{2d^2}$$

Mathematica [A]
time = 0.72, size = 566, normalized size = 1.74

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2*
b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTanh[c + d*x] + 3*a^2*b*(2*d*e + f - 2
*c*f)*Log[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*Log[1 + c + d*x] + 1
2*a*b^2*f*((c + d*x)*ArcTanh[c + d*x] - ((1 - (c + d*x)^2)*ArcTanh[c + d*x]
```

```
^2)/2 - Log[1/Sqrt[1 - (c + d*x)^2]]) + 12*a*b^2*d*e*(ArcTanh[c + d*x]*((-1
 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLo
g[2, -E^(-2*ArcTanh[c + d*x])]) - 12*a*b^2*c*f*(ArcTanh[c + d*x]*((-1 + c +
 d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -
E^(-2*ArcTanh[c + d*x])]) + 2*b^3*f*(ArcTanh[c + d*x]*(3*(-1 + c + d*x)*Arc
Tanh[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^2 - 6*Log[1
 + E^(-2*ArcTanh[c + d*x])]) + 3*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]) + 4*
b^3*d*e*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] - 3*Log[1 + E^
(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c +
d*x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/2) - 4*b^3*c*f*(ArcTanh[c
 + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] - 3*Log[1 + E^(-2*ArcTanh[c + d*
x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + (3*PolyLo
g[3, -E^(-2*ArcTanh[c + d*x])])/2))/(4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.83, size = 19935, normalized size = 61.15

method	result	size
derivativedivides	Expression too large to display	19935
default	Expression too large to display	19935

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*f*x^2 + 3/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a^3
*x*e + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*e/d
 - 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*x*e + 2*(c*d - d)*b^3*e - (c^2*f - 2*c*
f + f)*b^3)*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e +
 b^3*d*f)*x + (b^3*d^2*f*x^2 + 2*b^3*d^2*x*e + 2*(c*d + d)*b^3*e - (c^2*f +
 2*c*f + f)*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)^2)/d^2 - integrate(-1/
8*((b^3*d^2*f*x^2 + (c*d - d)*b^3*e + (b^3*d^2*e + (c*d*f - d*f)*b^3)*x)*lo
g(d*x + c + 1)^3 + 6*(a*b^2*d^2*f*x^2 + (c*d - d)*a*b^2*e + (a*b^2*d^2*e +
(c*d*f - d*f)*a*b^2)*x)*log(d*x + c + 1)^2 - 3*(2*a*b^2*d^2*f*x^2 + (b^3*d^
2*f*x^2 + (c*d - d)*b^3*e + (b^3*d^2*e + (c*d*f - d*f)*b^3)*x)*log(d*x + c
```

```
+ 1)^2 + 2*(2*a*b^2*d^2*e + b^3*d*f)*x - ((c^2*f + 2*c*f + f)*b^3 - (4*a*b^
2*d^2*f + b^3*d^2*f)*x^2 - 2*(2*(c*d*f - d*f)*a*b^2 + (2*a*b^2*d^2 + b^3*d^
2)*e)*x - 2*(2*(c*d - d)*a*b^2 + (c*d + d)*b^3)*e)*log(d*x + c + 1))*log(-d
*x - c + 1))/(d^2*x + c*d - d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + (b^3*f*x + b^3*e)*arctanh(d*x + c)^3 + a^3*e + 3*(a*b^2*
f*x + a*b^2*e)*arctanh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctanh(d*x + c
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^3 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arctanh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\,\left(a + b\,\mathrm{atanh}(c + d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((e + f*x)*(a + b*atanh(c + d*x))^3, x)
```

3.47 $\int \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=132

$$\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\tanh^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{3b^2(a$$

```
[Out] (a+b*arctanh(d*x+c))^3/d+(d*x+c)*(a+b*arctanh(d*x+c))^3/d-3*b*(a+b*arctanh(
d*x+c))^2*ln(2/(-d*x-c+1))/d-3*b^2*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x
-c+1))/d+3/2*b^3*polylog(3,1-2/(-d*x-c+1))/d
```

Rubi [A]
time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$,
Rules used = {6238, 6021, 6131, 6055, 6095, 6205, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{3b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)^2}{d}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (a + b*ArcTanh[c + d*x])^3/d + ((c + d*x)*(a + b*ArcTanh[c + d*x])^3)/d - (
3*b*(a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)])/d - (3*b^2*(a + b*ArcT
anh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d + (3*b^3*PolyLog[3, 1 - 2/
(1 - c - d*x)])/(2*d)
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\tanh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\text{Subst}\left(\int \frac{x\left(a+b\tanh^{-1}(x)\right)^2}{1-x^2}\,dx,x,c+\right.}{d}$$

$$=\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\text{Subst}}{}$$

$$=\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\text{ta}\right.}{}$$

$$=\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\text{ta}\right.}{}$$

$$=\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tanh^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\text{ta}\right.}{}$$

Mathematica [A]
time = 0.18, size = 205, normalized size = 1.55

$$\frac{2a^3dx+6a^2bdx\tanh^{-1}(c+dx)-3a^2b(-1+c)\log(1-c-dx)+3a^2b(1+c)\log(1+c+dx)+6ab^2\left(\tanh^{-1}(c+dx)\left((-1+c+dx)\tanh^{-1}(c+dx)-2\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)\right)+2b^3\left(\tanh^{-1}(c+dx)^2\left((-1+c+dx)\tanh^{-1}(c+dx)-3\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+3\tanh^{-1}(c+dx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)+\frac{3}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(c+dx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + 6*a^2*b*d*x*ArcTanh[c + d*x] - 3*a^2*b*(-1 + c)*Log[1 - c - d*
x] + 3*a^2*b*(1 + c)*Log[1 + c + d*x] + 6*a*b^2*(ArcTanh[c + d*x]*((-1 + c
+ d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2,
-E^(-2*ArcTanh[c + d*x])]) + 2*b^3*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcT
anh[c + d*x] - 3*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*Pol
yLog[2, -E^(-2*ArcTanh[c + d*x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])]
)/2))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(130) = 260$.
time = 0.50, size = 284, normalized size = 2.15

method	result
derivativedivides	$(dx+c)a^3+(dx+c)b^3\operatorname{arctanh}(dx+c)^3+b^3\operatorname{arctanh}(dx+c)^3-3b^3\operatorname{arctanh}(dx+c)^2\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-3b^3\operatorname{arctanh}(dx+c)$
default	$(dx+c)a^3+(dx+c)b^3\operatorname{arctanh}(dx+c)^3+b^3\operatorname{arctanh}(dx+c)^3-3b^3\operatorname{arctanh}(dx+c)^2\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-3b^3\operatorname{arctanh}(dx+c)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+(d*x+c)*b^3*arctanh(d*x+c)^3+b^3*arctanh(d*x+c)^3-3*b^3*ar
ctanh(d*x+c)^2*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))-3*b^3*arctanh(d*x+c)*polylog
(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2*b^3*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2)
)+3*(d*x+c)*a*b^2*arctanh(d*x+c)^2+3*a*b^2*arctanh(d*x+c)^2-6*arctanh(d*x+c
)*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))*a*b^2-3*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)
^2))*a*b^2+3*(d*x+c)*a^2*b*arctanh(d*x+c)+3/2*a^2*b*ln(1-(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d
- 1/8*((b^3*d*x + b^3*(c - 1))*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d*x + (b^3*
d*x + b^3*(c + 1))*log(d*x + c + 1))*log(-d*x - c + 1)^2)/d - integrate(-1/
8*((b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c - 1
))*log(d*x + c + 1)^2 - 3*(4*a*b^2*d*x + (b^3*d*x + b^3*(c - 1))*log(d*x +
c + 1)^2 + 2*(b^3*(c + 1) + 2*a*b^2*(c - 1) + (2*a*b^2*d + b^3*d)*x)*log(d*
x + c + 1))*log(-d*x - c + 1))/(d*x + c - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arct
anh(d*x + c) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atanh}(c + d\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3,x)
```

```
[Out] int((a + b*atanh(c + d*x))^3, x)
```

3.48 $\int \frac{(a+b\tanh^{-1}(c+dx))^3}{e+fx}\,dx$

Optimal. Leaf size=308

$$-\frac{(a+b\tanh^{-1}(c+dx))^3\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\tanh^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{3b(a+b\tanh^{-1}(c$$

```
[Out] -(a+b*arctanh(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))^3*ln(2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2/(
d*x+c+1))/f-3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+
f)/(d*x+c+1))/f+3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2/(d*x+c+1))/f-3/2
*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(-c*f+d
*e+f)/(d*x+c+1))/f
```

Rubi [A]

time = 0.13, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6246, 6061}

$$-\frac{3b^2(a+b\tanh^{-1}(c+dx))\operatorname{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{3b^2\operatorname{Li}_3\left(1-\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))}{2f}-\frac{3b(a+b\tanh^{-1}(c+dx))^2\operatorname{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{(a+b\tanh^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{3b\operatorname{Li}_2\left(1-\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))^2}{2f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\tanh^{-1}(c+dx))^3}{f}-\frac{3b^3\operatorname{Li}_4\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{4f}+\frac{3b^3\operatorname{Li}_4\left(1-\frac{2}{c+dx+1}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTanh[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c
+ d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a
 + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a +
 b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 +
 c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[3, 1 - 2/(1 +
c + d*x)])/(2*f) - (3*b^2*(a + b*ArcTanh[c + d*x])*PolyLog[3, 1 - (2*d*(e +
 f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1
 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f
)*(1 + c + d*x))])/(4*f)
```

Rule 6061

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^3)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^3*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[3*b*
(a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b*(
a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))
]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTanh[c*x])*(PolyLog[3, 1 - 2/(1 + c*x)]
/(2*e)), x] - Simp[3*b^2*(a + b*ArcTanh[c*x])*(PolyLog[3, 1 - 2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(2*e)), x] + Simp[3*b^3*(PolyLog[4, 1 - 2/(1 + c*
```

```
x)]/(4*e)), x] - Simp[3*b^3*(PolyLog[4, 1 - 2*c*((d + e*x)/((c*d + e)*(1 +
c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{e+fx}\,dx = \frac{\mathrm{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3\log\left(\frac{\ldots}{(de\ldots}\right.}{f}$$

Mathematica [F]
time = 46.23, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x),x]
[Out] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.70, size = 3825, normalized size = 12.42

method	result	size
derivativedivides	Expression too large to display	3825
default	Expression too large to display	3825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/d*(-3/2*I*a*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^
2)))*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-
```

```
1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2-3
/2*I*a*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2)
)+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(
d*x+c+1)^2/(1-(d*x+c)^2)))^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d
*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f))+1/2*I*b^3
*d/f*Pi*arctanh(d*x+c)^3*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(f*c*
(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1
)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(f*c*(1+(d*x+
c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(
d*x+c)^2)+1)*f))+3/2*I*a*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I/(1+(d*x+c+1)^2/
(1-(d*x+c)^2)))*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-
(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+
c)^2)))*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^
2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f))+b^3*d*c/(c*f-d*e-f)*arctanh(d*
x+c)^3*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3*a*b^2*d*c
/(c*f-d*e-f)*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)
/(-c*f+d*e-f))+3/2*a*b^2*d^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1
)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3*a*b^2*d*c/(c*f-d*e-f)*arctanh(d*x+c)^2*ln
(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-b^3*d^2/f*e/(c*f-d*e
-f)*arctanh(d*x+c)^3*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f
))-3/2*b^3*d^2/f*e/(c*f-d*e-f)*arctanh(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d*x+
c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3/2*b^3*d^2/f*e/(c*f-d*e-f)*arctanh(d*x+
c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/2*I*b^3*
d/f*Pi*arctanh(d*x+c)^3*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+
1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/
(1-(d*x+c)^2)))^3+a^3*d*ln(c*f-d*e-f*(d*x+c))/f-3/4*b^3*d/f*polylog(4,-(d*x
+c+1)^2/(1-(d*x+c)^2))-3/4*b^3*d/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1
)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-3/2*a^2*b*d/f*dilog((-f*(d*x+c)-f)/(-c*f+d*
e-f))+3/2*a^2*b*d/f*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))+3/2*a*b^2*d/f*polylo
g(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2*a*b^2*d/(c*f-d*e-f)*polylog(3,(c*f-d*e-
f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+b^3*d*ln(c*f-d*e-f*(d*x+c))/f*ar
ctanh(d*x+c)^3-b^3*d/f*arctanh(d*x+c)^3*ln(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2)
)+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)-3/2*
b^3*d/f*arctanh(d*x+c)^2*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/2*b^3*d/f*
arctanh(d*x+c)*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+3/4*b^3*d*c/(c*f-d*e-f
)*polylog(4,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-b^3*d/(c*f-
d*e-f)*arctanh(d*x+c)^3*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*
e-f))-3/2*b^3*d/(c*f-d*e-f)*arctanh(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d*x+c+1
)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3/2*b^3*d/(c*f-d*e-f)*arctanh(d*x+c)*polylo
g(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-1/2*I*b^3*d/f*Pi*ar
ctanh(d*x+c)^3*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(
d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c
)^2)))^2*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)
^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f))-1/2*I*b^3*d/f*Pi*arctanh(d*x+
c)^3*csgn(I/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))*csgn(I*(f*c*(1+(d*x+c+1)^2/(1-(d
```

```
*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)
*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^2-3*a*b^2*d^2/f*e/(c*f-d*e-f)*arctanh(d*
x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-3*a*b^2*d^2
/f*e/(c*f-d*e-f)*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c
)^2)/(-c*f+d*e-f))+3/2*I*a*b^2*d/f*Pi*arctanh(d*x+c)^2*csgn(I*(f*c*(1+(d*x+
c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(
d*x+c)^2)+1)*f)/(1+(d*x+c+1)^2/(1-(d*x+c)^2)))^3+3/2*b^3*d*c/(c*f-d*e-f)*ar
ctanh(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f)
)-3/2*b^3*d*c/(c*f-d*e-f)*arctanh(d*x+c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/
(1-(d*x+c)^2)/(-c*f+d*e-f))-3/4*b^3*d^2/f*e/(c*f-d*e-f)*polylog(4,(c*f-d*e-
f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+3*a^2*b*d*ln(c*f-d*e-f*(d*x+c))/
f*arctanh(d*x+c)-3/2*a^2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*
f+d*e-f))+3/2*a^2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f
))+3*a*b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arctanh(d*x+c)^2-3*a*b^2*d/f*arctanh(d
*x+c)^2*ln(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(-(d*x+c+1)^2/(1-(d*x+c)^2)-1)
*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)-3*a*b^2*d/f*arctanh(d*x+c)*polylog(2
,-(d*x+c+1)^2/(1-(d*x+c)^2))-3/2*a*b^2*d*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f
)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-3*a*b^2*d/(c*f-d*e-f)*arctanh(d*x
+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-3*a*b^2*d/(c
*f-d*e-f)*arctanh(d*x+c)*polylog(2,(c*f-d*e-f)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(d*x + c + 1) - log(-d*x - c + 1
))^3/(f*x + e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c + 1))^2/(f*x +
e) + 3/2*a^2*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(c + dx))^3}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(f*x+e),x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3/(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)^3}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(e + f*x),x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(e + f*x), x)
```

3.49 $\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1089

$$-\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)}+\frac{3ab^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}+\frac{3b^3d\tanh^{-1}(c+dx)^2\log\left(\frac{2}{1-c-dx}\right)}{2f(de+f-cf)}-\frac{3a^2bd\,l}{2f(c}$$

```
[Out] -(a+b*arctanh(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*arctanh(d*x+c)*ln(2/(-d*x-c+1))
/f/(-c*f+d*e+f)+3/2*b^3*d*arctanh(d*x+c)^2*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-
3/2*a^2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*arctanh(d*x+c)*ln(2/(d*x+
c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*arctanh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-f)/
(-c*f+d*e+f)-3/2*b^3*d*arctanh(d*x+c)^2*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+3*b^
3*d*arctanh(d*x+c)^2*ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^2*b*d*
ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*a*b^2*d
*arctanh(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d
*e+f)-3*b^3*d*arctanh(d*x+c)^2*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f
+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d
*e+f)+3/2*b^3*d*arctanh(d*x+c)*polylog(2,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2
*a*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*polylog(2,1-2/(d
*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*arctanh(d*x+c)*polylog(2,1-2/(
d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*arctanh(d*x+c)*polylog(2,1-2/(d*x+c+1))/(-
c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x
+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*arctanh(d*x+c)*polylog(2,1-2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4*b^3*d*polylog(3,
1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*polylog(3,1-2/(d*x+c+1))/f/(-c*f+d
*e-f)-3/2*b^3*d*polylog(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*
d*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 2.07, antiderivative size = 1094, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6244, 6873, 6256, 6820, 12, 6857, 84, 6874, 6055, 2449, 2352, 6057, 2497, 6095, 6205, 6745, 6203, 6059}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcTanh[c + d*x])^3/(f*(e + f*x))) + (3*a*b^2*d*ArcTanh[c + d*x]*L
og[2/(1 - c - d*x)])/(f*(d*e + f - c*f)) + (3*b^3*d*ArcTanh[c + d*x]^2*Log[
2/(1 - c - d*x)])/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f
*(d*e + f - c*f)) - (3*a*b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d
```

```
*e - f - c*f)) + (6*a*b^2*d*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e +
f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcTanh[c + d*x]^2*Log[2/(1 + c + d*
x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcTanh[c + d*x]^2*Log[2/(1 + c + d*x
)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f
*(d*e - f - c*f)) - (3*a^2*b*d*Log[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (6*a*b^2*d*ArcTanh[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1
 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcTanh[c + d
*x]^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f
)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))
])/(2*f*(d*e + f - c*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 - 2/(1 -
c - d*x)])/(2*f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x
)])/(2*f*(d*e - f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d
*e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 -
 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcTanh[c + d*x]*PolyLo
g[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d
*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcTanh[c + d*x]*PolyLog[2, 1 - (2*d*
(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + (
3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*P
olyLog[3, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*
b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*(
d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6059

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6203

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6244

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)(1-(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\tanh^{-1}(c+dx)\right)^2}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{d\left(a+b\tanh^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \left(-\frac{a^2}{(-1+x)(1+x)(de-cf+fx)} - \frac{}{(-1\cdots}\right.\right.}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx, x, c\right.}{f}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{}{2(-de+(1\cdots}\right.\right.}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd\log(1-c-dx)}{2f(de+f-cf)} + \frac{3a^2bd\log(1+c+d\cdots}{2f(de-f-cf)}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\tan\cdots}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\tan\cdots}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\tanh^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\tan\cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.88, size = 2701, normalized size = 2.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] -(a^3/(f*(e + f*x))) - (3*a^2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (3*a^2*b*
d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])
/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f -
 f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*((d*e - c*f)/Sqrt[1 - (c + d*x
)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*(-(ArcTanh[c + d*x]^2/(E^ArcT
anh[(d*e - c*f)/f]*f*Sqrt[1 - (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTanh[c +
 d*x]^2)/(Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1
 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])) + ((d*e - c*f)*(I*P
i*Log[1 + E^(2*ArcTanh[c + d*x])] - 2*ArcTanh[c + d*x]*Log[1 - E^(-2*(ArcTa
nh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - I*Pi*(ArcTanh[c + d*x] + Log[1/Sq
rt[1 - (c + d*x)^2]]) - 2*ArcTanh[(d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1
- E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - Log[I*Sinh[ArcTanh[
(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f
)/f] + ArcTanh[c + d*x]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*(d
*e - c*f)*(e + f*x)^2) + (b^3*(1 - (c + d*x)^2)*((d*e - c*f)/Sqrt[1 - (c +
d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*((d*(c + d*x)*ArcTanh[c +
d*x]^3)/((d*e - c*f)*Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (
c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])) - (3*d*(
(ArcTanh[c + d*x]^2*(-(f*ArcTanh[c + d*x]) + (d*e - c*f)*Log[(d*e)/Sqrt[1 -
 (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d
*x)^2]]))/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (ArcTanh[c + d*x]*((-I)*d*e
*Pi*ArcTanh[c + d*x] + I*c*f*Pi*ArcTanh[c + d*x] - 2*f*ArcTanh[c + d*x]^2 +
 (Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]^2)/E^ArcTa
nh[(d*e - c*f)/f] + I*d*e*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - I*c*f*Pi*Log
[1 + E^(2*ArcTanh[c + d*x])] - 2*d*e*ArcTanh[c + d*x]*Log[1 - E^(-2*(ArcTan
h[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + 2*c*f*ArcTanh[c + d*x]*Log[1 - E^(
-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - I*d*e*Pi*Log[1/Sqrt[1 -
(c + d*x)^2]] + I*c*f*Pi*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*d*e*ArcTanh[c + d
*x]*Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c +
 d*x))/Sqrt[1 - (c + d*x)^2]] - 2*c*f*ArcTanh[c + d*x]*Log[(d*e)/Sqrt[1 - (
c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x
)^2]] - 2*(d*e - c*f)*ArcTanh[(d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^
(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - Log[I*Sinh[ArcTanh[(d*e
 - c*f)/f] + ArcTanh[c + d*x]]]) + (d*e - c*f)*PolyLog[2, E^(-2*(ArcTanh[(d
*e - c*f)/f] + ArcTanh[c + d*x]))])))/((d*e + f - c*f)*(d*e - (1 + c)*f)) +
(2*(2*d*e + (-2 - 2*c + Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]/E^ArcTa
nh[(d*e - c*f)/f])*f)*ArcTanh[c + d*x]^3 - 6*(d*e - c*f)*ArcTanh[c + d*x]^2
```

```
*Log[-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + 6*(d*e - c*f
)*ArcTanh[c + d*x]*((-I)*Pi*(ArcTanh[c + d*x] - Log[1 + E^(2*ArcTanh[c + d*
x])] + Log[(1 + E^(2*ArcTanh[c + d*x]))/(2*E^ArcTanh[c + d*x])]) - 2*ArcTan
h[(d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f]
+ ArcTanh[c + d*x]))] - Log[(I/2)*E^(-ArcTanh[(d*e - c*f)/f] - ArcTanh[c +
d*x])*(-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])))])) + 6*(d*e
- c*f)*ArcTanh[c + d*x]^2*Log[d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E
^(2*ArcTanh[c + d*x]) + c*E^(2*ArcTanh[c + d*x]))*f] - 6*(d*e - c*f)*ArcTan
h[c + d*x]^2*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + Ar
cTanh[c + d*x]))] - Log[-1 + E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x
]))] + Log[d*e*(1 + E^(2*ArcTanh[c + d*x])) - (1 + c - E^(2*ArcTanh[c + d*x
]) + c*E^(2*ArcTanh[c + d*x]))*f] - Log[(d*e*(1 + E^(2*ArcTanh[c + d*x])) -
 (1 + c - E^(2*ArcTanh[c + d*x]) + c*E^(2*ArcTanh[c + d*x]))*f)/(2*E^ArcTan
h[c + d*x])]) - 2*(d*e - c*f)*(2*ArcTanh[c + d*x]^3 + 3*ArcTanh[c + d*x]^2*
Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - 3*ArcTanh[c +
 d*x]^2*Log[1 - E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*ArcTanh[
c + d*x]^2*Log[1 + E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] - 3*ArcTa
nh[c + d*x]*PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]
- 6*ArcTanh[c + d*x]*PolyLog[2, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*
x])] - 6*ArcTanh[c + d*x]*PolyLog[2, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c
+ d*x])] + 6*PolyLog[3, -E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])] + 6
*PolyLog[3, E^(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])]) - d*e*(4*ArcTan
h[c + d*x]^3 - 6*ArcTanh[c + d*x]^2*Log[1 - E^(2*(ArcTanh[(d*e - c*f)/f] +
ArcTanh[c + d*x]))] - 6*ArcTanh[c + d*x]*PolyLog[2, E^(2*(ArcTanh[(d*e - c*
f)/f] + ArcTanh[c + d*x]))] + 3*PolyLog[3, E^(2*(ArcTanh[(d*e - c*f)/f] + A
rcTanh[c + d*x]))]) + c*f*(4*ArcTanh[c + d*x]^3 - 6*ArcTanh[c + d*x]^2*Log[
1 - E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - 6*ArcTanh[c + d*x]
*PolyLog[2, E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] + 3*PolyLog[
3, E^(2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]) + (d*e - c*f)*(4*Arc
Tanh[c + d*x]^3 - 6*ArcTanh[c + d*x]^2*Log[1 + ...
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.01, size = 5732, normalized size = 5.26

method	result	size
derivativedivides	Expression too large to display	5732
default	Expression too large to display	5732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)
*f^2 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*a
rctanh(d*x + c)/(f^2*x + f*e))*a^2*b - a^3/(f^2*x + f*e) + 1/8*(((c*d*f - d
*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*d*f^2 + d*f^2)*b^3)*x)*
log(-d*x - c + 1)^3 - 3*(4*a*b^2*c*d*f*e - 2*a*b^2*d^2*e^2 - 2*(c^2*f^2 - f
^2)*a*b^2 + ((c*d*f + d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*
d*f^2 - d*f^2)*b^3)*x)*log(d*x + c + 1))*log(-d*x - c + 1)^2)/(2*c*d*f^2*e^
2 - d^2*f*e^3 - (c^2*f^4 - 2*c*d*f^3*e + d^2*f^2*e^2 - f^4)*x - (c^2*f^3 -
f^3)*e) - integrate(1/8*(((c*d*f - d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*
d^2*f*e - (c*d*f^2 + d*f^2)*b^3)*x)*log(d*x + c + 1)^3 + 6*((c*d*f - d*f)*a
*b^2*e - (c^2*f^2 - f^2)*a*b^2 + (a*b^2*d^2*f*e - (c*d*f^2 + d*f^2)*a*b^2)*
x)*log(d*x + c + 1)^2 + 3*(4*a*b^2*d^2*e^2 - 4*(c*d*f + d*f)*a*b^2*e - ((c*
d*f - d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*d*f^2 + d*f^2)*b
^3)*x)*log(d*x + c + 1)^2 + 4*(a*b^2*d^2*f*e - (c*d*f^2 + d*f^2)*a*b^2)*x -
 2*(b^3*d^2*f^2*x^2 - 2*(c^2*f^2 - f^2)*a*b^2 - (2*(c*d*f^2 + d*f^2)*a*b^2
- (c*d*f^2 + d*f^2)*b^3 - (2*a*b^2*d^2*f + b^3*d^2*f)*e)*x + (2*(c*d*f - d*
f)*a*b^2 + (c*d*f + d*f)*b^3)*e)*log(d*x + c + 1))*log(-d*x - c + 1))/((c*d
*f^4 - d^2*f^3*e + d*f^4)*x^3 + (c^2*f^4 - 2*d^2*f^2*e^2 - f^4 + (c*d*f^3 +
 3*d*f^3)*e)*x^2 - (d^2*f*e^3 + (c*d*f^2 - 3*d*f^2)*e^2 - 2*(c^2*f^3 - f^3)
*e)*x - (c*d*f - d*f)*e^3 + (c^2*f^2 - f^2)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}{(c + dx)})^3}{(e + fx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*atanh(c + d*x))**3/(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^3/(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atanh}(c+d\,x))^3}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*atanh(c + d*x))^3/(e + f*x)^2, x)
```

3.50 $\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^3,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^3, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tanh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3, x]
```

Maple [A]
time = 2.54, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arctanh}\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*f*x + b^3*e)*(f*x + e)^m*log(-d*x - c + 1)^3/(f*(m + 1)) + (f*x +
 e)^(m + 1)*a^3/(f*(m + 1)) + integrate(1/8*((b^3*d*f*(m + 1)*x + (c*f*(m +
 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^3 + 6*(a*b^2*d*f*(m + 1)*x + (c*f*(m
 + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1)^2 + 3*(b^3*d*e + 2*(c*f*(m + 1)
- f*(m + 1))*a*b^2 + (2*a*b^2*d*f*(m + 1) + b^3*d*f)*x + (b^3*d*f*(m + 1)*x
 + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)^2 + 1
2*(a^2*b*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a^2*b)*log(d*x + c + 1)
- 3*(4*a^2*b*d*f*(m + 1)*x + 4*(c*f*(m + 1) - f*(m + 1))*a^2*b + (b^3*d*f*(
m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^2 + 4*(a*b^2*d*f
*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1))*log(-d*x -
c + 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atanh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(d*x + c) + a)^3*(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{atanh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atanh(c + d*x))^3,x)
[Out] int((e + f*x)^m*(a + b*atanh(c + d*x))^3, x)
```

3.51 $\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^2, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTanh[x])^2, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tanh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2, x]
```

Maple [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\text{arctanh}\,(dx+c)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
[Out] int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/4*(b^2*f*x + b^2*e)*(f*x + e)^m*log(-d*x - c + 1)^2/(f*(m + 1)) + (f*x +
e)^(m + 1)*a^2/(f*(m + 1)) - integrate(-1/4*((b^2*d*f*(m + 1)*x + (c*f*(m +
 1) - f*(m + 1))*b^2)*log(d*x + c + 1)^2 + 4*(a*b*d*f*(m + 1)*x + (c*f*(m +
 1) - f*(m + 1))*a*b)*log(d*x + c + 1) - 2*(b^2*d*e + 2*(c*f*(m + 1) - f*(m
 + 1))*a*b + (2*a*b*d*f*(m + 1) + b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c*f*(m
 + 1) - f*(m + 1))*b^2)*log(d*x + c + 1))*log(-d*x - c + 1))*(f*x + e)^m/(d
*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*(f*x + e)^
m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atanh(d*x+c))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)^2*(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{atanh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atanh(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*atanh(c + d*x))^2, x)
```

3.52 $\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=162

$$\frac{(e+fx)^{1+m}\left(a+b\tanh^{-1}(c+dx)\right)}{f(1+m)}+\frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-f-cf}\right)}{2f(de-(1+c)f)(1+m)(2+m)}-\frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2\right.}{2f(de+f-cf)}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arctanh(d*x+c))/f/(1+m)+1/2*b*d*(f*x+e)^(2+m)*hypergeom(
[1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e-f))/f/(d*e-(1+c)*f)/(1+m)/(2+m)-1/2*b*d*
(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e+f))/f/(-c*f+d*e+
f)/(1+m)/(2+m)
```

Rubi [A]
time = 0.19, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6246, 6063, 726, 70}

$$\frac{(e+fx)^{m+1}\left(a+b\tanh^{-1}(c+dx)\right)}{f(m+1)}+\frac{bd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de-(c+1)f)}-\frac{bd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf+f}\right)}{2f(m+1)(m+2)(-cf+de+f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTanh[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(
2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)])/(
2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeomet
ric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)])/(2*f*(d*e + f - c*f
)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 6063

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tanh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tanh^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x^2}\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tanh^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\text{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+}}{2(1-x)}\right.\right.}{}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tanh^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x}\right.}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tanh^{-1}(c+dx)\right)}{f(1+m)} + \frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2+\right.}{2f(de-(1+c)f)(}$$

Mathematica [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\tanh^{-1}(c+dx)\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]),x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]), x]
```

Maple [F]
time = 1.93, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arctanh}\,(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*b*((f*x + e)*(f*x + e)^m*log(-d*x - c + 1)/(f*(m + 1)) - integrate((d*
f*x + d*e + (d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1))*log(d*x + c + 1))*(f*
x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)) + (f*x + e)^(m + 1)
*a/(f*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(c + dx)) (e + fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atanh(d*x+c)),x)
```

```
[Out] Integral((a + b*atanh(c + d*x))*(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{atanh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atanh(c + d*x)),x)
[Out] int((e + f*x)^m*(a + b*atanh(c + d*x)), x)
```

3.53 $\int \frac{\tanh^{-1}(a+bx)}{c+dx^3}\,dx$

Optimal. Leaf size=780

$$-\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}-\frac{(-1)^{2/3}\log(1-a-bx)\,\mathrm{lo}}{6c^{2/}}$$

```
[Out] -1/6*ln(-b*x-a+1)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(1-a)*d^(1/3)))/c^(2/
3)/d^(1/3)+1/6*ln(b*x+a+1)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)-(1+a)*d^(1/3
)))/c^(2/3)/d^(1/3)-1/6*(-1)^(2/3)*ln(-b*x-a+1)*ln(b*(c^(1/3)-(-1)^(1/3)*d^
(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(2/
3)*ln(b*x+a+1)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/3)*(1
+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/3)*ln(-b*x-a+1)*ln(b*(c^(1/3)+(-1
)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/
6*(-1)^(1/3)*ln(b*x+a+1)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)-(-1
)^(2/3)*(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*polylog(2,d^(1/3)*(-b*x-a+1)/(b
*c^(1/3)+(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(2/3)*polylog(2,-(-1)^(1/
3)*d^(1/3)*(-b*x-a+1)/(b*c^(1/3)-(-1)^(1/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)
+1/6*(-1)^(1/3)*polylog(2,(-1)^(2/3)*d^(1/3)*(-b*x-a+1)/(b*c^(1/3)+(-1)^(2/
3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*polylog(2,-d^(1/3)*(b*x+a+1)/(b*c^(1
/3)-(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(2/3)*polylog(2,(-1)^(1/3)*d^(
1/3)*(b*x+a+1)/(b*c^(1/3)+(-1)^(1/3)*(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-
1)^(1/3)*polylog(2,-(-1)^(2/3)*d^(1/3)*(b*x+a+1)/(b*c^(1/3)-(-1)^(2/3)*(1+a
)*d^(1/3)))/c^(2/3)/d^(1/3)
```

Rubi [A]

time = 0.98, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6250, 2456, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d*x^3),x]
```

```
[Out] -1/6*(Log[1 - a - b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (1 - a)*d
^(1/3))])/(c^(2/3)*d^(1/3)) + (Log[1 + a + b*x]*Log[(b*(c^(1/3) + d^(1/3)*x
))/(b*c^(1/3) - (1 + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(2/3)*Log[1
- a - b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(1/3)
*(1 - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*Log[1 + a + b*x]*Log[
(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(1 + a)*d^(1/3
))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) + (
```

```
-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(1 - a)*d^(1/3))])/(6*c^(2/3)
*d^(1/3)) - ((-1)^(1/3)*Log[1 + a + b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/
3)*x))/(b*c^(1/3) - (-1)^(2/3)*(1 + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - Pol
yLog[2, (d^(1/3)*(1 - a - b*x))/(b*c^(1/3) + (1 - a)*d^(1/3))]/(6*c^(2/3)*d
^(1/3)) - ((-1)^(2/3)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(1 - a - b*x))/(b*c^
(1/3) - (-1)^(1/3)*(1 - a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/3)*Po
lyLog[2, ((-1)^(2/3)*d^(1/3)*(1 - a - b*x))/(b*c^(1/3) + (-1)^(2/3)*(1 - a)
*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + PolyLog[2, -((d^(1/3)*(1 + a + b*x))/(b*c
^(1/3) - (1 + a)*d^(1/3)))]/(6*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*PolyLog[2, ((
-1)^(1/3)*d^(1/3)*(1 + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(1 + a)*d^(1/3))])
/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/3)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(1 + a
+ b*x))/(b*c^(1/3) - (-1)^(2/3)*(1 + a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+dx^3}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+dx^3}\,dx\right) + \frac{1}{2}\int \frac{\log(1+a+bx)}{c+dx^3}\,dx$$

$$= -\left(\frac{1}{2}\int \left(-\frac{\log(1-a-bx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}\,x\right)} - \frac{\log(1-a-bx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\,\sqrt[3]{d}\,x\right)} - \frac{\log(1}{3c^{2/3}\left(-\sqrt[3]{c}\right.}\right.\right.$$

$$= \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}-\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} + \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\,\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} + \frac{\int \frac{\log(1-a-bx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} - \frac{\int \frac{\log(1+a+b}{-\sqrt[3]{c}-\sqrt[3]{}}}{6c^{2/3}}$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} -$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} -$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} -$$

<u>Mathematica</u> [A]
time = 0.53, size = 623, normalized size = 0.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d*x^3),x]
```

```
[Out] (-(Log[1 - a - b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-1 + a)*d^(
1/3))]) + Log[1 + a + b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (1 +
a)*d^(1/3))] - (-1)^(2/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(
1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-1 + a)*d^(1/3))] + (-1)^(2/3)*Log[1 + a
+ b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(1
+ a)*d^(1/3))] + (-1)^(1/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d
^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-1 + a)*d^(1/3))] - (-1)^(1/3)*Log[1 +
a + b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(
1 + a)*d^(1/3))] - PolyLog[2, -((d^(1/3)*(-1 + a + b*x))/(b*c^(1/3) - (-1 +
 a)*d^(1/3)))] - (-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-1 + a + b*x))/
```

```
(b*c^(1/3) + (-1)^(1/3)*(-1 + a)*d^(1/3))] + (-1)^(1/3)*PolyLog[2, ((-1)^(2
/3)*d^(1/3)*(-1 + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(-1 + a)*d^(1/3))] +
 PolyLog[2, -((d^(1/3)*(1 + a + b*x))/(b*c^(1/3) - (1 + a)*d^(1/3)))] + (-1
)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(1 + a + b*x))/(b*c^(1/3) + (-1)^(1/
3)*(1 + a)*d^(1/3))] - (-1)^(1/3)*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(1 + a + b
*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(1 + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.96, size = 743, normalized size = 0.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*b^3/d*sum(1/(-_R^2+2*_R*a-a^2)*ln(b*x-_R+a),_R=RootOf(_Z^3*d-3*_Z
^2*a*d+3*_Z*a^2*d-a^3*d+b^3*c))*arctanh(b*x+a)+1/3*b^3/d*(arctanh(b*x+a)*su
m(1/(-_R^2+2*_R*a-a^2)*ln(b*x-_R+a),_R=RootOf(_Z^3*d-3*_Z^2*a*d+3*_Z*a^2*d-
a^3*d+b^3*c))-3*d*(2/3*sum(1/(_R1^4*a^3*d-_R1^4*b^3*c-3*_R1^4*a^2*d+3*_R1^4
*a*d+2*_R1^2*a^3*d-2*_R1^2*b^3*c-_R1^4*d-2*_R1^2*a^2*d-2*_R1^2*a*d+a^3*d-b^
3*c+2*_R1^2*d+a^2*d-a*d-d)*(arctanh(b*x+a)*ln((_R1-(b*x+a+1)/(-(b*x+a)^2+1)
^(1/2))/_R1)+dilog((_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)),_R1=RootOf((a
^3*d-b^3*c-3*a^2*d+3*a*d-d)*_Z^6+(3*a^3*d-3*b^3*c-3*a^2*d-3*a*d+3*d)*_Z^4+(
3*a^3*d-3*b^3*c+3*a^2*d-3*a*d-3*d)*_Z^2+a^3*d-b^3*c+3*d*a^2+3*d*a+d))+2/3*s
um(_R1^2/(_R1^4*a^3*d-_R1^4*b^3*c-3*_R1^4*a^2*d+3*_R1^4*a*d+2*_R1^2*a^3*d-2
*_R1^2*b^3*c-_R1^4*d-2*_R1^2*a^2*d-2*_R1^2*a*d+a^3*d-b^3*c+2*_R1^2*d+a^2*d-
a*d-d)*(arctanh(b*x+a)*ln((_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)+dilog((
_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)),_R1=RootOf((a^3*d-b^3*c-3*a^2*d+3
*a*d-d)*_Z^6+(3*a^3*d-3*b^3*c-3*a^2*d-3*a*d+3*d)*_Z^4+(3*a^3*d-3*b^3*c+3*a^
2*d-3*a*d-3*d)*_Z^2+a^3*d-b^3*c+3*d*a^2+3*d*a+d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(b*x + a)/(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)/(d*x^3 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d*x^3),x)
```

```
[Out] int(atanh(a + b*x)/(c + d*x^3), x)
```

3.54 $\int \frac{\tanh^{-1}(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=481

$$-\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1-a-bx)\log}{4\sqrt{-}}$$

```
[Out] -1/4*ln(-b*x-a+1)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(1-a)*d^(1/2)))
/(-c)^(1/2)/d^(1/2)+1/4*ln(b*x+a+1)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/
2)+(1+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln(-b*x-a+1)*ln(b*((-c)^(1/2)+x*d
^(1/2))/(b*(-c)^(1/2)+(1-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*ln(b*x+a+1)*ln
(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(1+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-
1/4*polylog(2,-(-b*x-a+1)*d^(1/2)/(b*(-c)^(1/2)-(1-a)*d^(1/2)))/(-c)^(1/2)/
d^(1/2)+1/4*polylog(2,(-b*x-a+1)*d^(1/2)/(b*(-c)^(1/2)+(1-a)*d^(1/2)))/(-c)
^(1/2)/d^(1/2)-1/4*polylog(2,-(b*x+a+1)*d^(1/2)/(b*(-c)^(1/2)-(1+a)*d^(1/2)
))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(b*x+a+1)*d^(1/2)/(b*(-c)^(1/2)+(1+a)*d
^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$,
Rules used = {6250, 2456, 2441, 2440, 2438}

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,(-a-bx+1)}{\sqrt{d}\,(1-a)+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx+1)}{\sqrt{d}\,(a+1)+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(-a-bx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(a+bx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(-a-bx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{(1-a)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log(a+bx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(Log[1 - a - b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (1 - a)
*Sqrt[d])]])/(Sqrt[-c]*Sqrt[d]) + (Log[1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[
d]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])]])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 - a -
 b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d])]])/(4*Sq
rt[-c]*Sqrt[d]) - (Log[1 + a + b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[
-c] - (1 + a)*Sqrt[d])]])/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 -
a - b*x))/(b*Sqrt[-c] - (1 - a)*Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2
, (Sqrt[d]*(1 - a - b*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d])]/(4*Sqrt[-c]*Sqrt[
d]) - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d]))]
/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] + (1
 + a)*Sqrt[d])]/(4*Sqrt[-c]*Sqrt[d])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+dx^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\sqrt{-c}\,\log(1-a-bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\log(1-a-bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\sqrt{-c}\,\cdots}{2c\left(\sqrt{\cdots}\right.}\right.$$

$$= \frac{\int \frac{\log(1-a-bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}}+\frac{\int \frac{\log(1-a-bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right.\cdots}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right.\cdots}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right.\cdots}{4\sqrt{-c}\,\sqrt{d}}$$

<u>Mathematica</u> [A]
time = 0.22, size = 365, normalized size = 0.76

$$\frac{-\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right)+\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)+\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(-1+a)\sqrt{d}}\right)-\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)+\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}\,(-1+a+bx)}{b\sqrt{-c}-(-1+a)\sqrt{d}}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{d}\,(-1+a+bx)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right)-\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}\,(1+a+bx)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)+\mathrm{PolyLog}\left(2,\frac{\sqrt{d}\,(1+a+bx)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d*x^2),x]
```

```
[Out] (-(Log[1 - a - b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-1 + a)*S
qrt[d])]) + Log[1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (
1 + a)*Sqrt[d])] + Log[1 - a - b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[
-c] - (-1 + a)*Sqrt[d])] - Log[1 + a + b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/
(b*Sqrt[-c] - (1 + a)*Sqrt[d])] + PolyLog[2, -((Sqrt[d]*(-1 + a + b*x))/(b*
Sqrt[-c] - (-1 + a)*Sqrt[d]))] - PolyLog[2, (Sqrt[d]*(-1 + a + b*x))/(b*Sqr
t[-c] + (-1 + a)*Sqrt[d])] - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-
c] - (1 + a)*Sqrt[d]))] + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] +
(1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(377) = 754$.
time = 4.04, size = 1289, normalized size = 2.68

method	result
risch	$\frac{\ln(-bx-a+1)\ln\left(\frac{b\sqrt{-dc}-d(-bx-a+1)-da+d}{b\sqrt{-dc}-da+d}\right)}{4\sqrt{-dc}} - \frac{\ln(-bx-a+1)\ln\left(\frac{b\sqrt{-dc}+d(-bx-a+1)+da-d}{b\sqrt{-dc}+da-d}\right)}{4\sqrt{-dc}} + \frac{\operatorname{dilog}\left(\frac{b\sqrt{}}{}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*(-d*c*b^2)^(1/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a
)^2+1)/(-d*a^2-b^2*c-2*(-d*c*b^2)^(1/2)+d))*a^2*arctanh(b*x+a)/(d*a^2+b^2*c
+2*(-d*c*b^2)^(1/2)-d)+1/2*(-d*c*b^2)^(1/2)*b^2/d*ln(1-(a^2*d+b^2*c-2*a*d+d
)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2-b^2*c-2*(-d*c*b^2)^(1/2)+d))*arctanh(b
*x+a)/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)-1/2*(-d*c*b^2)^(1/2)/c*a^2*arctanh
(b*x+a)^2/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)-1/2*(-d*c*b^2)^(1/2)*b^2/d*arc
tanh(b*x+a)^2/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)+1/4*(-d*c*b^2)^(1/2)/c*pol
ylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2-b^2*c-2*(-d
*c*b^2)^(1/2)+d))*a^2/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)+1/4*(-d*c*b^2)^(1/
2)*b^2/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2
-b^2*c-2*(-d*c*b^2)^(1/2)+d))/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)-b^2/(d*a^2
+b^2*c+2*(-d*c*b^2)^(1/2)-d)*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+
a)^2+1)/(-d*a^2-b^2*c-2*(-d*c*b^2)^(1/2)+d))*arctanh(b*x+a)-1/2*(-d*c*b^2)^
(1/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2-b^2*c
-2*(-d*c*b^2)^(1/2)+d))*arctanh(b*x+a)/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)+b
^2/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)*arctanh(b*x+a)^2+1/2*(-d*c*b^2)^(1/2)
/c*arctanh(b*x+a)^2/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)-1/2*b^2/(d*a^2+b^2*c
+2*(-d*c*b^2)^(1/2)-d)*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a
)^2+1)/(-d*a^2-b^2*c-2*(-d*c*b^2)^(1/2)+d))-1/4*(-d*c*b^2)^(1/2)/c*polylog(
2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2-b^2*c-2*(-d*c*b^
2)^(1/2)+d))/(d*a^2+b^2*c+2*(-d*c*b^2)^(1/2)-d)-1/2*(-d*c*b^2)^(1/2)/c/d*ar
ctanh(b*x+a)*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^2+1)/(-d*a^2-
b^2*c+2*(-d*c*b^2)^(1/2)+d))+1/2*(-d*c*b^2)^(1/2)/c/d*arctanh(b*x+a)^2-1/4*
(-d*c*b^2)^(1/2)/c/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)^2/(-(b*x+a)^
2+1)/(-d*a^2-b^2*c+2*(-d*c*b^2)^(1/2)+d)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.59, size = 591, normalized size = 1.23

$$\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)\operatorname{artanh}(bx+a)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d}, \frac{(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right) - \arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d}, \frac{(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)\right)\log(dx^2+c) - \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{b^2dx^2+2(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right) + \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{b^2dx^2+2(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right) - i\operatorname{Li}_2\left(\frac{(a-1)bdx+b^2c+(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{b^2c+2(-ia+i)b\sqrt{c}\sqrt{d}-(a^2-2a+1)d}\right) + i\operatorname{Li}_2\left(\frac{(a-1)bdx+b^2c-(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{b^2c-2(-ia+i)b\sqrt{c}\sqrt{d}-(a^2-2a+1)d}\right) + i\operatorname{Li}_2\left(\frac{(a+1)bdx+b^2c+(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{b^2c+2(-ia-i)b\sqrt{c}\sqrt{d}-(a^2+2a+1)d}\right) - i\operatorname{Li}_2\left(\frac{(a+1)bdx+b^2c-(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{b^2c-2(-ia-i)b\sqrt{c}\sqrt{d}-(a^2+2a+1)d}\right)}{4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arctan(d*x/sqrt(c*d))*arctanh(b*x + a)/sqrt(c*d) + 1/4*((arctan2((b^2*x + (
a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (a^
2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*b)*
sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*a +
1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/sqrt(
c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2
*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b*d*x +
(a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*b*d*x +
b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a + I)*b*s
qrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b^2*c - (I*
b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a + I)*b*sqrt(c)*sqrt
(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I*b^2*x + (-I
*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (a^2
+ 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a - I)*b)*
sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (a^2 + 2*a + 1)*
d)))/sqrt(c*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)/(d*x^2 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(atanh(a + b*x)/(c + d*x^2), x)
```

3.55 $\int \frac{\tanh^{-1}(a+bx)}{c+dx} dx$

Optimal. Leaf size=120

$$-\frac{\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d}+\frac{\tanh^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2d}-\frac{\text{PolyLog}}{}$$

[Out] -arctanh(b*x+a)*ln(2/(b*x+a+1))/d+arctanh(b*x+a)*ln(2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d+1/2*polylog(2,1-2/(b*x+a+1))/d-1/2*polylog(2,1-2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6246, 6057, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc-ad+d)(a+bx+1)}\right)}{2d}+\frac{\tanh^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d}+\frac{\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{2d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\tanh^{-1}(a+bx)}{d}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a + b*x]/(c + d*x),x]

[Out] -((ArcTanh[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcTanh[a + b*x]*Log[(2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2, 1 - 2/(1 + a + b*x)]/(2*d) - PolyLog[2, 1 - (2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))]/(2*d)

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1-\right.}{}$$

$$= -\frac{\tanh^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\tanh^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2(1-}{2d}$$

Mathematica [A]
time = 0.01, size = 138, normalized size = 1.15

$$-\frac{\log(1-a-bx)\log\left(-\frac{b(c+dx)}{-bc-(1-a)d}\right)}{2d} + \frac{\log(1+a+bx)\log\left(\frac{b(c+dx)}{bc-(1+a)d}\right)}{2d} - \frac{\text{PolyLog}\left(2,-\frac{d(1-a-bx)}{-bc-d+ad}\right)}{2d} + \frac{\text{PolyLog}\left(2,\frac{d(1+a+bx)}{-bc+d+ad}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d*x),x]
```

```
[Out] -1/2*(Log[1 - a - b*x]*Log[-((b*(c + d*x))/(-(b*c) - (1 - a)*d))])/d + (Log
[1 + a + b*x]*Log[(b*(c + d*x))/(b*c - (1 + a)*d)])/(2*d) - PolyLog[2, -((d
*(1 - a - b*x))/(-(b*c) - d + a*d))]/(2*d) + PolyLog[2, (d*(1 + a + b*x))/(
-(b*c) + d + a*d)]/(2*d)
```

Maple [A]

time = 12.58, size = 184, normalized size = 1.53

method	result
risch	$-\frac{\operatorname{dilog}\left(\frac{d(-bx-a+1)+da-bc-d}{da-bc-d}\right)}{2d}-\frac{\ln(-bx-a+1)\ln\left(\frac{d(-bx-a+1)+da-bc-d}{da-bc-d}\right)}{2d}+\frac{\operatorname{dilog}\left(\frac{d(bx+a+1)-da+bc-d}{-da+bc-d}\right)}{2d}+\frac{\ln(}{}$
derivativedivides	$\frac{\frac{b\ln(da-bc-d(bx+a))\operatorname{arctanh}(bx+a)}{d}+\frac{b\left(\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)+d}{-da+bc+d}\right)+\ln(da-bc-d(bx+a))\ln\left(\frac{-d(bx+a)+d}{-da+bc+d}\right)\right)d}{2}-\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)-}{-da+bc-d}\right.\right.}{}\right.}{d^2}}{b}$
default	$\frac{\frac{b\ln(da-bc-d(bx+a))\operatorname{arctanh}(bx+a)}{d}+\frac{b\left(\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)+d}{-da+bc+d}\right)+\ln(da-bc-d(bx+a))\ln\left(\frac{-d(bx+a)+d}{-da+bc+d}\right)\right)d}{2}-\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)-}{-da+bc-d}\right.\right.}{}\right.}{d^2}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(d*a-b*c-d*(b*x+a))/d*arctanh(b*x+a)+b/d^2*(1/2*(dilog((-d*(b*x+a)
+d)/(-a*d+b*c+d))+ln(d*a-b*c-d*(b*x+a))*ln((-d*(b*x+a)+d)/(-a*d+b*c+d)))*d-
1/2*(dilog((-d*(b*x+a)-d)/(-a*d+b*c-d))+ln(d*a-b*c-d*(b*x+a))*ln((-d*(b*x+a
)-d)/(-a*d+b*c-d)))*d))
```

Maxima [A]

time = 0.26, size = 192, normalized size = 1.60

$$-\frac{1}{2}b\left(\frac{\log(bx+a-1)\log\left(\frac{bdx+ad-d}{bc-ad+d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd}-\frac{\log(bx+a+1)\log\left(\frac{bdx+ad+d}{bc-ad-d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd}\right)-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(dx+c)}{2d}+\frac{\operatorname{artanh}(bx+a)\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a - 1)*log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + dilo
g(-(b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (log(b*x + a + 1)*log((b*d*x
 + a*d + d)/(b*c - a*d - d) + 1) + dilog(-(b*d*x + a*d + d)/(b*c - a*d - d)
))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + c)/d
+ arctanh(b*x + a)*log(d*x + c)/d
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(b*x + a)/(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(atanh(a + b*x)/(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d*x),x)
```

```
[Out] int(atanh(a + b*x)/(c + d*x), x)
```

3.56 $\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x}} \, dx$

Optimal. Leaf size=186

$$\frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{d\log(1+a+bx)\log\left(-\frac{b(d+cx)}{c+ac-bd}\right)}{2c^2}+\frac{d\log(1-a$$

```
[Out] 1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-1/2*d*ln(b*x+
a+1)*ln(-b*(c*x+d)/(a*c-b*d+c))/c^2+1/2*d*ln(-b*x-a+1)*ln(b*(c*x+d)/(-a*c+b
*d+c))/c^2+1/2*d*polylog(2,c*(-b*x-a+1)/(-a*c+b*d+c))/c^2-1/2*d*polylog(2,c
*(b*x+a+1)/(a*c-b*d+c))/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6250, 2456, 2436, 2332, 2441, 2440, 2438}

$$\frac{d\text{Li}_2\left(\frac{c(-a-bx+1)}{-ac+c+bd}\right)}{2c^2}-\frac{d\text{Li}_2\left(\frac{c(a+bx+1)}{ac+c-bd}\right)}{2c^2}+\frac{d\log(-a-bx+1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2}-\frac{d\log(a+bx+1)\log\left(-\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2}+\frac{(-a-bx+1)\log(-a-bx+1)}{2bc}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d/x),x]
```

```
[Out] ((1 - a - b*x)*Log[1 - a - b*x])/(2*b*c) + ((1 + a + b*x)*Log[1 + a + b*x])
/(2*b*c) - (d*Log[1 + a + b*x]*Log[-((b*(d + c*x))/(c + a*c - b*d))])/(2*c^
2) + (d*Log[1 - a - b*x]*Log[(b*(d + c*x))/(c - a*c + b*d)])/(2*c^2) + (d*P
olyLog[2, (c*(1 - a - b*x))/(c - a*c + b*d)])/(2*c^2) - (d*PolyLog[2, (c*(1
 + a + b*x))/(c + a*c - b*d)])/(2*c^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+\frac{d}{x}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x}}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{c}-\frac{d\log(1-a-bx)}{c(d+cx)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{c}-\frac{d}{}\right.$$

$$= -\frac{\int \log(1-a-bx)\,dx}{2c}+\frac{\int \log(1+a+bx)\,dx}{2c}+\frac{d\int \frac{\log(1-a-bx)}{d+cx}\,dx}{2c}-\frac{d\int \frac{\log(1+a+bx)}{d+cx}}{2c}$$

$$= -\frac{d\log(1+a+bx)\log\left(-\frac{b(d+cx)}{c+ac-bd}\right)}{2c^2}+\frac{d\log(1-a-bx)\log\left(\frac{b(d+cx)}{c-ac+bd}\right)}{2c^2}+\frac{\mathrm{Subst}(\int \log}{}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{d\log(1+a+bx)\log}{2c^2}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{d\log(1+a+bx)\log}{2c^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.16, size = 759, normalized size = 4.08

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d/x),x]
```

```
[Out] (-2*a^2*c^2*ArcTanh[a + b*x] + 2*a*b*c*d*ArcTanh[a + b*x] + I*a*b*c*d*Pi*Ar
cTanh[a + b*x] - I*b^2*d^2*Pi*ArcTanh[a + b*x] - 2*a*b*c^2*x*ArcTanh[a + b*
x] + 2*b^2*c*d*x*ArcTanh[a + b*x] - 2*a*b*c*d*ArcTanh[a - (b*d)/c]*ArcTanh[
a + b*x] + 2*b^2*d^2*ArcTanh[a - (b*d)/c]*ArcTanh[a + b*x] - b*c*d*ArcTanh[
a + b*x]^2 - a*b*c*d*ArcTanh[a + b*x]^2 + b^2*d^2*ArcTanh[a + b*x]^2 + b*c*
d*Sqrt[1 - a^2 + (2*a*b*d)/c - (b^2*d^2)/c^2]*E^ArcTanh[a - (b*d)/c]*ArcTan
h[a + b*x]^2 - 2*a*b*c*d*ArcTanh[a - (b*d)/c]*Log[1 - E^(2*(ArcTanh[a - (b*
d)/c] - ArcTanh[a + b*x]))] + 2*b^2*d^2*ArcTanh[a - (b*d)/c]*Log[1 - E^(2*(
ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] + 2*a*b*c*d*ArcTanh[a + b*x]*Log
[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] - 2*b^2*d^2*ArcTanh[a
 + b*x]*Log[1 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] - 2*a*b*c*
d*ArcTanh[a + b*x]*Log[1 + E^(-2*ArcTanh[a + b*x])] + 2*b^2*d^2*ArcTanh[a +
 b*x]*Log[1 + E^(-2*ArcTanh[a + b*x])] - I*a*b*c*d*Pi*Log[1 + E^(2*ArcTanh[
a + b*x])] + I*b^2*d^2*Pi*Log[1 + E^(2*ArcTanh[a + b*x])] + 2*a*c^2*Log[1/S
qrt[1 - (a + b*x)^2]] - 2*b*c*d*Log[1/Sqrt[1 - (a + b*x)^2]] + I*a*b*c*d*Pi
*Log[1/Sqrt[1 - (a + b*x)^2]] - I*b^2*d^2*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] +
 2*a*b*c*d*ArcTanh[a - (b*d)/c]*Log[(-I)*Sinh[ArcTanh[a - (b*d)/c] - ArcTan
h[a + b*x]]] - 2*b^2*d^2*ArcTanh[a - (b*d)/c]*Log[(-I)*Sinh[ArcTanh[a - (b*
d)/c] - ArcTanh[a + b*x]]] + b*d*(-(a*c) + b*d)*PolyLog[2, E^(2*(ArcTanh[a
- (b*d)/c] - ArcTanh[a + b*x]))] + b*d*(a*c - b*d)*PolyLog[2, -E^(-2*ArcTan
h[a + b*x])])/(2*b*c^2*(-(a*c) + b*d))
```

Maple [A]
time = 8.19, size = 305, normalized size = 1.64

method	result
risch	$-\frac{\ln(-bx-a+1)x}{2c}-\frac{\ln(-bx-a+1)a}{2bc}+\frac{\ln(-bx-a+1)}{2bc}-\frac{1}{bc}+\frac{d\,\mathrm{dilog}\left(\frac{c(-bx-a+1)+ac-db-c}{ac-db-c}\right)}{2c^2}+\frac{d\ln(-bx-a+1$
derivativedivides	$\frac{\frac{\mathrm{arctanh}(bx+a)(bx+a)}{c}-\frac{\mathrm{arctanh}(bx+a)db\ln(ac-db-c(bx+a))}{c^2}+-\frac{bd\ln(ac-db-c(bx+a))\ln\left(\frac{-c(bx+a)+c}{-ac+db+c}\right)}{2c}-\frac{bd\,\mathrm{dilog}\left(\frac{-c(bx+a)+c}{-ac+db+c}\right.}{2c}}{}$
default	$\frac{\frac{\mathrm{arctanh}(bx+a)(bx+a)}{c}-\frac{\mathrm{arctanh}(bx+a)db\ln(ac-db-c(bx+a))}{c^2}+-\frac{bd\ln(ac-db-c(bx+a))\ln\left(\frac{-c(bx+a)+c}{-ac+db+c}\right)}{2c}-\frac{bd\,\mathrm{dilog}\left(\frac{-c(bx+a)+c}{-ac+db+c}\right.}{2c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(b*x+a)/c*(b*x+a)-arctanh(b*x+a)*d*b/c^2*ln(a*c-d*b-c*(b*x+a))+
1/c*(-1/2*b*d/c*ln(a*c-d*b-c*(b*x+a))*ln((-c*(b*x+a)+c)/(-a*c+b*d+c))-1/2*b
*d/c*dilog((-c*(b*x+a)+c)/(-a*c+b*d+c))+1/2*b*d/c*ln(a*c-d*b-c*(b*x+a))*ln(
(-c*(b*x+a)-c)/(-a*c+b*d-c))+1/2*b*d/c*dilog((-c*(b*x+a)-c)/(-a*c+b*d-c))+1
/2*ln(a^2*c^2-2*b*a*d*c+d^2*b^2-2*a*c*(a*c-d*b-c*(b*x+a))+2*b*d*(a*c-d*b-c*
(b*x+a))-c^2+(a*c-d*b-c*(b*x+a))^2)))
```

Maxima [A]
time = 0.27, size = 192, normalized size = 1.03

$$\frac{1}{2}\,b\left(\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd+c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right)\right)d}{bc^2}-\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd-c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right)\right)d}{bc^2}+\frac{(a+1)\log\left(bx+a+1\right)}{b^2c}-\frac{(a-1)\log\left(bx+a-1\right)}{b^2c}\right)+\left(\frac{x}{c}-\frac{d\log\left(cx+d\right)}{c^2}\right)\mathrm{artanh}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*x
 + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*c
- b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a + 1
)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c - d*l
og(c*x + d)/c^2)*arctanh(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arctanh(b*x + a)/(c*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{atanh}\,(a+bx)}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x),x)
```

```
[Out] Integral(x*atanh(a + b*x)/(c*x + d), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(c + d/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d/x),x)
```

```
[Out] int(atanh(a + b*x)/(c + d/x), x)
```

3.57 $\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=545

$$\frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{\sqrt{d}\,\log(1-a-bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}$$

```
[Out] 1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c+1/4*ln(-b*x-a
+1)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c
)^(3/2)+1/4*ln(b*x+a+1)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)-b*d^
(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(-b*x-a+1)*ln(b*(x*(-c)^(1/2)+d^(1/2))/((1
-a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(b*x+a+1)*ln(b*(-x*(-c)
^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*polylo
g(2,(-b*x-a+1)*(-c)^(1/2)/((-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)
^(3/2)+1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)-b*d^(1/2)))*d^(
1/2)/(-c)^(3/2)-1/4*polylog(2,(-b*x-a+1)*(-c)^(1/2)/((1-a)*(-c)^(1/2)+b*d^(
1/2)))*d^(1/2)/(-c)^(3/2)-1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1
/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

Rubi [A]
time = 0.65, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6250, 2456, 2436, 2332, 2441, 2440, 2438}

$$\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(-a-bx+1)}{-\sqrt{-c}\,a+\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(-a-bx+1)}{\sqrt{-c}\,(1-a)+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+1)}{\sqrt{-c}\,(a+1)+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\,\log(-a-bx+1)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\log(-a-bx+1)\log\left(\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\log(a+bx+1)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(a+1)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\,\log(a+bx+1)\log\left(-\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{(-a-bx+1)\log(-a-bx+1)}{2bc}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((1 - a - b*x)*Log[1 - a - b*x])/(2*b*c) + ((1 + a + b*x)*Log[1 + a + b*x])
/(2*b*c) + (Sqrt[d]*Log[1 - a - b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((1 -
 a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[d]*Log[1 + a + b*x]*Log
[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2))
 + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((1 + a)*Sqrt
[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[d]*Log[1 - a - b*x]*Log[(b*(Sqr
t[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt
[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/(Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])
])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*S
qrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 +
a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLo
g[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2
))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+\frac{d}{x^2}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{c}-\frac{d\log(1-a-bx)}{c\,(d+cx^2)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{c}-\frac{d}{}\right.$$

$$= -\frac{\int \log(1-a-bx)\,dx}{2c}+\frac{\int \log(1+a+bx)\,dx}{2c}+\frac{d\int \frac{\log(1-a-bx)}{d+cx^2}\,dx}{2c}-\frac{d\int \frac{\log(1+a+bx)}{d+cx^2}}{2c}$$

$$= \frac{\text{Subst}(\int \log(x)\,dx, x, 1-a-bx)}{2bc}+\frac{\text{Subst}(\int \log(x)\,dx, x, 1+a+bx)}{2bc}+\frac{d\int \left(\frac{1}{2\sqrt{d}}\right.}{}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{\sqrt{d}\int \frac{\log(1-a-bx)}{\sqrt{d}-\sqrt{-c}\,x}\,d}{4c}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{\sqrt{d}\log(1-a-bx)}{4}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{\sqrt{d}\log(1-a-bx)}{4}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{\sqrt{d}\log(1-a-bx)}{4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.08, size = 1456, normalized size = 2.67

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((a + b*x)*ArcTanh[a + b*x] - Log[1/Sqrt[1 - (a + b*x)^2]])/(b*c) + (Sqrt[d
]*((2*I)*Sqrt[c]*ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]*x)/
Sqrt[d]] - (2*I)*a^2*Sqrt[c]*ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[
(Sqrt[c]*x)/Sqrt[d]] - (2*I)*Sqrt[c]*ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])]*
```

```
ArcTan[(Sqrt[c]*x)/Sqrt[d]] + (2*I)*a^2*Sqrt[c]*ArcTan[((1 + a)*Sqrt[c])/(b
*Sqrt[d])]*ArcTan[(Sqrt[c]*x)/Sqrt[d]] - 2*b*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqr
t[d]]^2 + (b*Sqrt[d]*Sqrt[((-1 + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x
)/Sqrt[d]]^2)/E^(I*ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])]) + (a*b*Sqrt[d]*S
qrt[((-1 + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*Arc
Tan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])]) + (b*Sqrt[d]*Sqrt[((1 + a)^2*c + b^2*d
)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((1 + a)*Sqrt[c])/(b*
Sqrt[d])]) - (a*b*Sqrt[d]*Sqrt[((1 + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[
c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])]) - 4*(-1 + a^2
)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*ArcTanh[a + b*x] + 2*Sqrt[c]*ArcTan[(
(-1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-1 + a)*Sqrt[c])
/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - 2*a^2*Sqrt[c]*ArcTan[((-1
+ a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-1 + a)*Sqrt[c])/(b*
Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + 2*Sqrt[c]*ArcTan[(Sqrt[c]*x)/S
qrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[
(Sqrt[c]*x)/Sqrt[d]]))] - 2*a^2*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 -
 E^((-2*I)*(ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqr
t[d]]))] - 2*Sqrt[c]*ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I
)*(ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] +
 2*a^2*Sqrt[c]*ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(Arc
Tan[((1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - 2*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((1 + a)*Sqrt[c]
)/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + 2*a^2*Sqrt[c]*ArcTan[(Sqr
t[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])] +
 ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - 2*Sqrt[c]*ArcTan[((-1 + a)*Sqrt[c])/(b*Sq
rt[d])]*Log[-Sin[ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x
)/Sqrt[d]]]] + 2*a^2*Sqrt[c]*ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Si
n[ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] +
2*Sqrt[c]*ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((1 + a)*Sq
rt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - 2*a^2*Sqrt[c]*ArcTan[
((1 + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((1 + a)*Sqrt[c])/(b*Sqrt[d]
)] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - I*(-1 + a^2)*Sqrt[c]*PolyLog[2, E^((-2
*I)*(ArcTan[((-1 + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))
] + I*(-1 + a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(ArcTan[((1 + a)*Sqrt[c])/(b*
Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))]))/(4*(-1 + a^2)*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.90, size = 10288, normalized size = 18.88

method	result
risch	$-\frac{\ln(-bx-a+1)x}{2c}-\frac{\ln(-bx-a+1)a}{2bc}+\frac{\ln(-bx-a+1)}{2bc}-\frac{1}{bc}-\frac{d\ln(-bx-a+1)\ln\left(\frac{b\sqrt{-dc}-c(-bx-a+1)-ac+c}{b\sqrt{-dc}-ac+c}\right)}{4c\sqrt{-dc}}$
derivativedivides	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 651, normalized size = 1.19

$$-\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}-\frac{x}{c}\right)\operatorname{artanh}(bx+a)+\frac{2\,(a+1)c\log(bx+a+1)-2\,(a-1)c\log(bx+a-1)+\left(b\arctan\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2\,(a+1)bcx+(a^2+2\,a+1)c}{b^2d+(a^2+2\,a+1)c}\right)-b\arctan\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2\,(a-1)bcx+(a^2-2\,a+1)c}{b^2d+(a^2-2\,a+1)c}\right)+i\,b\mathrm{Li}_2\left(\frac{(a-1)bcx+b^2d+(i\,b^2x+(-i\,a+i)b)\sqrt{c}\,\sqrt{d}}{2\,(-i\,a+i)b\sqrt{c}\,\sqrt{d}+b^2d-(a^2-2\,a+1)c}\right)-i\,b\mathrm{Li}_2\left(-\frac{(a-1)bcx+b^2d-(i\,b^2x+(-i\,a+i)b)\sqrt{c}\,\sqrt{d}}{2\,(-i\,a+i)b\sqrt{c}\,\sqrt{d}-b^2d+(a^2-2\,a+1)c}\right)-i\,b\mathrm{Li}_2\left(\frac{(a+1)bcx+b^2d+(i\,b^2x+(-i\,a-i)b)\sqrt{c}\,\sqrt{d}}{2\,(-i\,a-i)b\sqrt{c}\,\sqrt{d}+b^2d-(a^2+2\,a+1)c}\right)+i\,b\mathrm{Li}_2\left(-\frac{(a+1)bcx+b^2d-(i\,b^2x+(-i\,a-i)b)\sqrt{c}\,\sqrt{d}}{2\,(-i\,a-i)b\sqrt{c}\,\sqrt{d}-b^2d+(a^2+2\,a+1)c}\right)-\left(b\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\,\sqrt{d}}{b^2d+(a^2+2\,a+1)c},\frac{(a+1)bcx+(a^2+2\,a+1)c}{b^2d+(a^2+2\,a+1)c}\right)-b\arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\,\sqrt{d}}{b^2d+(a^2-2\,a+1)c},\frac{(a-1)bcx+(a^2-2\,a+1)c}{b^2d+(a^2-2\,a+1)c}\right)\right)\log(cx^2+d)\right)\sqrt{c}\,\sqrt{d}}{4\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctanh(b*x + a) + 1/4*(2*(a
 + 1)*c*log(b*x + a + 1) - 2*(a - 1)*c*log(b*x + a - 1) + (b*arctan(sqrt(c)
*x/sqrt(d))*log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d +
(a^2 + 2*a + 1)*c)) - b*arctan(sqrt(c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a - 1
)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*dilog(((a -
 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)
*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*a + 1)*c)) - I*b*dilog(-((a - 1)*b*c*
x + b^2*d - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)*b*sqrt(
c)*sqrt(d) - b^2*d + (a^2 - 2*a + 1)*c)) - I*b*dilog(((a + 1)*b*c*x + b^2*d
 + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sqrt(d
) + b^2*d - (a^2 + 2*a + 1)*c)) + I*b*dilog(-((a + 1)*b*c*x + b^2*d - (I*b^
2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sqrt(d) - b^2*
d + (a^2 + 2*a + 1)*c)) - (b*arctan2((b^2*x + (a + 1)*b)*sqrt(c)*sqrt(d)/(b
^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d + (a^
2 + 2*a + 1)*c)) - b*arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqrt(d)/(b^2*d + (
a^2 - 2*a + 1)*c), ((a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a
+ 1)*c)))*log(c*x^2 + d))*sqrt(c)*sqrt(d))/(b*c^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctanh(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(c + d/x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d/x^2),x)
```

```
[Out] int(atanh(a + b*x)/(c + d/x^2), x)
```

3.58 $\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^3}} \, dx$

Optimal. Leaf size=832

$$\frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{\sqrt[3]{d}\,\log(1+a+bx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{(1+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}+$$

```
[Out] 1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-1/6*d^(1/3)*l
n(b*x+a+1)*ln(-b*(d^(1/3)+c^(1/3)*x)/((1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6
*d^(1/3)*ln(-b*x-a+1)*ln(b*(d^(1/3)+c^(1/3)*x)/((1-a)*c^(1/3)+b*d^(1/3)))/c
^(4/3)+1/6*(-1)^(2/3)*d^(1/3)*ln(-b*x-a+1)*ln(-b*(d^(1/3)-(-1)^(1/3)*c^(1/3
)*x)/((-1)^(1/3)*(1-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(2/3)*d^(1/3)*l
n(b*x+a+1)*ln(b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(1+a)*c^(1/3)+b*
d^(1/3)))/c^(4/3)+1/6*(-1)^(1/3)*d^(1/3)*ln(b*x+a+1)*ln(-b*(d^(1/3)+(-1)^(2
/3)*c^(1/3)*x)/((-1)^(2/3)*(1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/3)
*d^(1/3)*ln(-b*x-a+1)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(1-a)
*c^(1/3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(2/3)*d^(1/3)*polylog(2,(-1)^(1/3)*c^
(1/3)*(-b*x-a+1)/((-1)^(1/3)*(1-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*d^(1/3)*
polylog(2,c^(1/3)*(-b*x-a+1)/((1-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1
/3)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(-b*x-a+1)/((-1)^(2/3)*(1-a)*c^(1/
3)+b*d^(1/3)))/c^(4/3)-1/6*d^(1/3)*polylog(2,c^(1/3)*(b*x+a+1)/((1+a)*c^(1/
3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/3)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*
(b*x+a+1)/((-1)^(2/3)*(1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(2/3)*d^(1
/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(b*x+a+1)/((-1)^(1/3)*(1+a)*c^(1/3)+b*d^(1
/3)))/c^(4/3)
```

Rubi [A]

time = 1.10, antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6250, 2456, 2436, 2332, 2441, 2440, 2438}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d/x^3),x]
```

```
[Out] ((1 - a - b*x)*Log[1 - a - b*x])/(2*b*c) + ((1 + a + b*x)*Log[1 + a + b*x])
/(2*b*c) - (d^(1/3)*Log[1 + a + b*x]*Log[-((b*(d^(1/3) + c^(1/3)*x))/((1 +
a)*c^(1/3) - b*d^(1/3)))])/(6*c^(4/3)) + (d^(1/3)*Log[1 - a - b*x]*Log[(b*(
d^(1/3) + c^(1/3)*x))/((1 - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-1)^(
2/3)*d^(1/3)*Log[1 - a - b*x]*Log[-((b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((
-1)^(1/3)*(1 - a)*c^(1/3) - b*d^(1/3)))])/(6*c^(4/3)) - ((-1)^(2/3)*d^(1/3)
```

```
*Log[1 + a + b*x]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(1 +
 a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-1)^(1/3)*d^(1/3)*Log[1 + a + b*
x]*Log[-((b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x))/((-1)^(2/3)*(1 + a)*c^(1/3) -
 b*d^(1/3)))])/(6*c^(4/3)) - ((-1)^(1/3)*d^(1/3)*Log[1 - a - b*x]*Log[(b*(d
^(1/3) + (-1)^(2/3)*c^(1/3)*x))/((-1)^(2/3)*(1 - a)*c^(1/3) + b*d^(1/3))])/
(6*c^(4/3)) + ((-1)^(2/3)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(1 - a - b
*x))/((-1)^(1/3)*(1 - a)*c^(1/3) - b*d^(1/3))])/(6*c^(4/3)) + (d^(1/3)*Poly
Log[2, (c^(1/3)*(1 - a - b*x))/((1 - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
- ((-1)^(1/3)*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(1 - a - b*x))/((-1)^(
2/3)*(1 - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - (d^(1/3)*PolyLog[2, (c^(1
/3)*(1 + a + b*x))/((1 + a)*c^(1/3) - b*d^(1/3))])/(6*c^(4/3)) + ((-1)^(1/3
)*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(1 + a + b*x))/((-1)^(2/3)*(1 + a)
*c^(1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(2/3)*d^(1/3)*PolyLog[2, ((-1)^
(1/3)*c^(1/3)*(1 + a + b*x))/((-1)^(1/3)*(1 + a)*c^(1/3) + b*d^(1/3))])/(6*
c^(4/3))
```

<u>Rule 2332</u>

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

<u>Rule 2436</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

<u>Rule 2440</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

<u>Rule 2441</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

<u>Rule 2456</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+\frac{d}{x^3}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x^3}}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{c}-\frac{d\log(1-a-bx)}{c\left(d+cx^3\right)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{c}-\frac{d}{}\right.$$

$$= -\frac{\int \log(1-a-bx)\,dx}{2c}+\frac{\int \log(1+a+bx)\,dx}{2c}+\frac{d\int \frac{\log(1-a-bx)}{d+cx^3}\,dx}{2c}-\frac{d\int \frac{\log(1+a+bx)}{d+cx^3}}{2c}$$

$$= \frac{\text{Subst}(\int \log(x)\,dx, x, 1-a-bx)}{2bc}+\frac{\text{Subst}(\int \log(x)\,dx, x, 1+a+bx)}{2bc}+\frac{d\int\left(-\frac{}{3d^{2/3}}\right.}{}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{\sqrt[3]{d}\int \frac{\log(1-a-bx)}{-\sqrt[3]{d}-\sqrt[3]{c}\,x}\,d}{6c}$$

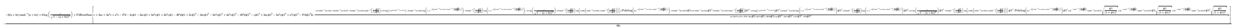
$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{\sqrt[3]{d}\log(1+a+bx)}{}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{\sqrt[3]{d}\log(1+a+bx)}{}$$

$$= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}-\frac{\sqrt[3]{d}\log(1+a+bx)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order

4 in optimal.
time = 4.66, size = 917, normalized size = 1.10



Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d/x^3),x]
```

```
[Out] -1/6*(-6*(a + b*x)*ArcTanh[a + b*x] + 6*Log[1/Sqrt[1 - (a + b*x)^2]] + b^3*
d*RootSum[c + 3*a*c + 3*a^2*c + a^3*c - b^3*d - 3*c*#1 - 3*a*c*#1 + 3*a^2*c
*#1 + 3*a^3*c*#1 - 3*b^3*d*#1 + 3*c*#1^2 - 3*a*c*#1^2 - 3*a^2*c*#1^2 + 3*a^
3*c*#1^2 - 3*b^3*d*#1^2 - c*#1^3 + 3*a*c*#1^3 - 3*a^2*c*#1^3 + a^3*c*#1^3 -
 b^3*d*#1^3 & , ((-I)*Pi*ArcTanh[a + b*x] - 2*ArcTanh[a + b*x]^2 - 2*ArcTan
h[a + b*x]*ArcTanh[(1 - #1)/(1 + #1)] + I*Pi*Log[1 + E^(2*ArcTanh[a + b*x])
] - 2*ArcTanh[a + b*x]*Log[1 - E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(
1 + #1)]))] - 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[1 - E^(-2*(ArcTanh[a + b*x]
+ ArcTanh[(1 - #1)/(1 + #1)]))] - I*Pi*Log[1/Sqrt[1 - (a + b*x)^2]] + 2*Arc
Tanh[(1 - #1)/(1 + #1)]*Log[I*Sinh[ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(1 +
 #1)]]] + PolyLog[2, E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)/(1 + #1)]))
] - 2*ArcTanh[a + b*x]^2*#1 + I*Pi*ArcTanh[a + b*x]*#1^2 + 2*ArcTanh[a + b*
x]*ArcTanh[(1 - #1)/(1 + #1)]*#1^2 - I*Pi*Log[1 + E^(2*ArcTanh[a + b*x])]*#
1^2 + 2*ArcTanh[a + b*x]*Log[1 - E^(-2*(ArcTanh[a + b*x] + ArcTanh[(1 - #1)
/(1 + #1)]))]*#1^2 + 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[1 - E^(-2*(ArcTanh[a
+ b*x] + ArcTanh[(1 - #1)/(1 + #1)]))]*#1^2 + I*Pi*Log[1/Sqrt[1 - (a + b*x)
^2]]*#1^2 - 2*ArcTanh[(1 - #1)/(1 + #1)]*Log[I*Sinh[ArcTanh[a + b*x] + ArcT
anh[(1 - #1)/(1 + #1)]]]*#1^2 - PolyLog[2, E^(-2*(ArcTanh[a + b*x] + ArcTan
h[(1 - #1)/(1 + #1)]))]*#1^2 + (2*ArcTanh[a + b*x]^2*Sqrt[#1/(1 + #1)^2])/E
^ArcTanh[(1 - #1)/(1 + #1)] + (4*ArcTanh[a + b*x]^2*#1*Sqrt[#1/(1 + #1)^2])
/E^ArcTanh[(1 - #1)/(1 + #1)] + (2*ArcTanh[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1)
^2])/E^ArcTanh[(1 - #1)/(1 + #1)])/(a*c + 2*a^2*c + a^3*c - b^3*d - 2*a*c*#
1 + 2*a^3*c*#1 - 2*b^3*d*#1 + a*c*#1^2 - 2*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*
#1^2) & ])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.98, size = 638, normalized size = 0.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(b*x+a)/c*(b*x+a)+1/2/c*ln(b*x+a-1)+1/2/c*ln(b*x+a+1)+2/3/c*d*b
^3*sum(_R1^2/(_R1^4*a^3*c-_R1^4*b^3*d-3*_R1^4*a^2*c+3*_R1^4*a*c+2*_R1^2*a^3
*c-2*_R1^2*b^3*d-_R1^4*c-2*_R1^2*a^2*c-2*_R1^2*a*c+a^3*c-b^3*d+2*_R1^2*c+a^
2*c-a*c-c)*(arctanh(b*x+a)*ln((_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)+dil
og((_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)),_R1=RootOf((a^3*c-b^3*d-3*a^2
*c+3*a*c-c)*_Z^6+(3*a^3*c-3*b^3*d-3*a^2*c-3*a*c+3*c)*_Z^4+(3*a^3*c-3*b^3*d+
3*a^2*c-3*a*c-3*c)*_Z^2+a^3*c-b^3*d+3*a^2*c+3*a*c+c))+2/3/c*d*b^3*sum(1/(_R
```

```
1^4*a^3*c-_R1^4*b^3*d-3*_R1^4*a^2*c+3*_R1^4*a*c+2*_R1^2*a^3*c-2*_R1^2*b^3*d
-_R1^4*c-2*_R1^2*a^2*c-2*_R1^2*a*c+a^3*c-b^3*d+2*_R1^2*c+a^2*c-a*c-c)*(arct
anh(b*x+a)*ln((_R1-(b*x+a+1)/(-(b*x+a)^2+1)^(1/2))/_R1)+dilog((_R1-(b*x+a+1
)/(-(b*x+a)^2+1)^(1/2))/_R1)),_R1=RootOf((a^3*c-b^3*d-3*a^2*c+3*a*c-c)*_Z^6
+(3*a^3*c-3*b^3*d-3*a^2*c-3*a*c+3*c)*_Z^4+(3*a^3*c-3*b^3*d+3*a^2*c-3*a*c-3*
c)*_Z^2+a^3*c-b^3*d+3*a^2*c+3*a*c+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="maxima")
[Out] integrate(arctanh(b*x + a)/(c + d/x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="fricas")
[Out] integral(x^3*arctanh(b*x + a)/(c*x^3 + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x**3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d/x^3),x)
```

```
[Out] int(atanh(a + b*x)/(c + d/x^3), x)
```

3.59 $\int \frac{\tanh^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=585

$$\frac{2\sqrt{1+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{\right.}$$

```
[Out] c*ln(-b*x-a+1)*ln(c+d*x^(1/2))/d^2-c*ln(b*x+a+1)*ln(c+d*x^(1/2))/d^2+c*ln(c
+d*x^(1/2))*ln(d*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-1-a)^(1/2)+c*b^(1/2)))
/d^2-c*ln(c+d*x^(1/2))*ln(d*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(1-a)^(1/2)+c*
b^(1/2)))/d^2+c*ln(c+d*x^(1/2))*ln(-d*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(-
1-a)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(-d*((1-a)^(1/2)+b^(1/2)*x^(
1/2))/(-d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d
*(-1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(-1-a)^(
1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d*(1-a)^(1/2)+c*b^
(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(1-a)^(1/2)+c*b^(1/2)))/d^
2-2*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/d/b^(1/2)+2*arctan(b^(
1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/d/b^(1/2)-ln(-b*x-a+1)*x^(1/2)/d+ln(b
*x+a+1)*x^(1/2)/d
```

Rubi [A]
time = 0.87, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6250, 2455, 2516, 2498, 327, 211, 2512, 266, 2463, 2441, 2440, 2438, 214}

$$\frac{2\sqrt{a+1}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}\,d}+\frac{c\,\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-1}\,d}\right)}{d^2}+\frac{c\,\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-a-1}\,d}\right)}{d^2}-\frac{c\,\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)}{d^2}-\frac{c\,\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{-a-1}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{-a-1}\,d+\sqrt{b}\,c}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{1-a}\,d+\sqrt{b}\,c}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{-a-1}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-1}\,d}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)}{d^2}+\frac{c\log(-a-bx+1)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log(a+bx+1)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{\sqrt{x}\,\log(-a-bx+1)}{d}+\frac{\sqrt{x}\,\log(a+bx+1)}{d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqrt
[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt
[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1
- a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*
(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c + d*Sq
rt[x]])/d^2 - (Sqrt[x]*Log[1 - a - b*x])/d + (c*Log[c + d*Sqrt[x]]*Log[1 -
a - b*x])/d^2 + (Sqrt[x]*Log[1 + a + b*x])/d - (c*Log[c + d*Sqrt[x]]*Log[1
+ a + b*x])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt
[-1 - a]*d)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sq
rt[-1 - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c -
```

```
Sqrt[1 - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c +
 Sqrt[1 - a]*d)])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+d\sqrt{x}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+d\sqrt{x}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x\log\left(1-a-bx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{x\log\left(1+a+bx^2\right)}{c+dx}\,dx, x,\right.$$

$$= -\text{Subst}\left(\int \left(\frac{\log\left(1-a-bx^2\right)}{d}-\frac{c\log\left(1-a-bx^2\right)}{d(c+dx)}\right)\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \left(\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \log\left(1-a-bx^2\right)\,dx, x, \sqrt{x}\right)}{d}+\frac{\text{Subst}\left(\int \log\left(1+a+bx^2\right)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{\sqrt{x}\,\log(1-a-bx)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log(1-a-bx)}{d^2}+\frac{\sqrt{x}\,\log(1+a+bx)}{d}$$

$$= -\frac{\sqrt{x}\,\log(1-a-bx)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log(1-a-bx)}{d^2}+\frac{\sqrt{x}\,\log(1+a+bx)}{d}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{x}\,\log(1-a-b}{d}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-}\right.}{\sqrt{b}\,c+\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-}\right.}{\sqrt{b}\,c+\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-}\right.}{\sqrt{b}\,c+\sqrt{}}\right.}{}$$

Mathematica [A]

time = 0.38, size = 549, normalized size = 0.94

$$\frac{\frac{2\sqrt{1+a}\,d\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}}-\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}}+c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)+c\log\left(\frac{d\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-d\sqrt{x}\log(1-a-bx)+c\log\left(c+d\sqrt{x}\right)\log(1-a-bx)+d\sqrt{x}\log(1+a+bx)-c\log\left(c+d\sqrt{x}\right)\log(1+a+bx)+c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-1-a}\,d}\right)+c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)-c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)-c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b] - (2*Sqrt[
1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b] + c*Log[(d*(Sqrt[-
1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]]
 - c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)]*L
og[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c
) + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[1 - a] + Sqrt[b]*S
qrt[x]))/(-(Sqrt[b]*c) + Sqrt[1 - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*Log
[1 - a - b*x] + c*Log[c + d*Sqrt[x]]*Log[1 - a - b*x] + d*Sqrt[x]*Log[1 + a
 + b*x] - c*Log[c + d*Sqrt[x]]*Log[1 + a + b*x] + c*PolyLog[2, (Sqrt[b]*(c
+ d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*
Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))
/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```

Maple [A]
time = 2.07, size = 773, normalized size = 1.32

method	result
derivativedivides	$\frac{2\,\mathrm{arctanh}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arctanh}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(-\frac{d^2\arctan\left(\frac{-2bc+2b\left(c+d\sqrt{x}\right)}{2\sqrt{ab\,d^2+d^2b}}\right)a}{2b\sqrt{ab\,d^2+d^2b}}-\frac{d^2\arctan\left(\frac{-2b}{2\sqrt{}}\right.}{2b\sqrt{ab}}\right.}{}$
default	$\frac{2\,\mathrm{arctanh}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arctanh}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(-\frac{d^2\arctan\left(\frac{-2bc+2b\left(c+d\sqrt{x}\right)}{2\sqrt{ab\,d^2+d^2b}}\right)a}{2b\sqrt{ab\,d^2+d^2b}}-\frac{d^2\arctan\left(\frac{-2b}{2\sqrt{}}\right.}{2b\sqrt{ab}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh(b*x+a)/d*x^(1/2)-2*arctanh(b*x+a)*c/d^2*ln(c+d*x^(1/2))-4*b/d^2*(
-1/2*d^2/b/(a*b*d^2+b*d^2)^(1/2)*arctan(1/2*(-2*b*c+2*b*(c+d*x^(1/2)))/(a*b
*d^2+b*d^2)^(1/2))*a-1/2*d^2/b/(a*b*d^2+b*d^2)^(1/2)*arctan(1/2*(-2*b*c+2*b
*(c+d*x^(1/2)))/(a*b*d^2+b*d^2)^(1/2))+1/2*d^2/b/(a*b*d^2-b*d^2)^(1/2)*arct
an(1/2*(-2*b*c+2*b*(c+d*x^(1/2)))/(a*b*d^2-b*d^2)^(1/2))*a-1/2*d^2/b/(a*b*d
^2-b*d^2)^(1/2)*arctan(1/2*(-2*b*c+2*b*(c+d*x^(1/2)))/(a*b*d^2-b*d^2)^(1/2)
)-1/4*c/b*ln(c+d*x^(1/2))*ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(
b*c+(-a*b*d^2-b*d^2)^(1/2)))-1/4*c/b*ln(c+d*x^(1/2))*ln((-b*c+b*(c+d*x^(1/2
```

```
))+(-a*b*d^2-b*d^2)^(1/2))/(-b*c+(-a*b*d^2-b*d^2)^(1/2)))-1/4*c/b*dilog((b*
c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(b*c+(-a*b*d^2-b*d^2)^(1/2)))-1/4
*c/b*dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(-b*c+(-a*b*d^2-b*
d^2)^(1/2)))+1/4*c/b*ln(c+d*x^(1/2))*ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^
2)^(1/2))/(b*c+(-a*b*d^2+b*d^2)^(1/2)))+1/4*c/b*ln(c+d*x^(1/2))*ln((-b*c+b*
(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(-b*c+(-a*b*d^2+b*d^2)^(1/2)))+1/4*c/
b*dilog((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(b*c+(-a*b*d^2+b*d^2)^
(1/2)))+1/4*c/b*dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(-b*c+(
-a*b*d^2+b*d^2)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
[Out] integrate(arctanh(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
[Out] integral((d*sqrt(x)*arctanh(b*x + a) - c*arctanh(b*x + a))/(d^2*x - c^2), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d*x**(1/2)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*x + a)/(d*sqrt(x) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d*x^(1/2)),x)
```

```
[Out] int(atanh(a + b*x)/(c + d*x^(1/2)), x)
```

3.60 $\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=661

$$-\frac{2\sqrt{1+a}\,d\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)\log(}{c^3}$$

```
[Out] 1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-d^2*ln(-b*x-a
+1)*ln(d+c*x^(1/2))/c^3+d^2*ln(b*x+a+1)*ln(d+c*x^(1/2))/c^3-d^2*ln(d+c*x^(1
/2))*ln(c*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-1-a)^(1/2)+d*b^(1/2)))/c^3+d^
2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(1-a)^(1/2)+d*b^(1/
2)))/c^3-d^2*ln(d+c*x^(1/2))*ln(c*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-1-a)^
(1/2)-d*b^(1/2)))/c^3+d^2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)+b^(1/2)*x^(1/2)
)/(c*(1-a)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-
1-a)^(1/2)-d*b^(1/2)))/c^3+d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1
/2)-d*b^(1/2)))/c^3-d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-1-a)^(1/2)+d*b
^(1/2)))/c^3+d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1/2)+d*b^(1/2)))
/c^3+2*d*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/c^2/b^(1/2)-2*d*a
rctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/c^2/b^(1/2)+d*ln(-b*x-a+1)*x
^(1/2)/c^2-d*ln(b*x+a+1)*x^(1/2)/c^2
```

Rubi [A]
time = 0.79, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6250, 2455, 2526, 2498, 327, 211, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438, 214}

$$-\frac{2\sqrt{a+1}\,d\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\text{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{-a-1}\,c-\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\text{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{-a-1}\,c+\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\text{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}\,c+\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}\,c-\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log(-a-bx+1)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d^2\log(a+bx+1)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d\sqrt{x}\log(-a-bx+1)}{c^2}-\frac{d\sqrt{x}\log(a+bx+1)}{c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}+\frac{(-a-bx+1)\log(-a-bx+1)}{2bc}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d^2
*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log
[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1
 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 - a] +
 Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 +
(d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*L
og[d + c*Sqrt[x]])/c^3 + (d*Sqrt[x]*Log[1 - a - b*x])/c^2 + ((1 - a - b*x)*
Log[1 - a - b*x])/(2*b*c) - (d^2*Log[d + c*Sqrt[x]]*Log[1 - a - b*x])/c^3 -
```

```
 (d*Sqrt[x]*Log[1 + a + b*x])/c^2 + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c
) + (d^2*Log[d + c*Sqrt[x]]*Log[1 + a + b*x])/c^3 - (d^2*PolyLog[2, -((Sqrt
[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2,
-((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyL
og[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)])/c^3 + (d^2*P
olyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/c^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 6250

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{c+\frac{d}{\sqrt{x}}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x\log\left(1-a-bx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{x\log\left(1+a+bx^2\right)}{c+\frac{d}{x}}\,dx, x,\right.$$

$$= -\text{Subst}\left(\int \left(-\frac{d\log\left(1-a-bx^2\right)}{c^2}+\frac{x\log\left(1-a-bx^2\right)}{c}+\frac{d^2\log\left(1-a-bx^2\right)}{c^2(d+cx)}\right)\right.$$

$$= -\frac{\text{Subst}\left(\int x\log\left(1-a-bx^2\right)\,dx, x, \sqrt{x}\right)}{c}+\frac{\text{Subst}\left(\int x\log\left(1+a+bx^2\right)\,dx, x, \sqrt{x}\right.}{c}$$

$$= \frac{d\sqrt{x}\,\log(1-a-bx)}{c^2}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1-a-bx)}{c^3}-\frac{d\sqrt{x}\,\log(1+a+bx}{c^2}$$

$$= \frac{d\sqrt{x}\,\log(1-a-bx)}{c^2}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1-a-bx)}{c^3}-\frac{d\sqrt{x}\,\log(1+a+bx}{c^2}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}+\frac{d\sqrt{x}\,\log(1-}{c^2}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-}\right.}{\sqrt{}}\right.}{}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-}\right.}{\sqrt{}}\right.}{}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-}\right.}{\sqrt{}}\right.}{}$$

Mathematica [A]
time = 0.44, size = 598, normalized size = 0.90

$$\frac{4cd\left(\sqrt{x}-\frac{\sqrt{1+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}}\right)-4cd\left(\sqrt{x}-\frac{\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}}\right)+2cd\sqrt{x}\log(1-a-bx)-\frac{c^2(-1+a+bx)\log(1-a-bx)}{b}-2d^2\log\left(d+c\sqrt{x}\right)\log(1-a-bx)-2cd\sqrt{x}\log(1+a+bx)+\frac{c^2(1+a+bx)\log(1+a+bx)}{b}+2d^2\log\left(d+c\sqrt{x}\right)\log(1+a+bx)-2d^2\left(\left(\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)+\log\left(\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c-\sqrt{b}\,d}\right)\right)\log\left(d+c\sqrt{x}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)\right)+2d^2\left(\left(\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)+\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)\right)\log\left(d+c\sqrt{x}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{1-a}\,c+\sqrt{b}\,d}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] (4*c*d*(Sqrt[x] - (Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[
b]) - 4*c*d*(Sqrt[x] - (Sqrt[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])
/Sqrt[b]) + 2*c*d*Sqrt[x]*Log[1 - a - b*x] - (c^2*(-1 + a + b*x)*Log[1 - a
- b*x])/b - 2*d^2*Log[d + c*Sqrt[x]]*Log[1 - a - b*x] - 2*c*d*Sqrt[x]*Log[1
 + a + b*x] + (c^2*(1 + a + b*x)*Log[1 + a + b*x])/b + 2*d^2*Log[d + c*Sqrt
[x]]*Log[1 + a + b*x] - 2*d^2*((Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(S
qrt[-1 - a]*c + Sqrt[b]*d)] + Log[(c*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(Sqr
t[-1 - a]*c - Sqrt[b]*d)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*
Sqrt[x]))/(-(Sqrt[-1 - a]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]) + 2*d^2*((Log[(c*(Sqrt[1 - a] - Sqrt[
b]*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)] + Log[(c*(Sqrt[1 - a] + Sqrt[b]*S
qrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(-(Sqrt[1 - a]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]
*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)]))/(2*c^3)
```

Maple [A]

time = 1.60, size = 1001, normalized size = 1.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(b*x+a)/c*x-2*arctanh(b*x+a)/c^2*d*x^(1/2)+2*arctanh(b*x+a)*d^2/c^3*
ln(d+c*x^(1/2))-4*b/c^2*(1/4/c*d^2/b*ln(d+c*x^(1/2))*ln((d*b-b*(d+c*x^(1/2)
)+(-a*b*c^2-b*c^2)^(1/2))/(d*b+(-a*b*c^2-b*c^2)^(1/2)))+1/4/c*d^2/b*ln(d+c*
x^(1/2))*ln((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-d*b+(-a*b*c^2-b
*c^2)^(1/2)))+1/4/c*d^2/b*dilog((d*b-b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2)
)/(d*b+(-a*b*c^2-b*c^2)^(1/2)))+1/4/c*d^2/b*dilog((-d*b+b*(d+c*x^(1/2))+(-a
*b*c^2-b*c^2)^(1/2))/(-d*b+(-a*b*c^2-b*c^2)^(1/2)))-1/4/c*d^2/b*ln(d+c*x^(1
/2))*ln((d*b-b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(d*b+(-a*b*c^2+b*c^2)^
(1/2)))-1/4/c*d^2/b*ln(d+c*x^(1/2))*ln((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2+b*c^
2)^(1/2))/(-d*b+(-a*b*c^2+b*c^2)^(1/2)))-1/4/c*d^2/b*dilog((d*b-b*(d+c*x^(1
/2))+(-a*b*c^2+b*c^2)^(1/2))/(d*b+(-a*b*c^2+b*c^2)^(1/2)))-1/4/c*d^2/b*dilo
g((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(-d*b+(-a*b*c^2+b*c^2)^(1/2
)))-1/8*c/b^2*a*ln(a*c^2+d^2*b-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2+c^2)+1
/2*c/b*a*d/(a*b*c^2+b*c^2)^(1/2)*arctan(1/2*(-2*d*b+2*b*(d+c*x^(1/2)))/(a*b
*c^2+b*c^2)^(1/2))-1/8*c/b^2*ln(a*c^2+d^2*b-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1
/2))^2+c^2)+1/2*c/b*d/(a*b*c^2+b*c^2)^(1/2)*arctan(1/2*(-2*d*b+2*b*(d+c*x^(
1/2)))/(a*b*c^2+b*c^2)^(1/2))+1/8*c/b^2*a*ln(a*c^2+d^2*b-2*b*d*(d+c*x^(1/2)
)+b*(d+c*x^(1/2))^2-c^2)-1/2*c/b*a*d/(a*b*c^2-b*c^2)^(1/2)*arctan(1/2*(-2*d
*b+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2))-1/8*c/b^2*ln(a*c^2+d^2*b-2*b*d
*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2-c^2)+1/2*c/b*d/(a*b*c^2-b*c^2)^(1/2)*arcta
n(1/2*(-2*d*b+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
[Out] integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
[Out] integral((c*x*arctanh(b*x + a) - d*sqrt(x)*arctanh(b*x + a))/(c^2*x - d^2),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(c+d/x**(1/2)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
[Out] integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*x)/(c + d/x^(1/2)),x)
[Out] int(atanh(a + b*x)/(c + d/x^(1/2)), x)
```

3.61 $\int \frac{\tanh^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=335

$$\frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(}\right.}{\sqrt{b^2-4ac}}$$

```
[Out] arctanh(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(
b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctanh(e*x+d)*ln(2*e*(b+2*c*x+(
-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+
b^2)^(1/2)-1/2*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(1/2)))/(
e*x+d+1)/(2*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*polyl
og(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d+1)/(2*c*(1-d)+
e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {632, 212, 6860, 6246, 6057, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2c+be-\sqrt{b^2-4ac}\,e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\text{Li}_2\left(\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(d+ex+1)\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTanh[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) +
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[d
 + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sq
rt[b^2 - 4*a*c]*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (
2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tanh^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tanh^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)$$

$$= \frac{(2c)\int \frac{\tanh^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tanh^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2c)\text{Subst}\left(\int \frac{\tanh^{-1}}{\frac{-2cd+\left(b+\sqrt{b^2 \cdots}\right.}{e}}\right.}{\sqrt{b^2 \cdots}}$$

$$= \frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}\,e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(d+ex)\log\left(\frac{\cdots}{\left(2c(\cdots\right.}\right.}{\sqrt{\cdots}}$$

$$= \frac{\tanh^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}\,e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(d+ex)\log\left(\frac{\cdots}{\left(2c(\cdots\right.}\right.}{\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.37, size = 1208, normalized size = 3.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*e*E^(-ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*
e)] - ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)])*(b*e*(Sqrt[-((c*(
c*(1 + d)^2 + e*(-(b*(1 + d)) + a*e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c
```

```
*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] - Sqrt[-((c*(c*(-1 + d)^2 + e*(b -
b*d + a*e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2
- 4*a*c]*e)]) - 2*c*((-1 + d)*Sqrt[-((c*(c*(1 + d)^2 + e*(-(b*(1 + d)) + a*
e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c
]*e)] - (1 + d)*Sqrt[-((c*(c*(-1 + d)^2 + e*(b - b*d + a*e)))/((b^2 - 4*a*c
)*e^2))]*E^ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + E^(ArcTanh[
(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(2*c*(1 + d) - b*e)/(
Sqrt[b^2 - 4*a*c]*e)]))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2 - 2*(4*c^
2*(-1 + d^2) - 4*b*c*d*e + b^2*e^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*
(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] - ArcTanh[(2*c*(1 + d)
 - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + 2*ArcTanh[d + e*x] + Log[1 - E^(-2*(ArcTan
h[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]]))] - Log[1 - E^(-2*(ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a
*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))]) - 2*(4*c^2*(-1 + d^2)
- 4*b*c*d*e + b^2*e^2)*(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]
*(Log[1 - E^(-2*(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] - Log[I*Sinh[ArcTanh[(2*c*(-1 + d) -
b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]]]) + A
rcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]*(-Log[1 - E^(-2*(ArcTanh[
(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]]))] + Log[I*Sinh[ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]
 + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]]])) + (4*c^2*(-1 + d^2) - 4*b*c*d
*e + b^2*e^2)*PolyLog[2, E^(-2*(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*
a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] - (4*c^2*(-1 + d^2) -
4*b*c*d*e + b^2*e^2)*PolyLog[2, E^(-2*(ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^
2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))])/(2*Sqrt[b^2 - 4
*a*c]*(-2*c*(-1 + d) + b*e)*(-2*c*(1 + d) + b*e))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2131 vs. $2(307) = 614$.

time = 5.38, size = 2132, normalized size = 6.36

method	result
risch	$\frac{e\ln(-ex-d+1)\ln\left(\frac{-2c(-ex-d+1)+be-2dc+\sqrt{-4ac\,e^2+b^2e^2}+2c}{be-2dc+2c+\sqrt{-4ac\,e^2+b^2e^2}}\right)}{2\sqrt{-4ac\,e^2+b^2e^2}} - \frac{e\ln(-ex-d+1)\ln\left(\frac{2c(-ex-d+1)-be+2dc}{-be+2dc+\sqrt{-}}\right)}{2\sqrt{-4ac\,e^2+}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*((e^2*(-4*a*c+b^2))^(1/2)*e^2/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2
*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(
1/2)+c))*a*arctanh(e*x+d)/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)-(e
```

```
^2*(-4*a*c+b^2))^(1/2)*e/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(
e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*
b*d*arctanh(e*x+d)/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*
a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^
2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*c*d^2*arc
tanh(e*x+d)/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2
))^(1/2)*e^2/(4*a*c-b^2)*a*arctanh(e*x+d)^2/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c
+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2))^(1/2)*e/(4*a*c-b^2)*b*d*arctanh(e*x+d)^2
/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1/2)/(4
*a*c-b^2)*c*d^2*arctanh(e*x+d)^2/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2
)-c)+1/2*(e^2*(-4*a*c+b^2))^(1/2)*e^2/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*
d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*
c-b^2))^(1/2)+c))*a/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)-1/2*(e^2
*(-4*a*c+b^2))^(1/2)*e/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c
)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c
))*b*d/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)+1/2*(e^2*(-4*a*c+b^2)
)^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(
-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*c*d^2/(a*e^2
-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)-e^2/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*
c+b^2))^(1/2)-c)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)
^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*arctanh(e*x+d)-(e^2*
(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d
+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*c*arc
tanh(e*x+d)/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)+e^2/(a*e^2-b*d*e
+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)*arctanh(e*x+d)^2+(e^2*(-4*a*c+b^2))^(1/2
)/(4*a*c-b^2)*c*arctanh(e*x+d)^2/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2
)-c)-1/2*e^2/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2))^(1/2)-c)*polylog(2,(a*e^
2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2-(
-e^2*(4*a*c-b^2))^(1/2)+c))-1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylo
g(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*
e-c*d^2-(-e^2*(4*a*c-b^2))^(1/2)+c))*c/(a*e^2-b*d*e+c*d^2+(e^2*(-4*a*c+b^2)
)^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arctanh(e*x+d)*ln(1-(a*e^2-
b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2+(-e
^2*(4*a*c-b^2))^(1/2)+c))+(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arctanh(e*x+
d)^2-1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+
b*e-2*c*d+c)*(e*x+d+1)^2/(-(e*x+d)^2+1)/(-a*e^2+b*d*e-c*d^2+(-e^2*(4*a*c-b^
2))^(1/2)+c)))
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x*e + d)/(c*x^2 + b*x + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e*x + d)/(c*x^2 + b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + e*x)/(a + b*x + c*x^2),x)
```

```
[Out] int(atanh(d + e*x)/(a + b*x + c*x^2), x)
```

3.62 $\int \frac{(ce+dex)(a+b\tanh^{-1}(c+dx))}{1-(c+dx)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d}+\frac{be\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2d}$$

```
[Out] -1/2*e*(a+b*arctanh(d*x+c))^2/b/d+e*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d
+1/2*b*e*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

Rubi [A]
time = 0.18, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {12, 6131, 6055, 2449, 2352}

$$-\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\tanh^{-1}(c+dx)\right)}{d}+\frac{be\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((c*e + d*e*x)*(a + b*ArcTanh[c + d*x]))/(1 - (c + d*x)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcTanh[c + d*x])^2)/(b*d) + (e*(a + b*ArcTanh[c + d*x])*Log
[2/(1 - c - d*x)])/d + (b*e*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ce+dex)\left(a+b\tanh^{-1}(c+dx)\right)}{1-(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{ex(a+b\tanh^{-1}(x))}{1-x^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{e\text{Subst}\left(\int \frac{x(a+b\tanh^{-1}(x))}{1-x^2}\,dx, x, c+dx\right)}{d}\\ &= -\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{1-x}\,dx, x, c+d\right.}{d}\\ &= -\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-}\right.}{d}\\ &= -\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-}\right.}{d}\\ &= -\frac{e\left(a+b\tanh^{-1}(c+dx)\right)^2}{2bd}+\frac{e\left(a+b\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-}\right.}{d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.92

$$e\left(-\frac{a\log\left(1-(c+dx)^2\right)}{2d}-\frac{b\left(-\tanh^{-1}(c+dx)\left(\tanh^{-1}(c+dx)+2\log\left(1+e^{-2\tanh^{-1}(c+dx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(c+dx)}\right)\right)}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*e + d*e*x)*(a + b*ArcTanh[c + d*x]))/(1 - (c + d*x)^2),x]
```

```
[Out] e*(-1/2*(a*Log[1 - (c + d*x)^2])/d - (b*(-(ArcTanh[c + d*x]*(ArcTanh[c + d*
x] + 2*Log[1 + E^(-2*ArcTanh[c + d*x])])) + PolyLog[2, -E^(-2*ArcTanh[c + d
*x])]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(81) = 162$.
time = 2.03, size = 168, normalized size = 2.02

method	result
risch	$\frac{e\ln(-dx-c+1)^2 b}{8d} + \frac{e\ln(-dx-c+1)\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)b}{4d} - \frac{e\,\mathrm{dilog}\left(-\frac{dx}{2}-\frac{c}{2}+\frac{1}{2}\right)b}{4d} - \frac{ea\ln((-dx-c+1)(-dx-c-1))}{2d} -$
derivativedivides	$\frac{-\frac{ae\ln(dx+c-1)}{2}-\frac{ae\ln(dx+c+1)}{2}-\frac{be\,\mathrm{arctanh}(dx+c)\ln(dx+c-1)}{2}-\frac{be\,\mathrm{arctanh}(dx+c)\ln(dx+c+1)}{2}+\frac{be\,\mathrm{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2}+\frac{be\ln(dx+c}{}}{d}$
default	$\frac{-\frac{ae\ln(dx+c-1)}{2}-\frac{ae\ln(dx+c+1)}{2}-\frac{be\,\mathrm{arctanh}(dx+c)\ln(dx+c-1)}{2}-\frac{be\,\mathrm{arctanh}(dx+c)\ln(dx+c+1)}{2}+\frac{be\,\mathrm{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2}+\frac{be\ln(dx+c}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a*e*ln(d*x+c-1)-1/2*a*e*ln(d*x+c+1)-1/2*b*e*arctanh(d*x+c)*ln(d*x
+c-1)-1/2*b*e*arctanh(d*x+c)*ln(d*x+c+1)+1/2*b*e*dilog(1/2*d*x+1/2*c+1/2)+1
/4*b*e*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/8*b*e*ln(d*x+c-1)^2-1/4*b*e*ln(-
1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/4*b*e*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/
2*c+1/2)+1/8*b*e*ln(d*x+c+1)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*b*c*(log(d*x + c + 1)/d - log(d*x + c - 1)/d)*arctanh(d*x + c)*e - 1/2*
a*d*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2)*e + 1/2*a
*c*(log(d*x + c + 1)/d - log(d*x + c - 1)/d)*e + 1/8*b*d*((2*(c + 1)*log(d*
x + c + 1)*log(-d*x - c + 1) - (c - 1)*log(-d*x - c + 1)^2)/d^2 - 4*integra
te(1/2*(c^2 + (c*d + 3*d)*x + 2*c + 1)*log(d*x + c + 1)/(d^3*x^2 + 2*c*d^2*
x + c^2*d - d), x))*e - 1/8*(log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d*
x + c - 1) + log(d*x + c - 1)^2)*b*c*e/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral(-((b*d*x + b*c)*arctanh(d*x + c)*e + (a*d*x + a*c)*e)/(d^2*x^2 + 2
*c*d*x + c^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-e\left(\int \frac{ac}{c^2+2cdx+d^2x^2-1}\,dx+\int \frac{adx}{c^2+2cdx+d^2x^2-1}\,dx+\int \frac{bc\,\mathrm{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2-1}\,dx+\int \frac{bdx\,\mathrm{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2-1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))/(1-(d*x+c)**2),x)
```

```
[Out] -e*(Integral(a*c/(c**2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(a*d*x/(c**
2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(b*c*atanh(c + d*x)/(c**2 + 2*c*
d*x + d**2*x**2 - 1), x) + Integral(b*d*x*atanh(c + d*x)/(c**2 + 2*c*d*x +
d**2*x**2 - 1), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate(-(d*e*x + c*e)*(b*arctanh(d*x + c) + a)/((d*x + c)^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{atanh}(c+d\,x)\right)}{{(c+d\,x)}^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1),x)
```

```
[Out] int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
         fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
         gamma,loggamma,digamma,zeta,polylog,LambertW,
         elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```